Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.3.1-a+b-sin-^m-c+d-sin-^n-A+B-sin-

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [358]. This is test number [76].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (358)	% 0. (0)
Mathematica	% 97.21 (348)	% 2.79 (10)
Maple	% 80.73 (289)	% 19.27 (69)
Maxima	% 36.87 (132)	% 63.13 (226)
Fricas	% 76.82 (275)	% 23.18 (83)
Sympy	% 16.76 (60)	% 83.24 (298)
Giac	% 44.69 (160)	% 55.31 (198)

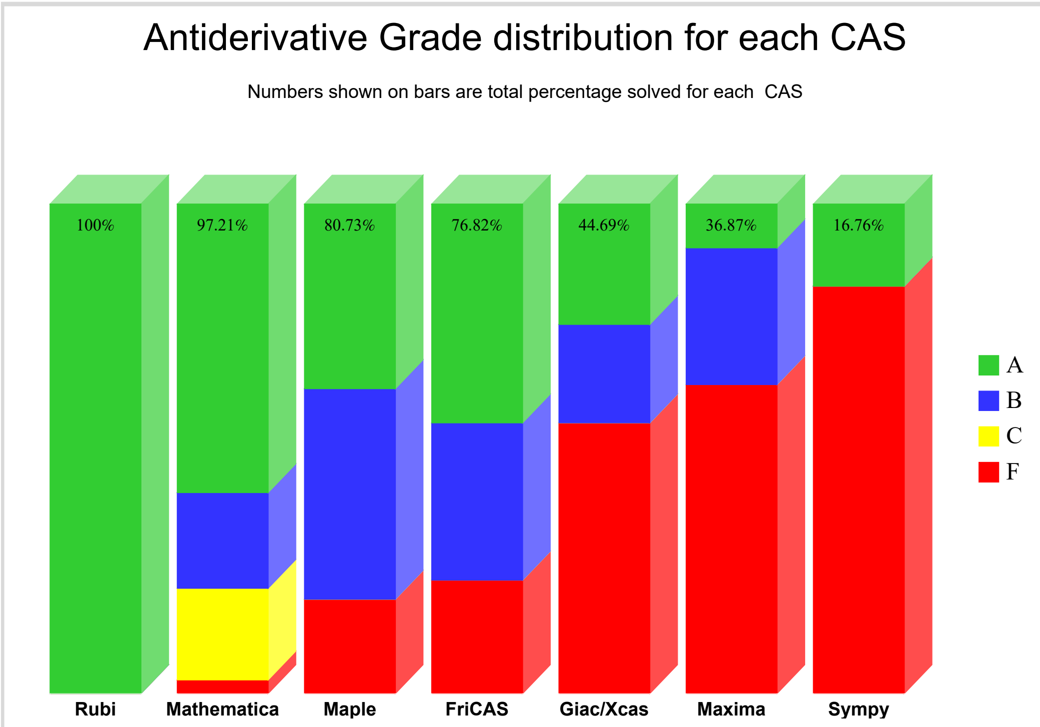
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

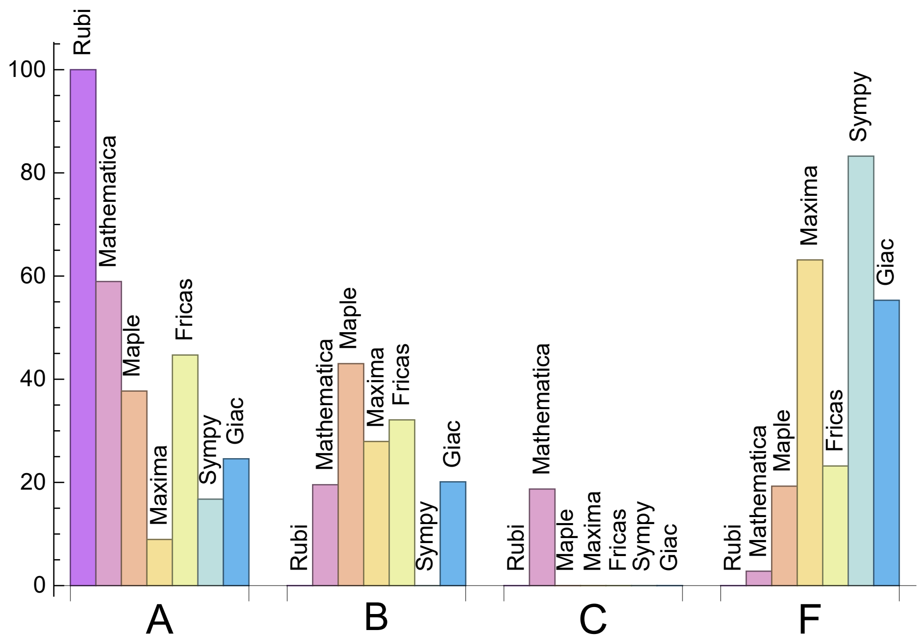
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	58.94	19.55	18.72	2.79
Maple	37.71	43.02	0.	19.27
Maxima	8.94	27.93	0.	63.13
Fricas	44.69	32.12	0.	23.18
Sympy	16.76	0.	0.	83.24
Giac	24.58	20.11	0.	55.31

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	179.38	1.	156.	1.
Mathematica	3.73	470.59	2.66	223.	1.43
Maple	1.45	38711.6	52.99	274.	1.97
Maxima	1.29	1063.95	7.45	692.	5.63
Fricas	3.74	1231.55	6.14	585.	4.47
Sympy	17.86	1237.02	9.7	876.	6.5
Giac	1.47	539.41	3.4	328.	2.31

1.4 list of integrals that has no closed form antiderivative

{358}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {7, 8, 10, 11, 12, 88, 195, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 215, 216, 217, 243, 304, 305, 306, 313, 320, 326, 327, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

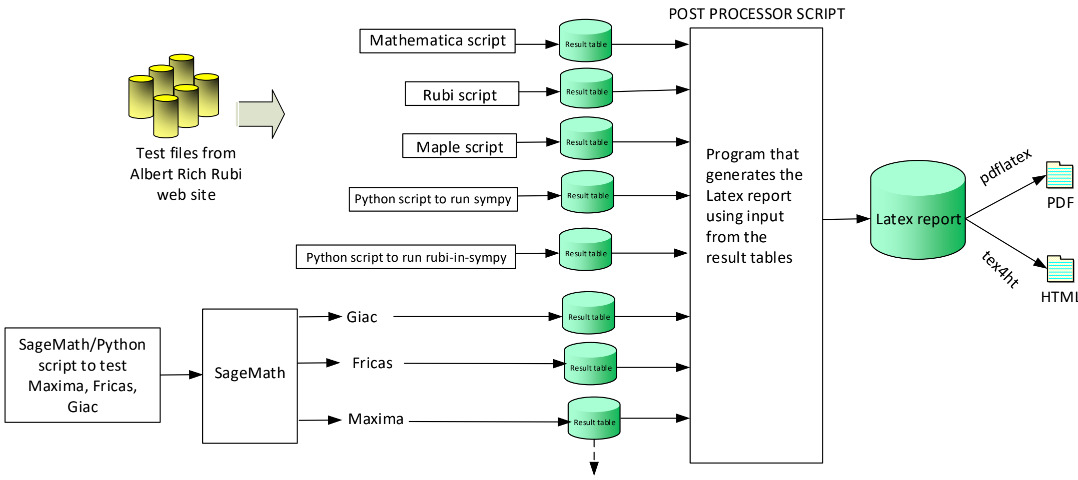
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 10, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 101, 108, 109, 110, 111, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 206, 207, 208, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 235, 238, 239, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 298, 299, 302, 303, 305, 306, 332, 334, 336, 337, 341, 350, 351, 352, 358 }

B grade: { 8, 11, 12, 14, 21, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 89, 90, 98, 99, 100, 115, 160, 170, 171, 172, 209, 232, 233, 234, 236, 237, 240, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 297, 301, 304, 335, 339, 340, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

C grade: { 3, 9, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 195, 198, 199, 200, 201, 202, 205, 210, 214, 215, 216, 217, 248, 249, 250, 290, 291, 292, 300, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338 }

F grade: { 13, 196, 197, 328, 329, 330, 331, 333, 348, 349}

2.1.3 Maple

A grade: { 18, 20, 21, 23, 24, 25, 32, 35, 36, 37, 43, 50, 51, 55, 56, 57, 58, 59, 63, 65, 68, 69, 74, 75, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 137, 139, 140, 141, 142, 147, 149, 150, 151, 152, 160, 161, 162, 163, 177, 185, 191, 194, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261, 268, 269, 275, 281, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 358 }

B grade: { 16, 17, 19, 22, 26, 27, 28, 29, 30, 31, 33, 34, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 52, 53, 54, 60, 61, 62, 64, 66, 67, 70, 71, 72, 73, 76, 77, 78, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 131, 135, 136, 138, 143, 144, 145, 146, 148, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 235, 248, 249, 250, 255, 256, 257, 260, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352, 353, 354, 355, 356, 357 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 79, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

2.1.4 Maxima

A grade: { 17, 18, 20, 30, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261

B grade: { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

2.1.5 FriCAS

A grade: { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 102, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351, 358 }

B grade: { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 112, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 43, 44, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 74, 75, 77, 224, 225, 226, 227, 238, 239, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 260, 261, 266, 267, 268, 273, 274, 275, 281, 282 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 24, 25, 33, 34, 35, 36, 37, 45, 46, 47, 48, 49, 50, 51, 52, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 248, 249, 250, 255, 256, 257, 262, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

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2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	248	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.841	2.263	3.333	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	204	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.492	1.501	2.673	0.	0.	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	392	0	0	0	0	0
normalized size	1	1.	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	3.778	1.806	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	157	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.867	1.033	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	212	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.488	1.277	1.539	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	260	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.846	4.424	1.724	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	596	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.872	18.294	0.499	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	478	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.494	15.405	0.417	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	409	0	0	0	0	0
normalized size	1	1.	2.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	65.847	0.458	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	250	0	0	0	0	0
normalized size	1	1.	1.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.396	4.709	0.383	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	523	0	0	0	0	0
normalized size	1	1.	2.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.675	13.103	0.378	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	5918	0	0	0	0	0
normalized size	1	1.	26.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.453	22.213	4.251	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	10.886	4.162	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	101	0	0
normalized size	1	1.	2.89	0.	0.	2.73	0.	0.
time (sec)	N/A	0.119	1.512	0.546	0.	1.503	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	101	0	0
normalized size	1	1.	1.	0.	0.	2.89	0.	0.
time (sec)	N/A	0.094	0.363	0.454	0.	1.466	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	147	493	0	1729	0	501
normalized size	1	1.	0.96	3.22	0.	11.3	0.	3.27
time (sec)	N/A	0.393	0.87	0.106	0.	1.839	0.	1.211

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	131	342	454	302	853	248
normalized size	1	1.	0.72	1.88	2.49	1.66	4.69	1.36
time (sec)	N/A	0.295	0.935	0.036	0.977	1.484	10.528	1.208

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	95	208	270	248	486	196
normalized size	1	1.	0.67	1.46	1.9	1.75	3.42	1.38
time (sec)	N/A	0.25	0.812	0.033	0.966	1.432	5.81	1.198

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	105	74	185	242	189	396	154
normalized size	1	1.08	0.76	1.91	2.49	1.95	4.08	1.59
time (sec)	N/A	0.186	0.65	0.029	0.966	1.517	2.628	1.171

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	74	99	112	138	78
normalized size	1	1.	0.98	1.51	2.02	2.29	2.82	1.59
time (sec)	N/A	0.083	0.154	0.023	0.968	1.302	1.229	1.142

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	125	113	358	289	830	167
normalized size	1	1.	2.23	2.02	6.39	5.16	14.82	2.98
time (sec)	N/A	0.169	0.862	0.1	1.456	1.405	10.484	1.199

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	160	160	616	402	711	124
normalized size	1	1.	2.22	2.22	8.56	5.58	9.88	1.72
time (sec)	N/A	0.225	0.607	0.102	1.484	1.421	19.942	1.143

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	147	115	995	466	1035	188
normalized size	1	1.	1.41	1.11	9.57	4.48	9.95	1.81
time (sec)	N/A	0.237	0.683	0.108	1.061	1.353	29.88	1.168

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	174	159	1458	637	0	252
normalized size	1	1.	1.23	1.12	10.27	4.49	0.	1.77
time (sec)	N/A	0.285	0.825	0.119	1.094	1.36	0.	1.193

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	200	203	1924	794	0	360
normalized size	1	1.	1.14	1.15	10.93	4.51	0.	2.05
time (sec)	N/A	0.307	0.825	0.136	1.155	1.478	0.	1.209

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	219	569	771	381	1586	375
normalized size	1	1.	0.96	2.48	3.37	1.66	6.93	1.64
time (sec)	N/A	0.368	1.938	0.037	1.023	1.682	40.239	1.247

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	163	463	621	323	1210	329
normalized size	1	1.	0.86	2.45	3.29	1.71	6.4	1.74
time (sec)	N/A	0.296	1.52	0.035	1.001	1.593	23.363	1.21

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	137	365	486	257	910	281
normalized size	1	1.	0.93	2.48	3.31	1.75	6.19	1.91
time (sec)	N/A	0.216	1.051	0.03	0.988	1.509	16.908	1.163

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	54	166	221	176	372	159
normalized size	1	1.	0.61	1.87	2.48	1.98	4.18	1.79
time (sec)	N/A	0.137	0.143	0.026	0.968	1.463	4.941	1.21

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	186	242	190	396	150
normalized size	1	1.	0.68	1.9	2.47	1.94	4.04	1.53
time (sec)	N/A	0.149	0.786	0.027	0.972	1.404	1.799	1.168

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	191	299	842	423	2365	220
normalized size	1	1.	1.63	2.56	7.2	3.62	20.21	1.88
time (sec)	N/A	0.289	1.235	0.113	1.483	1.398	9.736	1.172

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	238	198	1133	568	2474	182
normalized size	1	1.	2.18	1.82	10.39	5.21	22.7	1.67
time (sec)	N/A	0.284	0.607	0.116	1.531	1.433	26.478	1.199

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	278	249	1538	655	0	215
normalized size	1	1.	2.48	2.22	13.73	5.85	0.	1.92
time (sec)	N/A	0.277	0.696	0.127	1.579	1.427	0.	1.217

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	191	161	2121	652	0	309
normalized size	1	1.	2.55	2.15	28.28	8.69	0.	4.12
time (sec)	N/A	0.229	0.912	0.131	1.173	1.412	0.	1.24

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	261	205	2817	833	0	406
normalized size	1	1.	2.27	1.78	24.5	7.24	0.	3.53
time (sec)	N/A	0.286	1.196	0.153	1.259	1.457	0.	1.237

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	285	249	3515	1027	0	504
normalized size	1	1.	1.83	1.6	22.53	6.58	0.	3.23
time (sec)	N/A	0.374	1.542	0.153	1.377	1.466	0.	1.232

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	313	293	4212	1204	0	601
normalized size	1	1.	1.59	1.49	21.38	6.11	0.	3.05
time (sec)	N/A	0.465	3.592	0.185	1.527	1.426	0.	1.291

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	255	651	892	455	1948	468
normalized size	1	1.	0.96	2.46	3.37	1.72	7.35	1.77
time (sec)	N/A	0.391	4.287	0.148	1.042	1.754	83.288	1.488

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	232	611	833	381	1753	406
normalized size	1	1.	1.05	2.75	3.75	1.72	7.9	1.83
time (sec)	N/A	0.322	2.523	0.036	1.035	1.82	52.216	1.345

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	209	568	771	315	1579	369
normalized size	1	1.	1.15	3.14	4.26	1.74	8.72	2.04
time (sec)	N/A	0.234	1.885	0.032	1.017	1.596	36.119	1.291

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	64	263	356	221	682	219
normalized size	1	1.	0.55	2.25	3.04	1.89	5.83	1.87
time (sec)	N/A	0.148	0.226	0.024	0.993	1.46	15.922	1.302

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	133	364	486	258	910	275
normalized size	1	1.	0.96	2.64	3.52	1.87	6.59	1.99
time (sec)	N/A	0.2	1.034	0.03	0.993	1.506	11.675	1.175

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	95	208	270	248	486	196
normalized size	1	1.	0.68	1.49	1.93	1.77	3.47	1.4
time (sec)	N/A	0.222	0.824	0.031	0.969	1.35	6.062	1.134

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	223	449	1538	533	4255	316
normalized size	1	1.	1.43	2.88	9.86	3.42	27.28	2.03
time (sec)	N/A	0.31	1.508	0.121	1.54	1.501	32.466	1.178

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	280	399	1871	687	0	315
normalized size	1	1.	1.72	2.45	11.48	4.21	0.	1.93
time (sec)	N/A	0.348	0.849	0.131	1.61	1.43	0.	1.239

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	316	323	2275	821	0	305
normalized size	1	1.	2.07	2.11	14.87	5.37	0.	1.99
time (sec)	N/A	0.342	1.069	0.138	1.637	1.459	0.	1.238

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	356	374	2859	891	0	288
normalized size	1	1.	2.36	2.48	18.93	5.9	0.	1.91
time (sec)	N/A	0.33	1.152	0.142	1.719	1.564	0.	1.207

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	283	205	3646	824	0	406
normalized size	1	1.	3.68	2.66	47.35	10.7	0.	5.27
time (sec)	N/A	0.236	2.447	0.152	1.337	1.461	0.	1.265

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	313	249	4577	1014	0	504
normalized size	1	1.	2.65	2.11	38.79	8.59	0.	4.27
time (sec)	N/A	0.289	2.812	0.164	1.496	1.557	0.	1.239

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	339	293	5505	1215	0	601
normalized size	1	1.	2.17	1.88	35.29	7.79	0.	3.85
time (sec)	N/A	0.375	5.085	0.184	1.672	1.505	0.	1.301

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	378	337	6433	1400	0	698
normalized size	1	1.	1.92	1.71	32.65	7.11	0.	3.54
time (sec)	N/A	0.444	6.645	0.201	1.891	1.631	0.	1.264

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	274	678	2425	651	0	463
normalized size	1	1.	1.44	3.57	12.76	3.43	0.	2.44
time (sec)	N/A	0.363	2.292	0.134	1.58	1.505	0.	1.237

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	220	449	1512	533	4255	317
normalized size	1	1.	1.4	2.86	9.63	3.39	27.1	2.02
time (sec)	N/A	0.318	1.363	0.121	1.534	1.502	31.734	1.202

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	188	299	821	423	2365	221
normalized size	1	1.	1.59	2.53	6.96	3.58	20.04	1.87
time (sec)	N/A	0.278	1.274	0.106	1.498	1.419	14.927	1.189

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	127	113	346	289	830	165
normalized size	1	1.	2.23	1.98	6.07	5.07	14.56	2.89
time (sec)	N/A	0.155	0.562	0.096	1.452	1.424	7.146	1.153

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	57	47	58	83	55
normalized size	1	1.	1.	1.63	1.34	1.66	2.37	1.57
time (sec)	N/A	0.136	0.028	0.056	0.965	1.357	4.166	1.201

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	108	93	359	171	578	138
normalized size	1	1.	1.71	1.48	5.7	2.71	9.17	2.19
time (sec)	N/A	0.202	0.569	0.076	0.992	1.401	16.241	1.192

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	157	145	571	265	1732	239
normalized size	1	1.	1.54	1.42	5.6	2.6	16.98	2.34
time (sec)	N/A	0.257	0.842	0.085	1.026	1.32	31.958	1.229

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	240	189	836	350	0	320
normalized size	1	1.	1.69	1.33	5.89	2.46	0.	2.25
time (sec)	N/A	0.307	1.098	0.087	1.072	1.306	0.	1.196

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	354	778	4026	922	0	556
normalized size	1	1.	1.48	3.24	16.77	3.84	0.	2.32
time (sec)	N/A	0.41	1.986	0.161	1.767	1.527	0.	1.23

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	311	549	2827	786	0	498
normalized size	1	1.	1.73	3.05	15.71	4.37	0.	2.77
time (sec)	N/A	0.362	1.259	0.148	1.659	1.823	0.	1.222

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	274	399	1860	687	0	315
normalized size	1	1.	1.69	2.46	11.48	4.24	0.	1.94
time (sec)	N/A	0.332	0.846	0.129	1.588	1.814	0.	1.199

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	234	198	1125	568	2474	184
normalized size	1	1.	2.17	1.83	10.42	5.26	22.91	1.7
time (sec)	N/A	0.277	0.572	0.117	1.528	1.707	30.298	1.183

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	156	160	610	401	711	124
normalized size	1	1.	2.17	2.22	8.47	5.57	9.88	1.72
time (sec)	N/A	0.207	0.56	0.104	1.489	1.624	16.259	1.196

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	110	97	358	171	578	138
normalized size	1	1.	1.77	1.56	5.77	2.76	9.32	2.23
time (sec)	N/A	0.197	0.481	0.072	1.009	1.569	16.253	1.174

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	53	145	63	103	651	117
normalized size	1	1.	0.85	2.34	1.02	1.66	10.5	1.89
time (sec)	N/A	0.14	0.118	0.066	0.987	1.612	17.455	1.198

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	237	183	879	262	0	317
normalized size	1	1.	2.55	1.97	9.45	2.82	0.	3.41
time (sec)	N/A	0.22	0.961	0.095	1.073	1.622	0.	1.232

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	285	233	1127	371	0	398
normalized size	1	1.	2.11	1.73	8.35	2.75	0.	2.95
time (sec)	N/A	0.27	0.924	0.116	1.093	1.653	0.	1.237

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	329	277	1347	439	0	479
normalized size	1	1.	1.88	1.58	7.7	2.51	0.	2.74
time (sec)	N/A	0.325	1.105	0.118	1.15	1.777	0.	1.271

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	388	649	4431	1080	0	508
normalized size	1	1.	1.6	2.67	18.23	4.44	0.	2.09
time (sec)	N/A	0.412	2.627	0.184	1.81	1.893	0.	1.288

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	348	474	3232	961	0	412
normalized size	1	1.	1.73	2.36	16.08	4.78	0.	2.05
time (sec)	N/A	0.392	1.537	0.155	1.719	1.834	0.	1.235

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	308	323	2267	821	0	305
normalized size	1	1.	2.01	2.11	14.82	5.37	0.	1.99
time (sec)	N/A	0.331	1.048	0.141	1.63	1.735	0.	1.286

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	272	249	1531	656	0	215
normalized size	1	1.	2.47	2.26	13.92	5.96	0.	1.95
time (sec)	N/A	0.265	0.684	0.123	1.575	1.742	0.	1.213

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	139	115	990	464	1035	186
normalized size	1	1.	1.35	1.12	9.61	4.5	10.05	1.81
time (sec)	N/A	0.225	0.772	0.11	1.046	1.61	34.11	1.205

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	156	145	571	263	1732	236
normalized size	1	1.	1.53	1.42	5.6	2.58	16.98	2.31
time (sec)	N/A	0.249	0.792	0.087	1.011	1.613	54.091	1.212

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	237	185	878	262	0	317
normalized size	1	1.	2.63	2.06	9.76	2.91	0.	3.52
time (sec)	N/A	0.204	0.986	0.08	1.063	1.752	0.	1.207

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	227	81	138	1506	181
normalized size	1	1.	0.77	2.7	0.96	1.64	17.93	2.15
time (sec)	N/A	0.152	0.198	0.08	0.976	1.866	104.988	1.249

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	325	271	1376	350	0	479
normalized size	1	1.	2.69	2.24	11.37	2.89	0.	3.96
time (sec)	N/A	0.223	1.091	0.1	1.147	2.092	0.	1.213

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	373	0	1621	459	0	560
normalized size	1	1.	2.3	0.	10.01	2.83	0.	3.46
time (sec)	N/A	0.289	1.321	180.	1.19	2.038	0.	1.254

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	401	365	1872	543	0	641
normalized size	1	1.	1.96	1.78	9.13	2.65	0.	3.13
time (sec)	N/A	0.345	3.255	0.128	1.229	2.027	0.	1.279

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	149	119	0	749	0	0
normalized size	1	1.	0.75	0.6	0.	3.78	0.	0.
time (sec)	N/A	0.487	2.881	1.046	0.	1.789	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	123	103	0	591	0	0
normalized size	1	1.	0.78	0.66	0.	3.76	0.	0.
time (sec)	N/A	0.412	1.449	1.018	0.	1.759	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	104	81	0	454	0	0
normalized size	1	1.	0.9	0.7	0.	3.91	0.	0.
time (sec)	N/A	0.32	0.98	0.917	0.	1.718	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	87	63	0	336	0	0
normalized size	1	1.	1.19	0.86	0.	4.6	0.	0.
time (sec)	N/A	0.24	0.423	0.963	0.	1.663	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	166	159	0	690	0	541
normalized size	1	1.	1.36	1.3	0.	5.66	0.	4.43
time (sec)	N/A	0.336	1.265	1.253	0.	1.772	0.	2.415

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	157	227	0	852	0	720
normalized size	1	1.	1.37	1.97	0.	7.41	0.	6.26
time (sec)	N/A	0.318	1.553	0.958	0.	1.761	0.	3.558

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	199	268	0	1035	0	0
normalized size	1	1.	1.58	2.13	0.	8.21	0.	0.
time (sec)	N/A	0.335	2.175	1.45	0.	1.78	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	217	352	0	1289	0	0
normalized size	1	1.	1.33	2.16	0.	7.91	0.	0.
time (sec)	N/A	0.373	3.318	1.416	0.	1.712	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	1355	121	0	895	0	0
normalized size	1	1.	6.45	0.58	0.	4.26	0.	0.
time (sec)	N/A	0.554	6.703	1.12	0.	1.547	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	1173	105	0	740	0	0
normalized size	1	1.	7.02	0.63	0.	4.43	0.	0.
time (sec)	N/A	0.453	6.557	0.951	0.	1.531	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	106	83	0	575	0	0
normalized size	1	1.	0.88	0.69	0.	4.79	0.	0.
time (sec)	N/A	0.387	4.642	1.133	0.	1.437	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	463	0	0
normalized size	1	1.	1.1	0.8	0.	5.72	0.	0.
time (sec)	N/A	0.33	0.592	0.865	0.	1.506	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	175	197	0	822	0	771
normalized size	1	1.	1.09	1.22	0.	5.11	0.	4.79
time (sec)	N/A	0.442	1.194	1.164	0.	1.56	0.	1.973

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	355	282	0	988	0	0
normalized size	1	1.	2.02	1.6	0.	5.61	0.	0.
time (sec)	N/A	0.481	0.916	1.065	0.	1.528	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	344	386	0	1143	0	0
normalized size	1	1.	1.97	2.21	0.	6.53	0.	0.
time (sec)	N/A	0.478	1.2	1.443	0.	1.533	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	342	354	0	1353	0	0
normalized size	1	1.	1.95	2.02	0.	7.73	0.	0.
time (sec)	N/A	0.491	1.839	1.628	0.	1.599	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	357	440	0	1670	0	2141
normalized size	1	1.	1.61	1.98	0.	7.52	0.	9.64
time (sec)	N/A	0.511	2.674	1.628	0.	1.632	0.	6.215

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	1569	121	0	972	0	0
normalized size	1	1.	7.47	0.58	0.	4.63	0.	0.
time (sec)	N/A	0.534	6.886	1.05	0.	1.529	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	1351	105	0	849	0	0
normalized size	1	1.	8.39	0.65	0.	5.27	0.	0.
time (sec)	N/A	0.472	6.719	0.887	0.	1.54	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	1157	83	0	722	0	0
normalized size	1	1.	9.33	0.67	0.	5.82	0.	0.
time (sec)	N/A	0.407	6.527	0.946	0.	1.547	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	563	0	0
normalized size	1	1.	1.1	0.8	0.	6.95	0.	0.
time (sec)	N/A	0.305	1.041	0.968	0.	1.471	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	193	233	0	944	0	938
normalized size	1	1.	0.96	1.16	0.	4.72	0.	4.69
time (sec)	N/A	0.521	1.414	1.292	0.	1.482	0.	2.046

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	444	354	0	1099	0	0
normalized size	1	1.	2.04	1.62	0.	5.04	0.	0.
time (sec)	N/A	0.546	1.743	1.217	0.	1.535	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	434	434	0	1285	0	0
normalized size	1	1.	1.93	1.93	0.	5.71	0.	0.
time (sec)	N/A	0.549	2.308	1.602	0.	1.65	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	422	524	0	1434	0	0
normalized size	1	1.	1.94	2.41	0.	6.61	0.	0.
time (sec)	N/A	0.549	3.271	1.516	0.	1.697	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	355	432	0	1646	0	2140
normalized size	1	1.	1.64	1.99	0.	7.59	0.	9.86
time (sec)	N/A	0.557	4.672	1.585	0.	1.632	0.	7.619

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	485	526	0	1971	0	0
normalized size	1	1.	1.82	1.98	0.	7.41	0.	0.
time (sec)	N/A	0.587	6.862	1.792	0.	1.718	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	157	111	645	282	0	1127
normalized size	1	1.	0.78	0.56	3.22	1.41	0.	5.64
time (sec)	N/A	0.385	5.703	0.926	1.55	1.59	0.	2.089

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	134	95	521	230	0	957
normalized size	1	1.	0.84	0.6	3.28	1.45	0.	6.02
time (sec)	N/A	0.35	1.778	0.828	1.561	1.494	0.	1.929

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	113	73	397	158	0	786
normalized size	1	1.	0.96	0.62	3.36	1.34	0.	6.66
time (sec)	N/A	0.317	0.636	0.811	1.526	1.399	0.	1.592

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	235	101	0	475
normalized size	1	1.	0.6	0.73	3.22	1.38	0.	6.51
time (sec)	N/A	0.275	0.207	0.644	1.501	1.372	0.	1.576

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	140	130	0	447	0	536
normalized size	1	1.	1.54	1.43	0.	4.91	0.	5.89
time (sec)	N/A	0.283	0.46	1.197	0.	1.393	0.	1.753

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	284	225	0	608	0	878
normalized size	1	1.	2.09	1.65	0.	4.47	0.	6.46
time (sec)	N/A	0.33	0.562	0.98	0.	1.553	0.	2.407

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	404	350	0	753	0	0
normalized size	1	1.	2.24	1.94	0.	4.18	0.	0.
time (sec)	N/A	0.425	0.885	1.285	0.	1.513	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	953	143	1029	396	0	1754
normalized size	1	1.	3.94	0.59	4.25	1.64	0.	7.25
time (sec)	N/A	0.651	6.87	1.054	1.604	1.578	0.	2.538

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	159	121	905	331	0	1589
normalized size	1	1.	0.79	0.6	4.5	1.65	0.	7.91
time (sec)	N/A	0.558	2.966	0.915	1.569	1.512	0.	2.369

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	130	105	779	270	0	1339
normalized size	1	1.	0.84	0.68	5.06	1.75	0.	8.69
time (sec)	N/A	0.479	1.219	0.905	1.557	1.499	0.	2.161

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	113	81	651	207	0	1110
normalized size	1	1.	0.98	0.7	5.66	1.8	0.	9.65
time (sec)	N/A	0.408	0.692	0.84	1.542	1.518	0.	1.864

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	87	63	463	157	0	934
normalized size	1	1.	1.12	0.81	5.94	2.01	0.	11.97
time (sec)	N/A	0.313	0.281	1.041	1.525	1.593	0.	1.703

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	176	168	0	603	0	1046
normalized size	1	1.	1.3	1.24	0.	4.47	0.	7.75
time (sec)	N/A	0.354	0.534	1.105	0.	1.778	0.	1.879

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	300	258	0	560	0	0
normalized size	1	1.	1.71	1.47	0.	3.2	0.	0.
time (sec)	N/A	0.392	0.881	1.145	0.	1.811	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	430	426	0	717	0	1805
normalized size	1	1.	1.91	1.89	0.	3.19	0.	8.02
time (sec)	N/A	0.484	1.462	1.369	0.	1.853	0.	3.833

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	1276	420	0	2191
normalized size	1	1.	0.73	0.59	5.27	1.74	0.	9.05
time (sec)	N/A	0.647	4.426	0.92	1.652	1.902	0.	2.976

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	158	121	1153	367	0	2021
normalized size	1	1.	0.76	0.58	5.52	1.76	0.	9.67
time (sec)	N/A	0.567	2.763	1.097	1.651	1.812	0.	2.62

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	132	105	1027	313	0	1716
normalized size	1	1.	0.82	0.66	6.42	1.96	0.	10.72
time (sec)	N/A	0.48	1.26	1.176	1.605	1.727	0.	2.53

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	113	83	895	246	0	1391
normalized size	1	1.	0.93	0.69	7.4	2.03	0.	11.5
time (sec)	N/A	0.413	0.713	0.993	1.587	1.774	0.	1.991

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	89	65	682	196	0	1539
normalized size	1	1.	1.05	0.76	8.02	2.31	0.	18.11
time (sec)	N/A	0.315	0.307	1.001	1.563	1.626	0.	1.727

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	204	200	0	729	0	1511
normalized size	1	1.	1.17	1.15	0.	4.19	0.	8.68
time (sec)	N/A	0.438	0.785	1.209	0.	1.764	0.	2.144

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	357	308	0	732	0	0
normalized size	1	1.	1.59	1.38	0.	3.27	0.	0.
time (sec)	N/A	0.48	1.424	1.18	0.	1.9	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	479	410	0	659	0	0
normalized size	1	1.	1.86	1.59	0.	2.55	0.	0.
time (sec)	N/A	0.555	2.378	1.475	0.	1.84	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	118	174	0	342	0	0
normalized size	1	1.	1.26	1.85	0.	3.64	0.	0.
time (sec)	N/A	0.339	0.995	0.398	0.	1.81	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	102	129	0	293	0	0
normalized size	1	1.	1.09	1.37	0.	3.12	0.	0.
time (sec)	N/A	0.337	0.841	0.378	0.	1.72	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	91	0	227	0	0
normalized size	1	1.	0.89	0.97	0.	2.41	0.	0.
time (sec)	N/A	0.333	0.582	0.366	0.	1.709	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	63	57	0	157	0	0
normalized size	1	1.	0.68	0.62	0.	1.71	0.	0.
time (sec)	N/A	0.309	0.207	0.355	0.	1.654	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	120	394	236	0	0	0
normalized size	1	1.	1.2	3.94	2.36	0.	0.	0.
time (sec)	N/A	0.337	1.177	0.367	1.573	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	147	403	0	0	0	0
normalized size	1	1.	1.48	4.07	0.	0.	0.	0.
time (sec)	N/A	0.358	1.168	0.351	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	101	137	0	224	0	0
normalized size	1	1.	1.1	1.49	0.	2.43	0.	0.
time (sec)	N/A	0.345	0.55	0.352	0.	1.706	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	103	205	0	263	0	0
normalized size	1	1.	1.1	2.18	0.	2.8	0.	0.
time (sec)	N/A	0.337	0.628	0.364	0.	1.759	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	205	185	0	356	0	0
normalized size	1	1.	1.4	1.27	0.	2.44	0.	0.
time (sec)	N/A	0.358	1.59	0.324	0.	1.879	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	172	147	0	302	0	0
normalized size	1	1.	1.18	1.01	0.	2.07	0.	0.
time (sec)	N/A	0.361	1.693	0.303	0.	1.817	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	96	86	0	216	0	0
normalized size	1	1.	0.72	0.64	0.	1.61	0.	0.
time (sec)	N/A	0.348	0.78	0.27	0.	1.74	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	81	91	0	228	0	0
normalized size	1	1.	0.84	0.95	0.	2.38	0.	0.
time (sec)	N/A	0.325	0.578	0.326	0.	1.68	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	136	495	0	0	0	0
normalized size	1	1.	0.94	3.41	0.	0.	0.	0.
time (sec)	N/A	0.382	0.71	0.342	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	210	749	494	0	0	0
normalized size	1	1.	1.33	4.74	3.13	0.	0.	0.
time (sec)	N/A	0.385	0.905	0.288	1.58	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	198	594	0	0	0	0
normalized size	1	1.	1.33	3.99	0.	0.	0.	0.
time (sec)	N/A	0.387	0.978	0.28	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	125	223	0	309	0	0
normalized size	1	1.	1.3	2.32	0.	3.22	0.	0.
time (sec)	N/A	0.275	1.038	0.273	0.	2.029	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	123	217	0	336	0	0
normalized size	1	1.	0.84	1.49	0.	2.3	0.	0.
time (sec)	N/A	0.376	1.397	0.279	0.	2.094	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	126	339	0	393	0	0
normalized size	1	1.	0.82	2.2	0.	2.55	0.	0.
time (sec)	N/A	0.373	1.97	0.299	0.	2.091	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	223	203	0	371	0	0
normalized size	1	1.	1.13	1.03	0.	1.87	0.	0.
time (sec)	N/A	0.476	2.858	0.368	0.	2.347	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	113	114	0	279	0	0
normalized size	1	1.	0.63	0.63	0.	1.55	0.	0.
time (sec)	N/A	0.466	0.841	0.285	0.	2.184	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	165	147	0	304	0	0
normalized size	1	1.	1.16	1.04	0.	2.14	0.	0.
time (sec)	N/A	0.362	1.891	0.302	0.	2.091	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	102	129	0	293	0	0
normalized size	1	1.	1.06	1.34	0.	3.05	0.	0.
time (sec)	N/A	0.318	0.875	0.339	0.	1.956	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	177	591	0	0	0	0
normalized size	1	1.	0.92	3.06	0.	0.	0.	0.
time (sec)	N/A	0.463	1.569	0.356	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	231	845	0	0	0	0
normalized size	1	1.	1.1	4.02	0.	0.	0.	0.
time (sec)	N/A	0.485	1.752	0.272	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	207	1093	683	0	0	0
normalized size	1	1.	0.98	5.16	3.22	0.	0.	0.
time (sec)	N/A	0.49	1.227	0.265	1.649	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	204	832	0	0	0	0
normalized size	1	1.	1.04	4.24	0.	0.	0.	0.
time (sec)	N/A	0.488	1.237	0.299	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	145	309	0	400	0	0
normalized size	1	1.	1.51	3.22	0.	4.17	0.	0.
time (sec)	N/A	0.276	3.06	0.283	0.	1.507	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	368	0	451	0	0
normalized size	1	1.	1.	2.52	0.	3.09	0.	0.
time (sec)	N/A	0.376	4.204	0.305	0.	1.577	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	144	423	0	490	0	0
normalized size	1	1.	0.73	2.16	0.	2.5	0.	0.
time (sec)	N/A	0.483	5.747	0.332	0.	1.57	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	870	259	0	427	0	0
normalized size	1	1.	3.48	1.04	0.	1.71	0.	0.
time (sec)	N/A	0.567	7.139	0.306	0.	1.894	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	135	142	0	324	0	0
normalized size	1	1.	0.6	0.63	0.	1.43	0.	0.
time (sec)	N/A	0.559	1.673	0.33	0.	1.814	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	232	203	0	373	0	0
normalized size	1	1.	1.21	1.06	0.	1.94	0.	0.
time (sec)	N/A	0.46	2.197	0.359	0.	1.74	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	212	185	0	356	0	0
normalized size	1	1.	1.49	1.3	0.	2.51	0.	0.
time (sec)	N/A	0.359	1.856	0.329	0.	1.566	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	121	174	0	340	0	0
normalized size	1	1.	1.26	1.81	0.	3.54	0.	0.
time (sec)	N/A	0.325	1.023	0.359	0.	1.423	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	183	671	0	0	0	0
normalized size	1	1.	0.77	2.81	0.	0.	0.	0.
time (sec)	N/A	0.569	2.801	0.342	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	292	927	0	0	0	0
normalized size	1	1.	1.08	3.42	0.	0.	0.	0.
time (sec)	N/A	0.593	3.567	0.281	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	251	1189	0	0	0	0
normalized size	1	1.	0.95	4.52	0.	0.	0.	0.
time (sec)	N/A	0.595	2.595	0.28	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	244	1455	1011	0	0	0
normalized size	1	1.	0.92	5.51	3.83	0.	0.	0.
time (sec)	N/A	0.609	2.988	0.282	1.636	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	238	1019	0	0	0	0
normalized size	1	1.	0.96	4.13	0.	0.	0.	0.
time (sec)	N/A	0.599	2.748	0.323	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	434	389	0	489	0	0
normalized size	1	1.	4.52	4.05	0.	5.09	0.	0.
time (sec)	N/A	0.272	6.911	0.298	0.	2.414	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	442	393	0	535	0	0
normalized size	1	1.	3.03	2.69	0.	3.66	0.	0.
time (sec)	N/A	0.379	6.949	0.331	0.	2.29	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	442	505	0	585	0	0
normalized size	1	1.	2.19	2.5	0.	2.9	0.	0.
time (sec)	N/A	0.491	7.149	0.366	0.	2.135	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	436	560	0	613	0	0
normalized size	1	1.	1.77	2.28	0.	2.49	0.	0.
time (sec)	N/A	0.591	7.112	0.405	0.	2.199	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	185	595	0	0	0	0
normalized size	1	1.	0.94	3.02	0.	0.	0.	0.
time (sec)	N/A	0.463	1.314	0.378	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	504	0	0	0	0
normalized size	1	1.	1.	3.45	0.	0.	0.	0.
time (sec)	N/A	0.365	0.674	0.345	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	119	399	238	0	0	0
normalized size	1	1.	1.24	4.16	2.48	0.	0.	0.
time (sec)	N/A	0.322	1.138	0.34	1.566	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	165	0	0	0	0
normalized size	1	1.	0.86	1.46	0.	0.	0.	0.
time (sec)	N/A	0.364	0.328	0.323	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	191	302	0	869	0	0
normalized size	1	1.	1.85	2.93	0.	8.44	0.	0.
time (sec)	N/A	0.252	0.523	0.325	0.	2.048	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	222	465	0	1083	0	0
normalized size	1	1.	1.45	3.04	0.	7.08	0.	0.
time (sec)	N/A	0.355	0.618	0.344	0.	2.162	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	271	938	0	0	0	0
normalized size	1	1.	1.	3.46	0.	0.	0.	0.
time (sec)	N/A	0.577	3.512	0.288	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	212	853	0	0	0	0
normalized size	1	1.	1.01	4.06	0.	0.	0.	0.
time (sec)	N/A	0.478	1.586	0.273	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	190	760	495	0	0	0
normalized size	1	1.	1.19	4.78	3.11	0.	0.	0.
time (sec)	N/A	0.386	0.865	0.29	1.595	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	143	408	0	0	0	0
normalized size	1	1.	1.43	4.08	0.	0.	0.	0.
time (sec)	N/A	0.346	1.156	0.328	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	186	303	0	868	0	0
normalized size	1	1.	1.81	2.94	0.	8.43	0.	0.
time (sec)	N/A	0.256	0.552	0.331	0.	2.333	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	178	132	0	703	0	0
normalized size	1	1.	1.19	0.88	0.	4.69	0.	0.
time (sec)	N/A	0.373	0.672	0.269	0.	2.437	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	306	431	0	1060	0	0
normalized size	1	1.	1.41	1.99	0.	4.88	0.	0.
time (sec)	N/A	0.479	0.948	0.272	0.	2.624	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	573	1287	0	0	0	0
normalized size	1	1.	1.77	3.98	0.	0.	0.	0.
time (sec)	N/A	0.713	7.046	0.3	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	243	1205	0	0	0	0
normalized size	1	1.	0.92	4.58	0.	0.	0.	0.
time (sec)	N/A	0.607	2.556	0.285	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	199	1106	680	0	0	0
normalized size	1	1.	0.94	5.24	3.22	0.	0.	0.
time (sec)	N/A	0.494	1.144	0.269	1.599	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	179	604	0	0	0	0
normalized size	1	1.	1.2	4.05	0.	0.	0.	0.
time (sec)	N/A	0.392	0.975	0.28	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	99	135	0	223	0	0
normalized size	1	1.	1.05	1.44	0.	2.37	0.	0.
time (sec)	N/A	0.333	0.505	0.32	0.	1.984	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	214	465	0	1081	0	0
normalized size	1	1.	1.42	3.08	0.	7.16	0.	0.
time (sec)	N/A	0.354	0.66	0.336	0.	2.559	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	305	431	0	1058	0	0
normalized size	1	1.	1.47	2.07	0.	5.09	0.	0.
time (sec)	N/A	0.48	0.947	0.276	0.	2.535	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	246	151	0	783	0	0
normalized size	1	1.	1.	0.62	0.	3.2	0.	0.
time (sec)	N/A	0.57	0.94	0.297	0.	2.596	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	2903	0	0	0	0	0
normalized size	1	1.	16.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.315	14.173	2.369	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	180.084	3.02	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	180.043	2.553	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	462	0	0	0	0	0
normalized size	1	1.	3.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.289	4.204	1.549	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	1.823	1.133	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	7409	0	0	0	0	0
normalized size	1	1.	60.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.303	25.65	0.296	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	8371	0	0	0	0	0
normalized size	1	1.	56.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.33	23.328	0.754	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	9702	0	0	0	0	0
normalized size	1	1.	65.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	25.902	0.931	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0
normalized size	1	1.	1.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	4.868	0.321	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0
normalized size	1	1.	1.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.288	2.353	0.312	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	667	0	979	1354	0	0
normalized size	1	1.	2.43	0.	3.56	4.92	0.	0.
time (sec)	N/A	0.503	6.829	0.327	1.739	2.318	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	192	174	0	672	772	0	0
normalized size	1	1.16	1.05	0.	4.05	4.65	0.	0.
time (sec)	N/A	0.354	1.698	0.325	1.699	2.118	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	436	417	0	0
normalized size	1	1.	1.12	0.	4.19	4.01	0.	0.
time (sec)	N/A	0.282	0.443	0.306	1.619	2.143	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0
normalized size	1	1.	1.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.282	2.309	0.014	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	369	0	0	0	0	0
normalized size	1	1.	2.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	10.185	0.284	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	5387	0	0	0	0	0
normalized size	1	1.	40.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	23.804	0.293	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	353	0	0	506	0	0
normalized size	1	1.	1.32	0.	0.	1.9	0.	0.
time (sec)	N/A	0.427	12.362	0.56	0.	2.18	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	269	0	0	335	0	0
normalized size	1	1.	1.41	0.	0.	1.75	0.	0.
time (sec)	N/A	0.31	10.051	0.558	0.	2.296	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	211	0	0	213	0	0
normalized size	1	1.	1.85	0.	0.	1.87	0.	0.
time (sec)	N/A	0.222	8.502	0.515	0.	2.014	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	163	163	675	0	0	0	0	0
normalized size	1	1.	4.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.309	11.336	0.475	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	2552	0	0	0	0	0
normalized size	1	1.	16.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.268	16.9	1.58	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	3601	0	0	0	0	0
normalized size	1	1.	21.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	92.897	0.506	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	5163	0	0	0	0	0
normalized size	1	1.	29.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.336	53.132	0.522	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	63	0	0	185	0	0
normalized size	1	1.	1.85	0.	0.	5.44	0.	0.
time (sec)	N/A	0.274	0.529	2.334	0.	1.981	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	67	0	0	182	0	0
normalized size	1	1.	1.97	0.	0.	5.35	0.	0.
time (sec)	N/A	0.238	1.144	2.428	0.	2.026	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	184	0	0
normalized size	1	1.	2.	0.	0.	5.58	0.	0.
time (sec)	N/A	0.237	1.123	2.378	0.	2.095	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	0	0	184	0	0
normalized size	1	1.	1.74	0.	0.	5.26	0.	0.
time (sec)	N/A	0.238	0.545	2.238	0.	2.036	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	88	0	0
normalized size	1	1.	1.	0.	0.	2.44	0.	0.
time (sec)	N/A	0.132	0.464	2.552	0.	2.051	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	89	0	0
normalized size	1	1.	1.	0.	0.	2.41	0.	0.
time (sec)	N/A	0.123	0.464	2.543	0.	2.077	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	87	158	212	269	440	177
normalized size	1	1.	0.62	1.13	1.51	1.92	3.14	1.26
time (sec)	N/A	0.186	0.152	0.031	0.992	2.085	15.476	1.14

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	77	136	186	227	359	153
normalized size	1	1.	0.64	1.12	1.54	1.88	2.97	1.26
time (sec)	N/A	0.168	0.108	0.032	0.955	1.957	9.339	1.122

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	55	117	151	188	267	104
normalized size	1	1.	0.57	1.22	1.57	1.96	2.78	1.08
time (sec)	N/A	0.116	0.482	0.026	1.028	2.176	6.074	1.119

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	116	155	196	104
normalized size	1	1.	0.66	1.09	1.41	1.89	2.39	1.27
time (sec)	N/A	0.106	0.357	0.026	0.96	1.949	2.459	1.124

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	74	99	115	247	0	144
normalized size	1	1.	0.97	1.3	1.51	3.25	0.	1.89
time (sec)	N/A	0.104	0.153	0.05	0.969	2.015	0.	1.154

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	95	112	297	0	207
normalized size	1	1.	0.97	1.2	1.42	3.76	0.	2.62
time (sec)	N/A	0.179	0.186	0.046	0.993	1.951	0.	1.188

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	142	94	122	377	0	185
normalized size	1	1.	1.82	1.21	1.56	4.83	0.	2.37
time (sec)	N/A	0.121	0.035	0.058	0.971	2.028	0.	1.223

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	141	103	158	435	0	203
normalized size	1	1.	1.81	1.32	2.03	5.58	0.	2.6
time (sec)	N/A	0.131	0.461	0.057	0.989	1.938	0.	1.172

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	210	109	196	431	0	235
normalized size	1	1.	2.44	1.27	2.28	5.01	0.	2.73
time (sec)	N/A	0.148	0.069	0.058	0.974	1.984	0.	1.246

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	268	132	236	520	0	235
normalized size	1	1.	2.55	1.26	2.25	4.95	0.	2.24
time (sec)	N/A	0.234	0.074	0.06	0.994	2.094	0.	1.207

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	306	155	279	602	0	327
normalized size	1	1.	2.35	1.19	2.15	4.63	0.	2.52
time (sec)	N/A	0.197	0.08	0.062	0.991	1.951	0.	1.216

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	254	257	965	656	0	211
normalized size	1	1.	1.97	1.99	7.48	5.09	0.	1.64
time (sec)	N/A	0.208	0.927	0.112	1.494	2.073	0.	1.16

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	228	155	733	585	0	153
normalized size	1	1.	2.21	1.5	7.12	5.68	0.	1.49
time (sec)	N/A	0.188	0.789	0.102	1.508	1.96	0.	1.148

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	189	131	529	508	0	126
normalized size	1	1.	2.12	1.47	5.94	5.71	0.	1.42
time (sec)	N/A	0.173	0.763	0.099	1.485	1.764	0.	1.16

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	107	71	470	386	461	85
normalized size	1	1.	1.3	0.87	5.73	4.71	5.62	1.04
time (sec)	N/A	0.138	0.459	0.093	1.008	1.858	34.04	1.166

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	92	86	522	381	571	107
normalized size	1	1.	1.59	1.48	9.	6.57	9.84	1.84
time (sec)	N/A	0.114	0.239	0.087	1.016	1.861	14.17	1.129

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	313	130	585	817	0	134
normalized size	1	1.	3.19	1.33	5.97	8.34	0.	1.37
time (sec)	N/A	0.164	1.005	0.153	1.013	2.06	0.	1.178

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	167	169	701	1065	0	197
normalized size	1	1.	1.48	1.5	6.2	9.42	0.	1.74
time (sec)	N/A	0.398	3.093	0.169	1.022	2.107	0.	1.167

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	245	209	840	1327	0	243
normalized size	1	1.	1.78	1.51	6.09	9.62	0.	1.76
time (sec)	N/A	0.225	4.236	0.197	1.024	2.046	0.	1.176

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	348	249	953	1563	0	288
normalized size	1	1.	2.27	1.63	6.23	10.22	0.	1.88
time (sec)	N/A	0.246	6.223	0.208	1.023	2.202	0.	1.175

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	267	422	548	608	996	424
normalized size	1	1.	0.82	1.29	1.68	1.86	3.05	1.3
time (sec)	N/A	0.579	2.028	0.069	1.013	2.229	7.872	1.155

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	185	274	356	402	571	267
normalized size	1	1.	0.87	1.29	1.67	1.89	2.68	1.25
time (sec)	N/A	0.36	1.101	0.056	0.986	2.05	3.646	1.133

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	104	147	193	225	277	136
normalized size	1	1.	0.94	1.32	1.74	2.03	2.5	1.23
time (sec)	N/A	0.157	0.427	0.047	0.958	1.949	1.351	1.219

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	77	113	94	65
normalized size	1	1.	0.94	1.23	1.6	2.35	1.96	1.35
time (sec)	N/A	0.023	0.098	0.037	0.945	1.889	0.612	1.17

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	196	294	0	680	0	190
normalized size	1	1.	2.	3.	0.	6.94	0.	1.94
time (sec)	N/A	0.273	0.65	0.117	0.	1.988	0.	1.216

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	217	434	0	1416	0	275
normalized size	1	1.	1.75	3.5	0.	11.42	0.	2.22
time (sec)	N/A	0.325	1.315	0.141	0.	2.258	0.	1.271

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	345	2021	0	2151	0	802
normalized size	1	1.	1.96	11.48	0.	12.22	0.	4.56
time (sec)	N/A	0.421	2.632	0.164	0.	2.479	0.	1.297

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	464	437	745	977	844	1865	640
normalized size	1	1.	0.94	1.61	2.11	1.82	4.02	1.38
time (sec)	N/A	0.952	3.143	0.077	1.022	2.587	11.844	1.265

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	296	496	645	574	1129	420
normalized size	1	1.	0.88	1.48	1.92	1.71	3.36	1.25
time (sec)	N/A	0.703	1.526	0.066	0.992	2.265	5.849	1.304

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	160	278	362	343	571	232
normalized size	1	1.	0.96	1.67	2.18	2.07	3.44	1.4
time (sec)	N/A	0.271	0.747	0.051	0.966	2.037	2.224	1.246

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	106	117	154	176	199	119
normalized size	1	1.	1.13	1.24	1.64	1.87	2.12	1.27
time (sec)	N/A	0.061	0.32	0.04	0.977	1.959	0.918	1.257

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	177	713	0	1029	0	424
normalized size	1	1.	1.04	4.17	0.	6.02	0.	2.48
time (sec)	N/A	0.522	0.629	0.143	0.	2.296	0.	1.269

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	192	848	0	1589	0	672
normalized size	1	1.	0.97	4.28	0.	8.03	0.	3.39
time (sec)	N/A	0.581	1.005	0.161	0.	2.565	0.	1.352

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	226	1916	0	3087	0	949
normalized size	1	1.	1.05	8.91	0.	14.36	0.	4.41
time (sec)	N/A	0.623	1.389	0.187	0.	2.767	0.	1.403

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	604	528	1077	1426	1019	2878	764
normalized size	1	1.	0.87	1.78	2.36	1.69	4.76	1.26
time (sec)	N/A	1.486	4.763	0.092	1.082	2.69	22.494	1.348

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	355	725	950	713	1804	513
normalized size	1	1.	0.77	1.57	2.05	1.54	3.9	1.11
time (sec)	N/A	1.128	2.399	0.075	1.026	2.486	11.335	1.326

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	156	414	537	437	960	293
normalized size	1	1.	0.78	2.06	2.67	2.17	4.78	1.46
time (sec)	N/A	0.335	1.064	0.059	0.998	2.185	5.693	1.244

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	117	120	178	231	231	371	157
normalized size	1	0.92	0.94	1.4	1.82	1.82	2.92	1.24
time (sec)	N/A	0.101	0.486	0.046	0.966	1.944	1.963	1.29

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	233	1357	0	1393	0	833
normalized size	1	1.	0.95	5.52	0.	5.66	0.	3.39
time (sec)	N/A	0.895	0.964	0.165	0.	2.563	0.	1.289

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	244	1534	0	2242	0	794
normalized size	1	1.	0.86	5.42	0.	7.92	0.	2.81
time (sec)	N/A	0.941	1.487	0.189	0.	2.868	0.	1.315

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	830	2906	0	3568	0	1331
normalized size	1	1.	2.72	9.53	0.	11.7	0.	4.36
time (sec)	N/A	0.934	3.182	0.217	0.	3.725	0.	1.345

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	788	1110	1517	1062	0	647
normalized size	1	1.	3.58	5.05	6.9	4.83	0.	2.94
time (sec)	N/A	0.361	1.309	0.102	1.574	2.457	0.	1.32

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	141	200	524	818	684	5583	300
normalized size	1	0.99	1.4	3.66	5.72	4.78	39.04	2.1
time (sec)	N/A	0.206	0.46	0.085	1.493	2.302	12.106	1.273

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	126	179	346	354	1244	216
normalized size	1	1.	1.88	2.67	5.16	5.28	18.57	3.22
time (sec)	N/A	0.203	0.468	0.065	1.471	1.974	5.036	1.232

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	79	65	105	166	109	54
normalized size	1	1.	2.26	1.86	3.	4.74	3.11	1.54
time (sec)	N/A	0.049	0.155	0.042	1.438	1.837	2.036	1.223

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	148	176	0	1297	0	153
normalized size	1	1.	1.47	1.74	0.	12.84	0.	1.51
time (sec)	N/A	0.17	0.325	0.114	0.	2.011	0.	1.246

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	209	615	0	3247	0	983
normalized size	1	1.	1.15	3.4	0.	17.94	0.	5.43
time (sec)	N/A	0.349	1.236	0.141	0.	2.456	0.	1.307

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	313	2482	0	7112	0	1017
normalized size	1	1.	1.11	8.77	0.	25.13	0.	3.59
time (sec)	N/A	0.55	1.386	0.165	0.	3.196	0.	1.336

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	547	946	1866	1334	0	667
normalized size	1	1.	2.4	4.15	8.18	5.85	0.	2.93
time (sec)	N/A	0.523	3.516	0.105	1.581	2.207	0.	1.322

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	338	489	1122	863	5358	374
normalized size	1	1.	2.56	3.7	8.5	6.54	40.59	2.83
time (sec)	N/A	0.51	1.635	0.092	1.519	2.159	23.3	1.237

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	180	252	613	491	986	190
normalized size	1	1.	2.12	2.96	7.21	5.78	11.6	2.24
time (sec)	N/A	0.211	0.341	0.077	1.463	1.887	10.634	1.27

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	43	70	289	288	309	92
normalized size	1	1.	0.66	1.08	4.45	4.43	4.75	1.42
time (sec)	N/A	0.052	0.052	0.056	0.978	1.824	4.63	1.235

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	229	327	0	2755	0	350
normalized size	1	1.	1.51	2.15	0.	18.12	0.	2.3
time (sec)	N/A	0.419	0.638	0.131	0.	2.428	0.	1.266

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	313	770	0	6543	0	574
normalized size	1	1.	1.14	2.8	0.	23.79	0.	2.09
time (sec)	N/A	0.672	2.885	0.16	0.	3.029	0.	1.298

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	386	1522	2641	0	10961	0	1274
normalized size	1	1.	3.94	6.84	0.	28.4	0.	3.3
time (sec)	N/A	0.961	6.357	0.172	0.	4.146	0.	1.398

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	366	936	2271	1511	0	805
normalized size	1	1.	1.63	4.16	10.09	6.72	0.	3.58
time (sec)	N/A	0.809	6.057	0.106	1.644	2.315	0.	1.315

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	514	617	1528	1002	0	516
normalized size	1	1.	3.13	3.76	9.32	6.11	0.	3.15
time (sec)	N/A	0.461	0.897	0.098	1.597	2.181	0.	1.218

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	176	151	990	641	1819	301
normalized size	1	1.	1.39	1.19	7.8	5.05	14.32	2.37
time (sec)	N/A	0.227	0.672	0.085	1.04	1.823	23.643	1.201

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	114	522	466	899	176
normalized size	1	1.	0.62	1.12	5.12	4.57	8.81	1.73
time (sec)	N/A	0.075	0.078	0.064	1.014	1.771	11.353	1.292

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	502	606	0	4929	0	780
normalized size	1	1.	2.19	2.65	0.	21.52	0.	3.41
time (sec)	N/A	0.724	1.243	0.138	0.	2.633	0.	1.289

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	1253	1049	0	9643	0	1042
normalized size	1	1.	3.29	2.75	0.	25.31	0.	2.73
time (sec)	N/A	1.081	6.375	0.162	0.	4.056	0.	1.342

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	548	2918	0	16342	0	1717
normalized size	1	1.	1.08	5.74	0.	32.17	0.	3.38
time (sec)	N/A	1.447	4.571	0.193	0.	5.118	0.	1.557

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	305	242	0	1156	0	0
normalized size	1	1.	1.19	0.95	0.	4.52	0.	0.
time (sec)	N/A	0.46	1.273	1.207	0.	2.167	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	176	161	0	761	0	0
normalized size	1	1.	0.92	0.84	0.	3.96	0.	0.
time (sec)	N/A	0.339	0.746	1.049	0.	2.007	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	117	102	0	436	0	0
normalized size	1	1.	0.99	0.86	0.	3.69	0.	0.
time (sec)	N/A	0.249	0.365	0.954	0.	1.999	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	225	0	0
normalized size	1	1.	1.32	0.94	0.	3.63	0.	0.
time (sec)	N/A	0.058	0.125	0.926	0.	1.886	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	1539	0	0
normalized size	1	1.	9.03	1.39	0.	15.39	0.	0.
time (sec)	N/A	0.246	8.785	1.545	0.	9.332	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	2354	0	0
normalized size	1	1.	7.15	2.17	0.	18.68	0.	0.
time (sec)	N/A	0.262	8.72	1.881	0.	10.529	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	967	628	0	3954	0	0
normalized size	1	1.	5.04	3.27	0.	20.59	0.	0.
time (sec)	N/A	0.37	10.064	2.075	0.	16.44	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	390	312	0	1667	0	0
normalized size	1	1.	1.04	0.83	0.	4.46	0.	0.
time (sec)	N/A	0.92	4.593	1.015	0.	1.989	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	267	207	0	1094	0	0
normalized size	1	1.	0.91	0.7	0.	3.72	0.	0.
time (sec)	N/A	0.712	2.248	1.083	0.	1.848	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	144	150	0	660	0	0
normalized size	1	1.	0.87	0.91	0.	4.	0.	0.
time (sec)	N/A	0.316	1.076	0.996	0.	1.782	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	347	0	0
normalized size	1	1.	1.	0.76	0.	3.44	0.	0.
time (sec)	N/A	0.087	0.402	1.089	0.	1.657	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	356	291	0	2086	0	0
normalized size	1	1.	2.33	1.9	0.	13.63	0.	0.
time (sec)	N/A	0.503	3.431	1.621	0.	9.733	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	381	592	0	3245	0	0
normalized size	1	1.	1.99	3.1	0.	16.99	0.	0.
time (sec)	N/A	0.552	4.89	1.839	0.	11.553	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	416	895	0	5040	0	0
normalized size	1	1.	1.88	4.05	0.	22.81	0.	0.
time (sec)	N/A	0.61	5.161	2.233	0.	19.132	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1565	374	0	2237	0	0
normalized size	1	1.	2.93	0.7	0.	4.19	0.	0.
time (sec)	N/A	1.203	6.897	1.157	0.	2.23	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	891	257	0	1500	0	0
normalized size	1	1.	2.08	0.6	0.	3.5	0.	0.
time (sec)	N/A	1.066	6.617	0.928	0.	1.965	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	202	152	0	900	0	0
normalized size	1	1.	0.95	0.72	0.	4.25	0.	0.
time (sec)	N/A	0.368	4.205	1.167	0.	1.769	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	483	0	0
normalized size	1	1.	0.86	0.72	0.	3.5	0.	0.
time (sec)	N/A	0.112	1.523	0.897	0.	1.618	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	450	543	0	2961	0	0
normalized size	1	1.	2.06	2.49	0.	13.58	0.	0.
time (sec)	N/A	0.885	5.869	1.764	0.	17.634	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	460	933	0	4475	0	0
normalized size	1	1.	1.74	3.52	0.	16.89	0.	0.
time (sec)	N/A	0.938	5.893	2.13	0.	19.136	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	504	1587	0	6692	0	0
normalized size	1	1.	1.64	5.15	0.	21.73	0.	0.
time (sec)	N/A	0.972	8.12	2.445	0.	21.893	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	375	610	0	1543	0	2520
normalized size	1	1.	1.32	2.15	0.	5.43	0.	8.87
time (sec)	N/A	1.001	0.876	1.624	0.	1.977	0.	2.126

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	246	396	0	1119	0	1494
normalized size	1	1.	1.23	1.98	0.	5.6	0.	7.47
time (sec)	N/A	0.585	0.528	1.362	0.	1.832	0.	1.812

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	135	232	0	782	0	720
normalized size	1	1.	1.04	1.78	0.	6.02	0.	5.54
time (sec)	N/A	0.27	0.466	1.293	0.	1.839	0.	1.589

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	572	0	297
normalized size	1	1.	1.34	1.62	0.	7.24	0.	3.76
time (sec)	N/A	0.07	0.22	1.024	0.	1.946	0.	1.478

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	238	199	0	1831	0	0
normalized size	1	1.	1.75	1.46	0.	13.46	0.	0.
time (sec)	N/A	0.283	3.052	1.713	0.	10.033	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	374	899	0	4890	0	0
normalized size	1	1.	1.81	4.34	0.	23.62	0.	0.
time (sec)	N/A	0.617	6.791	2.365	0.	29.207	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	847	2275	0	9234	0	0
normalized size	1	1.	2.74	7.36	0.	29.88	0.	0.
time (sec)	N/A	1.054	10.686	3.1	0.	59.847	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	684	1030	0	1901	0	0
normalized size	1	1.	2.42	3.64	0.	6.72	0.	0.
time (sec)	N/A	1.	1.06	1.46	0.	2.032	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	357	694	0	1422	0	0
normalized size	1	1.	1.76	3.42	0.	7.	0.	0.
time (sec)	N/A	0.575	0.735	1.345	0.	1.871	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	246	389	0	1023	0	1077
normalized size	1	1.	1.85	2.92	0.	7.69	0.	8.1
time (sec)	N/A	0.279	0.436	0.986	0.	1.805	0.	2.51

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	755	0	599
normalized size	1	1.	1.72	2.02	0.	8.68	0.	6.89
time (sec)	N/A	0.078	0.194	1.084	0.	1.727	0.	1.91

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	419	624	0	3671	0	0
normalized size	1	1.	2.24	3.34	0.	19.63	0.	0.
time (sec)	N/A	0.59	3.001	1.467	0.	25.483	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	542	2049	0	7567	0	0
normalized size	1	1.	1.86	7.02	0.	25.91	0.	0.
time (sec)	N/A	1.018	9.072	2.48	0.	24.681	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	402	402	1395	4707	0	13168	0	0
normalized size	1	1.	3.47	11.71	0.	32.76	0.	0.
time (sec)	N/A	1.562	13.243	3.293	0.	46.456	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	523	1438	0	2394	0	0
normalized size	1	1.	1.7	4.67	0.	7.77	0.	0.
time (sec)	N/A	1.059	1.787	2.148	0.	2.399	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	544	982	0	1817	0	0
normalized size	1	1.	2.48	4.48	0.	8.3	0.	0.
time (sec)	N/A	0.579	1.133	1.888	0.	2.337	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	267	449	0	1343	0	0
normalized size	1	1.	1.77	2.97	0.	8.89	0.	0.
time (sec)	N/A	0.287	0.767	1.544	0.	2.083	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	1015	0	0
normalized size	1	1.	1.8	2.21	0.	8.06	0.	0.
time (sec)	N/A	0.107	0.366	1.338	0.	2.115	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	550	1418	0	5966	0	0
normalized size	1	1.	2.11	5.43	0.	22.86	0.	0.
time (sec)	N/A	0.984	5.342	2.361	0.	52.802	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	395	395	1318	4092	0	11614	0	0
normalized size	1	1.	3.34	10.36	0.	29.4	0.	0.
time (sec)	N/A	1.536	12.285	3.569	0.	98.914	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	519	519	2103	7322	0	19733	0	0
normalized size	1	1.	4.05	14.11	0.	38.02	0.	0.
time (sec)	N/A	2.152	13.883	5.305	0.	61.256	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	25.496	0.584	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	9.054	0.48	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	4.924	0.299	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.341	8.675	0.714	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	245	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.915	26.581	0.431	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F(-2)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	8.216	0.451	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	244	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.403	5.483	0.346	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	603	0	0	0	0	0
normalized size	1	1.	2.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.47	10.18	0.345	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	351	351	300	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.987	7.649	2.762	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	198	212	0	0	0	0	0
normalized size	1	0.99	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.362	3.405	2.088	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	1.769	0.014	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	473	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.292	7.036	1.326	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	293	293	654	0	0	0	0	0
normalized size	1	1.	2.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.618	5.671	1.797	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	467	467	654	0	0	0	0	0
normalized size	1	1.	1.4	0.	0.	0.	0.	0.
time (sec)	N/A	1.347	6.127	2.198	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	3281	0	0	0	0	0
normalized size	1	1.	11.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.628	8.098	0.355	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.555	12.064	0.346	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0
normalized size	1	1.	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.544	6.316	0.343	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	288	288	672	0	0	0	0	0
normalized size	1	1.	2.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.545	6.385	0.348	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	682	0	0	0	0	0
normalized size	1	1.	2.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.43	6.224	0.451	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	573	0	0	0	0	0
normalized size	1	1.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.483	6.897	0.44	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	9.61	0.434	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	4.762	0.45	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	140	0	0
normalized size	1	1.	1.	0.	0.	3.59	0.	0.
time (sec)	N/A	0.172	0.665	0.589	0.	1.991	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	142	0	0
normalized size	1	1.	1.	0.	0.	3.55	0.	0.
time (sec)	N/A	0.172	0.71	0.566	0.	1.829	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	188	1246	0	2761	0	1048
normalized size	1	1.	0.94	6.26	0.	13.87	0.	5.27
time (sec)	N/A	0.58	1.574	0.156	0.	2.235	0.	1.266

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	840	840	2012	6776582	0	0	0	0
normalized size	1	1.	2.4	8067.36	0.	0.	0.	0.
time (sec)	N/A	3.156	6.744	87.435	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	630	630	1871	3151745	0	0	0	0
normalized size	1	1.	2.97	5002.77	0.	0.	0.	0.
time (sec)	N/A	0.886	10.279	148.394	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	1919	99082	0	0	0	0
normalized size	1	1.	4.6	237.61	0.	0.	0.	0.
time (sec)	N/A	0.537	6.537	1.39	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	544	544	2236	198381	0	0	0	0
normalized size	1	1.	4.11	364.67	0.	0.	0.	0.
time (sec)	N/A	1.377	7.279	2.608	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	858	858	2807	827030	0	0	0	0
normalized size	1	1.	3.27	963.9	0.	0.	0.	0.
time (sec)	N/A	2.616	8.652	14.667	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	17.361	0.46	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [0.2857]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.	33	0.152
2	A	6	5	1.	33	0.152
3	A	5	4	1.	31	0.129
4	A	4	3	1.	33	0.091
5	A	5	3	1.	33	0.091
6	A	6	3	1.	33	0.091
7	A	6	5	1.	35	0.143
8	A	5	5	1.	35	0.143
9	A	4	4	1.	35	0.114
10	A	9	9	1.	35	0.257
11	A	10	10	1.	35	0.286
12	A	9	5	1.	33	0.152
13	A	4	3	1.	34	0.088
14	A	1	1	1.	43	0.023
15	A	1	1	1.	37	0.027

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	6	6	1.	31	0.194
17	A	7	6	1.	34	0.176
18	A	6	6	1.	34	0.176
19	A	5	5	1.08	34	0.147
20	A	4	4	1.	32	0.125
21	A	4	3	1.	34	0.088
22	A	4	4	1.	34	0.118
23	A	4	4	1.	34	0.118
24	A	5	5	1.	34	0.147
25	A	6	5	1.	34	0.147
26	A	8	6	1.	36	0.167
27	A	7	6	1.	36	0.167
28	A	6	5	1.	36	0.139
29	A	5	4	1.	36	0.111
30	A	5	5	1.	34	0.147
31	A	5	5	1.	36	0.139
32	A	5	5	1.	36	0.139
33	A	5	4	1.	36	0.111
34	A	3	3	1.	36	0.083
35	A	4	4	1.	36	0.111
36	A	5	4	1.	36	0.111
37	A	6	4	1.	36	0.111
38	A	9	6	1.	36	0.167
39	A	8	6	1.	36	0.167
40	A	7	5	1.	36	0.139
41	A	6	4	1.	36	0.111
42	A	6	5	1.	36	0.139
43	A	6	6	1.	34	0.176
44	A	6	6	1.	36	0.167
45	A	6	6	1.	36	0.167
46	A	6	5	1.	36	0.139
47	A	6	4	1.	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	3	3	1.	36	0.083
49	A	4	4	1.	36	0.111
50	A	5	4	1.	36	0.111
51	A	6	4	1.	36	0.111
52	A	7	7	1.	36	0.194
53	A	6	6	1.	36	0.167
54	A	5	5	1.	36	0.139
55	A	4	3	1.	34	0.088
56	A	4	4	1.	36	0.111
57	A	4	4	1.	36	0.111
58	A	5	5	1.	36	0.139
59	A	6	5	1.	36	0.139
60	A	8	7	1.	36	0.194
61	A	7	6	1.	36	0.167
62	A	6	6	1.	36	0.167
63	A	5	5	1.	36	0.139
64	A	4	4	1.	34	0.118
65	A	4	4	1.	36	0.111
66	A	4	3	1.	36	0.083
67	A	4	3	1.	36	0.083
68	A	5	4	1.	36	0.111
69	A	6	4	1.	36	0.111
70	A	8	6	1.	36	0.167
71	A	7	6	1.	36	0.167
72	A	6	5	1.	36	0.139
73	A	5	4	1.	36	0.111
74	A	4	4	1.	34	0.118
75	A	5	5	1.	36	0.139
76	A	4	3	1.	36	0.083
77	A	4	3	1.	36	0.083
78	A	4	3	1.	36	0.083
79	A	5	4	1.	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	6	4	1.	36	0.111
81	A	6	4	1.	36	0.111
82	A	5	4	1.	36	0.111
83	A	4	4	1.	36	0.111
84	A	3	3	1.	36	0.083
85	A	5	5	1.	36	0.139
86	A	5	5	1.	36	0.139
87	A	5	5	1.	36	0.139
88	A	6	6	1.	36	0.167
89	A	6	4	1.	38	0.105
90	A	5	4	1.	38	0.105
91	A	4	4	1.	38	0.105
92	A	3	3	1.	38	0.079
93	A	6	5	1.	38	0.132
94	A	6	5	1.	38	0.132
95	A	6	6	1.	38	0.158
96	A	6	5	1.	38	0.132
97	A	7	6	1.	38	0.158
98	A	6	4	1.	38	0.105
99	A	5	4	1.	38	0.105
100	A	4	4	1.	38	0.105
101	A	3	3	1.	38	0.079
102	A	7	5	1.	38	0.132
103	A	7	5	1.	38	0.132
104	A	7	6	1.	38	0.158
105	A	7	6	1.	38	0.158
106	A	7	5	1.	38	0.132
107	A	8	6	1.	38	0.158
108	A	6	4	1.	38	0.105
109	A	5	4	1.	38	0.105
110	A	4	4	1.	38	0.105
111	A	3	3	1.	38	0.079

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	4	4	1.	38	0.105
113	A	5	5	1.	38	0.132
114	A	6	6	1.	38	0.158
115	A	7	4	1.	38	0.105
116	A	6	4	1.	38	0.105
117	A	5	4	1.	38	0.105
118	A	4	4	1.	38	0.105
119	A	3	3	1.	38	0.079
120	A	5	5	1.	38	0.132
121	A	6	6	1.	38	0.158
122	A	7	7	1.	38	0.184
123	A	7	4	1.	38	0.105
124	A	6	4	1.	38	0.105
125	A	5	4	1.	38	0.105
126	A	4	4	1.	38	0.105
127	A	3	3	1.	38	0.079
128	A	6	5	1.	38	0.132
129	A	7	7	1.	38	0.184
130	A	8	7	1.	38	0.184
131	A	3	2	1.	40	0.05
132	A	3	2	1.	40	0.05
133	A	3	2	1.	40	0.05
134	A	3	2	1.	40	0.05
135	A	5	5	1.	40	0.125
136	A	5	5	1.	40	0.125
137	A	3	2	1.	40	0.05
138	A	3	2	1.	40	0.05
139	A	3	3	1.	40	0.075
140	A	3	3	1.	40	0.075
141	A	3	3	1.	40	0.075
142	A	3	2	1.	40	0.05
143	A	5	5	1.	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	5	5	1.	40	0.125
145	A	5	5	1.	40	0.125
146	A	2	2	1.	40	0.05
147	A	3	3	1.	40	0.075
148	A	3	3	1.	40	0.075
149	A	4	3	1.	40	0.075
150	A	4	3	1.	40	0.075
151	A	3	3	1.	40	0.075
152	A	3	2	1.	40	0.05
153	A	6	5	1.	40	0.125
154	A	6	5	1.	40	0.125
155	A	6	6	1.	40	0.15
156	A	6	5	1.	40	0.125
157	A	2	2	1.	40	0.05
158	A	3	3	1.	40	0.075
159	A	4	3	1.	40	0.075
160	A	5	3	1.	40	0.075
161	A	5	3	1.	40	0.075
162	A	4	3	1.	40	0.075
163	A	3	3	1.	40	0.075
164	A	3	2	1.	40	0.05
165	A	7	5	1.	40	0.125
166	A	7	5	1.	40	0.125
167	A	7	6	1.	40	0.15
168	A	7	6	1.	40	0.15
169	A	7	5	1.	40	0.125
170	A	2	2	1.	40	0.05
171	A	3	3	1.	40	0.075
172	A	4	3	1.	40	0.075
173	A	5	3	1.	40	0.075
174	A	6	5	1.	40	0.125
175	A	5	5	1.	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	5	5	1.	40	0.125
177	A	7	4	1.	40	0.1
178	A	3	3	1.	40	0.075
179	A	4	4	1.	40	0.1
180	A	7	5	1.	40	0.125
181	A	6	5	1.	40	0.125
182	A	5	5	1.	40	0.125
183	A	5	5	1.	40	0.125
184	A	3	3	1.	40	0.075
185	A	4	4	1.	40	0.1
186	A	5	4	1.	40	0.1
187	A	8	6	1.	40	0.15
188	A	7	6	1.	40	0.15
189	A	6	6	1.	40	0.15
190	A	5	5	1.	40	0.125
191	A	3	2	1.	40	0.05
192	A	4	4	1.	40	0.1
193	A	5	4	1.	40	0.1
194	A	6	4	1.	40	0.1
195	A	5	5	1.	36	0.139
196	A	5	5	1.	36	0.139
197	A	5	5	1.	36	0.139
198	A	5	5	1.	34	0.147
199	A	3	3	1.	23	0.13
200	A	5	5	1.	36	0.139
201	A	5	5	1.	36	0.139
202	A	5	5	1.	36	0.139
203	A	4	4	1.	38	0.105
204	A	4	4	1.	38	0.105
205	A	4	3	1.	38	0.079
206	A	3	3	1.16	38	0.079
207	A	3	2	1.	38	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	4	4	1.	38	0.105
209	A	4	4	1.	38	0.105
210	A	4	4	1.	38	0.105
211	A	4	3	1.	40	0.075
212	A	3	3	1.	40	0.075
213	A	2	2	1.	40	0.05
214	A	5	5	1.	40	0.125
215	A	5	5	1.	38	0.132
216	A	5	5	1.	40	0.125
217	A	5	5	1.	40	0.125
218	A	2	2	1.	46	0.043
219	A	2	2	1.	45	0.044
220	A	2	2	1.	44	0.045
221	A	2	2	1.	43	0.047
222	A	1	1	1.	47	0.021
223	A	1	1	1.	46	0.022
224	A	13	4	1.	32	0.125
225	A	12	4	1.	32	0.125
226	A	10	5	1.	30	0.167
227	A	5	5	1.	24	0.208
228	A	7	5	1.	30	0.167
229	A	9	7	1.	32	0.219
230	A	7	6	1.	32	0.188
231	A	7	4	1.	32	0.125
232	A	10	5	1.	32	0.156
233	A	12	6	1.	32	0.188
234	A	12	4	1.	32	0.125
235	A	11	6	1.	32	0.188
236	A	9	4	1.	32	0.125
237	A	8	3	1.	32	0.094
238	A	8	3	1.	30	0.1
239	A	3	3	1.	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	9	4	1.	30	0.133
241	A	15	9	1.	32	0.281
242	A	13	7	1.	32	0.219
243	A	15	7	1.	32	0.219
244	A	5	4	1.	33	0.121
245	A	4	4	1.	33	0.121
246	A	3	3	1.	31	0.097
247	A	1	1	1.	21	0.048
248	A	6	6	1.	33	0.182
249	A	6	6	1.	33	0.182
250	A	7	7	1.	33	0.212
251	A	6	5	1.	35	0.143
252	A	5	5	1.	35	0.143
253	A	4	4	1.	33	0.121
254	A	2	2	1.	23	0.087
255	A	7	7	1.	35	0.2
256	A	7	7	1.	35	0.2
257	A	7	7	1.	35	0.2
258	A	7	5	1.	35	0.143
259	A	6	5	1.	35	0.143
260	A	10	8	1.	33	0.242
261	A	8	6	0.92	23	0.261
262	A	8	7	1.	35	0.2
263	A	8	8	1.	35	0.229
264	A	8	7	1.	35	0.2
265	A	3	3	1.	35	0.086
266	A	2	2	0.99	35	0.057
267	A	4	4	1.	33	0.121
268	A	2	2	1.	23	0.087
269	A	5	5	1.	35	0.143
270	A	6	6	1.	35	0.171
271	A	7	6	1.	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	3	2	1.	35	0.057
273	A	5	5	1.	35	0.143
274	A	4	4	1.	33	0.121
275	A	2	2	1.	23	0.087
276	A	6	5	1.	35	0.143
277	A	7	6	1.	35	0.171
278	A	8	6	1.	35	0.171
279	A	6	5	1.	35	0.143
280	A	5	5	1.	35	0.143
281	A	4	4	1.	33	0.121
282	A	3	3	1.	23	0.13
283	A	7	5	1.	35	0.143
284	A	8	6	1.	35	0.171
285	A	9	6	1.	35	0.171
286	A	5	5	1.	37	0.135
287	A	4	4	1.	37	0.108
288	A	4	4	1.	35	0.114
289	A	2	2	1.	25	0.08
290	A	3	3	1.	37	0.081
291	A	3	3	1.	37	0.081
292	A	4	4	1.	37	0.108
293	A	6	6	1.	37	0.162
294	A	5	5	1.	37	0.135
295	A	5	5	1.	35	0.143
296	A	3	3	1.	25	0.12
297	A	4	4	1.	37	0.108
298	A	4	4	1.	37	0.108
299	A	4	4	1.	37	0.108
300	A	7	6	1.	37	0.162
301	A	6	5	1.	37	0.135
302	A	6	5	1.	35	0.143
303	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
304	A	5	4	1.	37	0.108
305	A	5	5	1.	37	0.135
306	A	5	4	1.	37	0.108
307	A	7	6	1.	37	0.162
308	A	6	6	1.	37	0.162
309	A	5	5	1.	35	0.143
310	A	3	3	1.	25	0.12
311	A	5	5	1.	37	0.135
312	A	6	6	1.	37	0.162
313	A	7	6	1.	37	0.162
314	A	7	7	1.	37	0.189
315	A	6	6	1.	37	0.162
316	A	5	5	1.	35	0.143
317	A	3	3	1.	25	0.12
318	A	6	6	1.	37	0.162
319	A	7	7	1.	37	0.189
320	A	8	7	1.	37	0.189
321	A	7	6	1.	37	0.162
322	A	6	6	1.	37	0.162
323	A	5	5	1.	35	0.143
324	A	4	4	1.	25	0.16
325	A	7	6	1.	37	0.162
326	A	8	7	1.	37	0.189
327	A	9	7	1.	37	0.189
328	A	7	4	1.	35	0.114
329	A	8	6	1.	33	0.182
330	A	7	5	1.	35	0.143
331	A	7	4	1.	35	0.114
332	A	11	7	1.	37	0.189
333	A	4	4	1.	37	0.108
334	A	7	7	1.	37	0.189
335	A	7	4	1.	37	0.108

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
336	A	6	6	1.	35	0.171
337	A	5	5	0.99	33	0.152
338	A	3	3	1.	23	0.13
339	A	6	6	1.	35	0.171
340	A	7	7	1.	35	0.2
341	A	8	7	1.	35	0.2
342	A	9	5	1.	37	0.135
343	A	9	5	1.	37	0.135
344	A	9	5	1.	37	0.135
345	A	9	5	1.	37	0.135
346	A	9	5	1.	35	0.143
347	A	7	6	1.	39	0.154
348	A	4	4	1.	36	0.111
349	A	4	4	1.	40	0.1
350	A	1	1	1.	55	0.018
351	A	1	1	1.	51	0.02
352	A	6	6	1.	35	0.171
353	A	7	7	1.	39	0.18
354	A	5	5	1.	39	0.128
355	A	3	3	1.	39	0.077
356	A	4	4	1.	39	0.103
357	A	5	5	1.	39	0.128
358	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=373

$$\frac{a^3(A(4n+11)+B(4n+9))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3\left(A\left(4n^2+21n+20\right)+B\right.}{}$$

```
[Out] -((a^3*(B*(27 + 14*n + 2*n^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[
e + f*x])^(1 + n))/(d*f*(2 + n)*(3 + n)*(4 + n))) + (a^3*(B*(15 + 19*n + 4*
n^2) + A*(20 + 21*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)
*(4 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e +
f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e
+ f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2]) - (a*B*Cos[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*(4 + n)) - ((A
*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^3 + a^3*Sin[
e + f*x]))/(d*f*(3 + n)*(4 + n))
```

Rubi [A] time = 0.840783, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2968, 3023, 2748, 2643}

$$\frac{a^3(A(4n+11)+B(4n+9))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^3\left(A\left(4n^2+21n+20\right)+B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a^3*(B*(27 + 14*n + 2*n^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[
e + f*x])^(1 + n))/(d*f*(2 + n)*(3 + n)*(4 + n))) + (a^3*(B*(15 + 19*n + 4*
n^2) + A*(20 + 21*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)
*(4 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e +
f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e
+ f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2]) - (a*B*Cos[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*(4 + n)) - ((A
*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^3 + a^3*Sin[
e + f*x]))/(d*f*(3 + n)*(4 + n))
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)} +$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)} -$$

$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)} -$$

$$= -\frac{a^3\left(B\left(27+14n+2n^2\right)+A\left(28+15n+2n^2\right)\right)\cos(e+}{df(2+n)(3+n)(4+n)}$$

$$= -\frac{a^3\left(B\left(27+14n+2n^2\right)+A\left(28+15n+2n^2\right)\right)\cos(e+}{df(2+n)(3+n)(4+n)}$$

$$= -\frac{a^3\left(B\left(27+14n+2n^2\right)+A\left(28+15n+2n^2\right)\right)\cos(e+}{df(2+n)(3+n)(4+n)}$$

Mathematica [A] time = 2.26348, size = 248, normalized size = 0.66

$$\frac{a^3\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(\sin(e+fx)\left(\frac{(3A+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{n+2}+\sin(e+fx)\left(\frac{3(A+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\mathrm{s}\right.}{n+3}\right.\right.\right.}{f\sqrt{\cos^2(e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((3*A + B)
```

```
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4 + n)
/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (5 + n)
/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n))))))/(f*Sqrt[Cos[e + f
*x]^2])
```

Maple [F] time = 3.333, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{3}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(Ba^{3}\cos\left(fx+e\right)^{4}-(3\,A+5\,B)a^{3}\cos\left(fx+e\right)^{2}+4\,(A+B)a^{3}-\left((A+3\,B)a^{3}\cos\left(fx+e\right)^{2}-4\,(A+B)a^{3}\right)\sin\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*sin(
f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x
)
```

3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=277

$$\frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^2(A(2n+3)+2B(n+1))\cos}{df}$$

```
[Out] -((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*
(2 + n)*(3 + n))) + (a^2*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeom
etric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n
))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) + B*(5 +
2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x
]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2])
 - (B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))
```

Rubi [A] time = 0.49217, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2976, 2968, 3023, 2748, 2643}

$$\frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}+\frac{a^2(A(2n+3)+2B(n+1))\cos}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```


```
[Out] -((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*
(2 + n)*(3 + n))) + (a^2*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeom
etric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n
))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) + B*(5 +
2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x
]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*(3 + n)*Sqrt[Cos[e + f*x]^2])
 - (B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)} +$$

$$= -\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)} +$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} -$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} -$$

$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)} +$$

Mathematica [A] time = 1.50093, size = 204, normalized size = 0.74

$$\frac{a^2\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(\sin(e+fx)\left(\frac{(2A+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{n+2}+\sin(e+fx)\left(\frac{(A+2B)\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\sin\right.}{n+3}\right.\right.\right.}{f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((2*A + B)
*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Si
n[e + f*x]*(((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e +
 f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Sin[e +
 f*x]^2]*Sin[e + f*x])/(4 + n)))))/(f*Sqrt[Cos[e + f*x]^2])
```

Maple [F] time = 2.673, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(a+a\sin\left(fx+e\right)\right)^2\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)\right)^n\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x
)
```

3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=191

$$\frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin(e+f}{df(n+1)(n+2)\sqrt{c}}$$

```
[Out] -((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + (a*(B*(1 + n
) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)
/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*Sqrt[Cos[e + f
*x]^2])
```

Rubi [A] time = 0.217322, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2968, 3023, 2748, 2643}

$$\frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{d^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin(e+f}{df(n+1)(n+2)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + (a*(B*(1 + n
) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n)*(2 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)
/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d^2*f*(2 + n)*Sqrt[Cos[e + f
*x]^2])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (d\sin(e+fx))^n\left(aA+(aA+aB)\sin(e+fx)+aB\sin^2(\right.$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{\int (d\sin(e+fx))^n(ad}{}$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{(a(A+B))\int (d\sin(e}{d}$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)} + \frac{a(B(1+n)+A(2+n}{}$$

Mathematica [C] time = 3.77841, size = 392, normalized size = 2.05

$$-\frac{a2^{-n-2}e^{ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n(\sin(e+fx)+1)\sin^{-n}(e+fx)(d\sin(e+fx))^n\left(\frac{2(A+B)e^{-i(e+f(n+1)}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(-2 - n)*a*E^(I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e + f*x))))/E^(I*(e + f*
x)))^n*((2*(A + B)*Hypergeometric2F1[(-1 - n)/2, -n, (1 - n)/2, E^((2*I)*(e
 + f*x))])/(E^(I*(e + f*(1 + n)*x))*(1 + n)) - (2*(A + B)*E^(I*(e - f*(-1 +
 n)*x))*Hypergeometric2F1[(1 - n)/2, -n, (3 - n)/2, E^((2*I)*(e + f*x))])/(
-1 + n) + I*((B*Hypergeometric2F1[-1 - n/2, -n, -n/2, E^((2*I)*(e + f*x))])
/(E^(I*(2*e + f*(2 + n)*x))*(2 + n)) + (B*E^((2*I)*(e + f*x))*n*Hypergeomet
ric2F1[1 - n/2, -n, 2 - n/2, E^((2*I)*(e + f*x))] - 2*(2*A + B)*(-2 + n)*Hy
pergeometric2F1[-n, -n/2, 1 - n/2, E^((2*I)*(e + f*x))])/(E^(I*f*n*x)*(-2 +
 n)*n)))*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x]))/((1 - E^((2*I)*(e + f*x)))^
n*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[e + f*x]^n))
```

Maple [F] time = 1.806, size = 0, normalized size = 0.

$$\int \left(d \sin \left(fx+e\right)\right)^{n} \left(a+a \sin \left(fx+e\right)\right) \left(A+B \sin \left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B \sin \left(fx+e\right)+A\right)\left(a \sin \left(fx+e\right)+a\right) \left(d \sin \left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(Ba \cos \left(fx+e\right)^{2}-(A+B)a \sin \left(fx+e\right)-(A+B)a\right) \left(d \sin \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=202

$$\frac{(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{ad^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{(-An+Bn+B)\cos(e+fx)(d\sin(e+f}{adf(n+1)\sqrt{\mathrm{co}}}$$

```
[Out] ((B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(a*d^2*f*(2 + n)*Sqrt[Cos[e
 + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*S
in[e + f*x]))
```

Rubi [A] time = 0.223897, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2978, 2748, 2643}

$$\frac{(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{ad^2f(n+2)\sqrt{\cos^2(e+fx)}}+\frac{(-An+Bn+B)\cos(e+fx)(d\sin(e+f}{adf(n+1)\sqrt{\mathrm{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(a*d^2*f*(2 + n)*Sqrt[Cos[e
 + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*S
in[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{\int (d\sin(e+fx))^n(ad(B-An+Bn)}{a}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))} + \frac{((A-B)(1+n))\int (d\sin(e+fx))^{1+}}{ad}$$

$$= \frac{(B-An+Bn)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{adf(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.866731, size = 157, normalized size = 0.78

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(\frac{(n+1)(A-B)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{(n+2)\sqrt{\cos^2(e+fx)}} + \frac{(-An+Bn+B)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{(n+1)\sqrt{\cos^2(e+fx)}} + \frac{}{\sin}\right.}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(((B - A*n + B*n)*Hypergeomet
ric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*Sqrt[Cos[e + f*
```

```
x]^2]) + ((A - B)*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[
e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[Cos[e + f*x]^2]) + (A - B)/(1 + Sin
[e + f*x])))/(a*f)
```

Maple [F] time = 1.033, size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=279

$$\frac{(n+1)(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{3a^2d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n(-2An+A+2B(n+1)}{}$$

```
[Out] -(n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + n)*S
qrt[Cos[e + f*x]^2]) + ((1 + n)*(B + 2*A*(1 - n) + 2*B*n)*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(
2 + n))/(3*a^2*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((B + 2*A*(1 - n) + 2*
B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + Sin[e + f*x]))
+ ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f*x
])^2)
```

Rubi [A] time = 0.4879, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2978, 2748, 2643}

$$\frac{(n+1)(2A(1-n)+2Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{3a^2d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n(-2An+A+2B(n+1)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```



```
[Out] -(n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + n)*S
qrt[Cos[e + f*x]^2]) + ((1 + n)*(B + 2*A*(1 - n) + 2*B*n)*Cos[e + f*x]*Hype
rgeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(
2 + n))/(3*a^2*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((B + 2*A*(1 - n) + 2*
B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*a^2*d*f*(1 + Sin[e + f*x]))
+ ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f*x
])^2)
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2} + \frac{\int \frac{(d\sin(e+fx))^n(ad(2A+B-An+Bn)+a(A-B)}{a+a\sin(e+fx)}}{3a^2d}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos(e+fx)(}{3df(a+a\sin}$$

$$= \frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))} + \frac{(A-B)\cos(e+fx)(}{3df(a+a\sin}$$

$$= -\frac{n(A-2An+2B(1+n))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e+fx)\right)(d\sin(}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 1.27717, size = 212, normalized size = 0.76

$$\frac{\sin(e+fx)\cos(e+fx)(d\sin(e+fx))^n\left(-\frac{n(-2An+A+2B(n+1))\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)}{(n+1)\sqrt{\cos^2(e+fx)}} + \frac{(n+1)(-2A(n-1)+2Bn+B)\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\right.}{(n+2)\sqrt{\cos^2(e+fx)}}\right.}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(-((n*(A - 2*A*n + 2*B*(1 + n
))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*S
qrt[Cos[e + f*x]^2])) + ((1 + n)*(B - 2*A*(-1 + n) + 2*B*n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[C
os[e + f*x]^2]) + (A - B)/(1 + Sin[e + f*x])^2 + ((-A + B)*n)/(1 + Sin[e +
f*x]) + (2*A + B - A*n + B*n)/(1 + Sin[e + f*x])))/(3*a^2*f)
```

Maple [F] time = 1.539, size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{\left(a+a\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a
^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x
)
```

3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=362

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{15a^3d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n\left(A\left(4n^2-9n\right.\right.}{}$$

```
[Out] -(n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(15*
a^3*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((1 - n)*(1 + n)*(7*A + 3*B - 4*A*n
 + 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
 f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(15*a^3*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e +
 f*x])^3) + ((A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(
15*a*d*f*(a + a*Sin[e + f*x])^2) + ((1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*Cos
[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*d*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.846187, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2978, 2748, 2643}

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{15a^3d^2f(n+2)\sqrt{\cos^2(e+fx)}}-\frac{n\left(A\left(4n^2-9n\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] -(n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(15*
a^3*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((1 - n)*(1 + n)*(7*A + 3*B - 4*A*n
 + 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
 f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(15*a^3*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^
2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e +
 f*x])^3) + ((A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(
15*a*d*f*(a + a*Sin[e + f*x])^2) + ((1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*Cos
[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*d*f*(a^3 + a^3*Sin[e + f*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{\int \frac{(d\sin(e+fx))^n(ad(4A+B-An+Bn)-a(A-B)}{(a+a\sin(e+fx))^2}}{5a^2d}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin}{15adf(a+a\sin(e+fx)}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin}{15adf(a+a\sin(e+fx)}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3} + \frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin}{15adf(a+a\sin(e+fx)}$$

$$= -\frac{n\left(B\left(3-n-4n^2\right)+A\left(2-9n+4n^2\right)\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(e}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 4.42434, size = 260, normalized size = 0.72

$$\frac{(d\sin(e+fx))^n\left(\frac{2\sin(e+fx)\cos(e+fx)\left(n\left(A\left(-4n^2+9n-2\right)+B\left(4n^2+n-3\right)\right){}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(e+fx)\right)+\frac{(n-1)(n+1)^2(A(4n-7)-B(4n+3))\sin(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\right.}{n+2}\right.}{(n+1)\sqrt{\cos^2(e+fx)}}\right.}{30a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] ((d*Sin[e + f*x])^n*((2*Cos[e + f*x]*Sin[e + f*x]*(n*(A*(-2 + 9*n - 4*n^2)
+ B*(-3 + n + 4*n^2))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e +
f*x]^2] + ((-1 + n)*(1 + n)^2*(A*(-7 + 4*n) - B*(3 + 4*n))*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(2 + n)))/((1 +
n)*Sqrt[Cos[e + f*x]^2]) + (3*(A - B)*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])^
3 + ((A*(5 - 2*n) + 2*B*n)*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])^2 + ((-1 +
n)*(A*(-7 + 4*n) - B*(3 + 4*n))*Sin[2*(e + f*x)])/(1 + Sin[e + f*x])))/(30*
a^3*f)
```

Maple [F] time = 1.724, size = 0, normalized size = 0.

$$\int \frac{\left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 - 4
*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x
)
```

3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=336

$$-\frac{2a^3\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1}{f(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a^3*(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 +
32*n^3))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*
Sin[e + f*x])^n)/(f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a
*Sin[e + f*x]]) - (2*a^3*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*
Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*
(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n)*(7 + 2*n)
) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2)
)/(d*f*(7 + 2*n))
```

Rubi [A] time = 0.871863, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2776, 67, 65}

$$-\frac{2a^3\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1}{f(2n+3)(2n+5)(2n+7)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a^3*(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 +
32*n^3))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*
Sin[e + f*x])^n)/(f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a
*Sin[e + f*x]]) - (2*a^3*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*
Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 + 2*n)*(7 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*
(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n)*(7 + 2*n)
) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2)
)/(d*f*(7 + 2*n))
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)}$$

$$= -\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))}{df(5+2n)(7+2n)}$$

$$= -\frac{2a^3\left(2B\left(35+23n+4n^2\right)+A\left(77+50n+8n^2\right)\right)\cos(}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin}}$$

$$= -\frac{2a^3\left(2B\left(35+23n+4n^2\right)+A\left(77+50n+8n^2\right)\right)\cos(}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin}}$$

$$= -\frac{2a^3\left(2B\left(35+23n+4n^2\right)+A\left(77+50n+8n^2\right)\right)\cos(}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin}}$$

$$= -\frac{2a^3\left(2B\left(115+203n+104n^2+16n^3\right)+A\left(301+478n\right.\right.}{f(3}$$

Mathematica [A] time = 18.2937, size = 596, normalized size = 1.77

$$\frac{2^{n+1}\tan\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)(a(\sin(e+fx)+1))^{5/2}\sin^{-n}(e+fx)\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)+1}\right)^n\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)^n(d\sin}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((5*A +
 2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2]
)/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 + n)/2, 9
/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((5*(3*
```

```
A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e + f*x)/2]
^2])/(4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1[9/2 + n,
(5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + Tan[(e + f*x)/2]*(((1
1*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -Tan[(e + f*x)
/2]^2])/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 + n)/2, (9 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(7 + n))))))))/(f*Sqrt[Sec[(e +
 f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]^n)
```

Maple [F] time = 0.499, size = 0, normalized size = 0.

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \left(a+a \sin \left(f x+e\right)\right)^{\frac{5}{2}} \left(A+B \sin \left(f x+e\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B \sin \left(f x+e\right)+A\right) \left(a \sin \left(f x+e\right)+a\right)^{\frac{5}{2}} \left(d \sin \left(f x+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left((A+2\,B)a^{2} \cos \left(fx+e\right)^{2}-2\,(A+B)a^{2}+\left(Ba^{2} \cos \left(fx+e\right)^{2}-2\,(A+B)a^{2}\right) \sin \left(fx+e\right)\right)\sqrt{a \sin \left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e
))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.8 $\int (d \sin(e+fx))^n (a+a\sin(e+fx))^{3/2} (A+B\sin(e+fx))\,dx$

Optimal. Leaf size=229

$$-\frac{2a^2\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx\right.}{f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-2*a^2*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2))*Cos[e + f*x]*Hyper
geometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*
n)*(5 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(3 + n)
 + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 +
2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n))
```

Rubi [A] time = 0.493589, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2981, 2776, 67, 65}

$$-\frac{2a^2\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx\right.}{f(2n+3)(2n+5)\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```



```
[Out] (-2*a^2*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2))*Cos[e + f*x]*Hyper
geometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*
n)*(5 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(2*B*(3 + n)
 + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*(5 +
2*n)*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
n)*Sqrt[a + a*Sin[e + f*x]])/(d*f*(5 + 2*n))
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a^2\left(2B\left(9+13n+4n^2\right)+A\left(25+30n+8n^2\right)\right)\cos(}{f(3+2n)(5}$$

Mathematica [B] time = 15.4048, size = 478, normalized size = 2.09

$$\frac{2^{n+1}\tan\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)(a(\sin(e+fx)+1))^{3/2}\sin^{-n}(e+fx)\left(\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)+1}\right)^n\left(\tan^2\left(\frac{1}{2}(e+fx)\right)+1\right)^n(d\,s}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
 + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Tan[(
e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[
(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e + f*x
)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2, -Tan[
(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2
F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2])/(4 + n) + Tan[(e +
f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n)/2, -Tan
[(e + f*x)/2]^2])/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)/2, (8 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(6 + n)))))))/(f*Sqrt[Sec[(e +
f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]^n)
```

Maple [F] time = 0.417, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{n} \left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}} \left(A+B\sin\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(d\sin\left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^
n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.9 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=137

$$-\frac{2a(A(2n+3)+2B(n+1))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(\ldots}{df(2n+3)\sqrt{a\sin\ldots}}$$

```
[Out] (-2*a*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3
/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[
a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(
3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.213069, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2981, 2776, 67, 65}

$$-\frac{2a(A(2n+3)+2B(n+1))\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(\ldots}{df(2n+3)\sqrt{a\sin\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3
/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[
a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(
3 + 2*n)*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \left(A+\frac{2B(1+n)}{3+2n}\right)\int$$

$$= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2\left(A+\frac{2B(1+n)}{3+2n}\right)\right.}{f\sqrt{a}}$$

$$= -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a^2\left(A+\frac{2B(1+n)}{3+2n}\right)\right.}{}$$

$$= -\frac{2a\left(A+\frac{2B(1+n)}{3+2n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+f\right.}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 65.8471, size = 409, normalized size = 2.99

$$-\frac{(1+i)2^{-n-2}e^{ifnx-\frac{3ie}{2}}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^{n}\sqrt{a(\sin(e+fx)+1)}\sin^{-n}(e+fx)(d\sin(e+fx))^{n}\left(2a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),
x]
```

```
[Out] ((-1 - I)*2^(-2 - n)*E^(((-3*I)/2)*e + I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e +
f*x))))/E^(I*(e + f*x)))^n*((2*B*Hypergeometric2F1[(-3 - 2*n)/4, -n, (1 - 2
*n)/4, E^((2*I)*(e + f*x))])/(E^((I/2)*f*(3 + 2*n)*x)*f*(3 + 2*n)) + 2*E^(I
*e)*(((-I)*(2*A + B)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E^((2
*I)*(e + f*x))])/(E^((I/2)*f*(1 + 2*n)*x)*(f + 2*f*n)) + (E^((I/2)*(2*e + f
*(1 - 2*n)*x))*(-((2*A + B)*(-3 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4, -n, (
5 - 2*n)/4, E^((2*I)*(e + f*x))]) + I*B*E^(I*(e + f*x))*(-1 + 2*n)*Hypergeo
metric2F1[(3 - 2*n)/4, -n, (7 - 2*n)/4, E^((2*I)*(e + f*x))]))/(f*(-3 + 2*n
)*(-1 + 2*n))))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])])/((1 - E^((2*
I)*(e + f*x)))^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*x]^n)
```

Maple [F] time = 0.458, size = 0, normalized size = 0.

$$\int \left(d \sin \left(f x + e\right)\right)^{n} \sqrt{a + a \sin \left(f x + e\right)}\, \left(A + B \sin \left(f x + e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B \sin \left(f x + e\right) + A\right) \sqrt{a \sin \left(f x + e\right) + a} \left(d \sin \left(f x + e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)
), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=152

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\,\mathrm{s}}{}$$

```
[Out] -(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2
]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x
]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*
(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.396167, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {2987, 2787, 2786, 2785, 130, 429, 2776, 67, 65}

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```


```
[Out] -(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2
]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x
]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*
(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !In
tegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = (A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int(d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}\,d}{a}$$

$$= \frac{\left((A-B)\sqrt{1+\sin(e+fx)}\right)\int \frac{(d\sin(e+fx))^n}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} + \frac{(aB\cos(e+fx))\,\text{Subst}\left(\int \frac{(}{\sqrt{}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a}}$$

$$= \frac{\left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n\sqrt{1+\sin(e+fx)}\right)\int \frac{\sin^n(e+fx)}{\sqrt{1+\sin(e+fx)}}\,dx}{\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{2B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};1-\sin(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 4.70877, size = 250, normalized size = 1.64

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}\sin^n(e+fx)\left(-\sin^2(e+fx)\right)^{-n}\left(1-\frac{1}{\sin(e+fx)+1}\right)^{-n}(d\sin(e+fx))^n\left(4(A-B)\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]*
(4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 +
 Sin[e + f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e
+ f*x])] - (A + B)*(1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2,
1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1))^n)
)/(4*a*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 + Sin[e
+ f*x])^(-1))^n)
```

Maple [F] time = 0.383, size = 0, normalized size = 0.

$$\int \left(d \sin \left(f x+e\right)\right)^{n} \left(A+B \sin \left(f x+e\right)\right) \frac{1}{\sqrt{a+a \sin \left(f x+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \sin \left(f x+e\right)+A\right) \left(d \sin \left(f x+e\right)\right)^{n}}{\sqrt{a \sin \left(f x+e\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=226

$$-\frac{(-4An+A+B(4n+3))\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{4af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x])
^(3/2)) - ((A - 4*A*n + B*(3 + 4*n))*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e +
f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(4*a*f*Sin[e +
 f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*(1 + 2*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e
 + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.674921, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2978, 2987, 2787, 2786, 2785, 130, 429, 2776, 67, 65}

$$-\frac{(-4An+A+B(4n+3))\cos(e+fx)\sin^{-n}(e+fx)F_1\left(\frac{1}{2};-n,1;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{4af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x])
^(3/2)) - ((A - 4*A*n + B*(3 + 4*n))*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e +
f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(4*a*f*Sin[e +
 f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*(1 + 2*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e
 + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(d\sin(e+fx))^n\left(ad(A+B-An+Bn)+\frac{1}{2}a(A-B\right.}{\sqrt{a+a\sin(e+fx)}}}{2a^2d}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{((A-B)(1+2n))\int(d\sin(e+fx))^n}{4a^2}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+\right.\right.}{2a^3}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A-B)d(1+2n)+a^2d(A+\right.\right.}{}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\right.}{2a}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)(1+2n)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\right.}{2a}$$

$$= \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}} - \frac{(A+3B-4An+4Bn)F_1\left(\frac{1}{2};-n,1;\right.}{}$$

Mathematica [B] time = 13.1029, size = 523, normalized size = 2.31

$$\frac{\sec(e+fx)(d\sin(e+fx))^n\left(A\left(a^2\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)^2(-\sin(e+fx))^{-n}F_1\left(1;\frac{1}{2},-n;2;\frac{1}{2}(\sin(e+fx)+1)\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1/
2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*
(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Si
n[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Si
n[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/
```

```
2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e +
 f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)) + A*((a^2*AppellF1
[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e +
f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Sin[e + f*x
])/(1 + Sin[e + f*x])]*(1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n,
-1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 +
 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[
e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^
(-1))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^{n}\left(A+B\sin\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/
(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sin\left(e+fx\right)\right)^{n}\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(
3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2
), x)
```

3.12 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=221

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1\right.}{f}$$

```
[Out] -((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Si
n[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*Appell
F1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f
*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/(f*Sin[e + f*x]^n)
```

Rubi [A] time = 0.453302, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2987, 2787, 2786, 2785, 133}

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^nF_1\left(\frac{1}{2};-n,\frac{1}{2}-m;\frac{3}{2};1\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Si
n[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*Appell
F1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f
*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m
)/(f*Sin[e + f*x]^n)
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

```
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = (A-B)\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx + \frac{B\int$$

$$= \left((A-B)(1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (d\,\text{s}$$

$$= \left((A-B)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\sin(e+fx))\right.$$

$$= -\frac{\left((A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n(1+\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+m}BF_1\left(\frac{1}{2};-n,-\frac{1}{2}-m;\frac{3}{2};1-\sin(e+fx),\frac{1}{2}(1-\sin\right.}{}$$

Mathematica [B] time = 22.213, size = 5918, normalized size = 26.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 4.251, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n \left(a+a\sin\left(fx+e\right)\right)^m \left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^m \left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] Exception raised: AttributeError
```

3.13 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=114

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},\frac{1}{2}-m;n+2;\sin(e\right.}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e +
 f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e +
f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

Rubi [A] time = 0.155902, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {3008, 135, 133}

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},\frac{1}{2}-m;n+2;\sin(e\right.}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e +
 f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e +
f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
```

```
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subs}}{f}$$

$$= \frac{\left(\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subs}}{f\sqrt{1-\sin(e}}$$

$$= \frac{\left(\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}(a-a\sin(e+fx))(a+\right.}{f\surd}$$

$$= \frac{F_1\left(1+n;-\frac{1}{2},\frac{1}{2}-m;2+n;\sin(e+fx),-\sin(e+fx)\right)\text{se}}{}$$

Mathematica [F] time = 10.8855, size = 0, normalized size = 0.

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m, x
]
```

Maple [F] time = 4.162, size = 0, normalized size = 0.

$$\int \left(d\sin\left(fx+e\right)\right)^n\left(a-a\sin\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

Rubi [A] time = 0.119467, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2974}

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c +
 d*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(c+dx))}{d}$$

Mathematica [B] time = 1.5116, size = 107, normalized size = 2.89

$$-\frac{2^n \sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)(-\sin(c+dx)+\cos(c+dx)+1)(a(\sin(c+dx)+1))^{-n-2}\left(\left(\sin\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*S
in[c + d*x]),x]
```

```
[Out] -((2^n*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/4]*(-Sin[(c + d*x)/4] + Sin[(3*(c + d*x))/4]))^n*(1 + Cos[c + d*x] - Sin[
c + d*x])*(a*(1 + Sin[c + d*x]))^(-2 - n))/d)
```

Maple [F] time = 0.546, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^n (a+a\sin(dx+c))^{-2-n} (-1-n-(-2-n)\sin(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int ((n+2)\sin(dx+c)-n-1)(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="maxima")
```

```
[Out] integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin(
d*x + c)^n, x)
```

Fricas [A] time = 1.5027, size = 101, normalized size = 2.73

$$-\frac{(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\cos(dx+c)\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x
)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int((n+2)\sin(dx+c)-n-1)(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="giac")
```

```
[Out] integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin(
d*x + c)^n, x)
```

3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$

Optimal. Leaf size=35

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

Rubi [A] time = 0.0944697, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2974}

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),x
]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx=-\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+dx))^m}{d}$$

Mathematica [A] time = 0.362636, size = 35, normalized size = 1.

$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a(\sin(c+dx)+1))^m}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d
*x]),x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a*(1 + Sin[c + d*x]))^m)/d)
```

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int (\sin(dx+c))^{-2-m}(a+a\sin(dx+c))^m(1+m-m\sin(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (m\sin(dx+c)-m-1)(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="maxima")
```

```
[Out] -integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m
 - 2), x)
```

Fricas [A] time = 1.46566, size = 101, normalized size = 2.89

$$-\frac{(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\cos(dx+c)\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="fricas")
```

```
[Out] -(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2)*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(m \sin(dx+c) - m - 1)(a \sin(dx+c) + a)^m \sin(dx+c)^{-m-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate(-(m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m
 - 2), x)
```

3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=153

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{x(Ab-2aB)}{b^3}-\frac{B\cos(e}{b^2}$$

```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f) - (
B*Cos[e + f*x])/(b^2*f) + (a^2*(A*b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```

Rubi [A] time = 0.393458, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2988, 3023, 2735, 2660, 618, 204}

$$-\frac{2a\left(a^2Ab-2a^3B+3ab^2B-2Ab^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3f\left(a^2-b^2\right)^{3/2}}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{x(Ab-2aB)}{b^3}-\frac{B\cos(e}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```


```
[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcT
an[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*f) - (
B*Cos[e + f*x])/(b^2*f) + (a^2*(A*b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f
*(a + b*Sin[e + f*x]))
```


Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
```

```
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{ab(Ab-aB)+\left(a^2-b^2\right)(Ab-aB)\sin(e+fx)+b\left(a^2-b^2\right)B\sin^{2}}{a+b\sin(e+fx)}}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\int \frac{ab^2(Ab-aB)+b\left(a^2-b^2\right)(Ab-2aB}{a+b\sin(e+fx)}}{b^3\left(a^2-b^2\right)}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(a\left(a^2Ab-\right.\right.}{}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} - \frac{\left(2a\left(a^2Ab-\right.\right.}{}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))} + \frac{\left(4a\left(a^2Ab-\right.\right.}{}$$

$$= \frac{(Ab-2aB)x}{b^3} - \frac{2a\left(a^2Ab-2Ab^3-2a^3B+3ab^2B\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f} - \frac{B\,\mathrm{c}}{}$$

Mathematica [A] time = 0.870006, size = 147, normalized size = 0.96

$$\frac{\frac{2a\left(-a^2Ab+2a^3B-3ab^2B+2Ab^3\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(e+fx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a^2b(Ab-aB)\cos(e+fx)}{(a-b)(a+b)(a+b\sin(e+fx))} + (e+fx)(Ab-2aB) - bB\cos(e+fx)}{b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*B
*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Si
n[e + f*x])))/(b^3*f)
```

Maple [B] time = 0.106, size = 493, normalized size = 3.2

$$-2\frac{B}{b^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}+2\frac{A\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{b^2f}-4\frac{B\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)a}{b^3f}+2\frac{}{f\left(\left(\tan\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x)
```

```
[Out] -2/f/b^2*B/(1+tan(1/2*f*x+1/2*e)^2)+2/f/b^2*A*arctan(tan(1/2*f*x+1/2*e))-4/
f/b^3*B*arctan(tan(1/2*f*x+1/2*e))*a+2/f*a/(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/
2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*A-2/f*a^2/b/(tan(1/2*f*x+1/2
*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)*B+2/f*a^2/b/
(tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*A-2/f*a^3/b^2/(
tan(1/2*f*x+1/2*e)^2*a+2*tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*B-2/f*a^3/b^2/(a
^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2))*A+4/
f*a/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^(1/2)
)*A+4/f*a^4/b^3/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e)+2*b)/(a^
2-b^2)^(1/2))*B-6/f*a^2/b/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e
)+2*b)/(a^2-b^2)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.83895, size = 1729, normalized size = 11.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a
*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A
*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*
a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x +
 e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e)^2 -
2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 +
 A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^
3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6
)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) +
 (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A
*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 +
2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e
))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x +
e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x
+ e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*
b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((
a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21139, size = 501, normalized size = 3.27

$$\frac{\frac{2\left(2\,Ba^4-Aa^3b-3\,Ba^2b^2+2\,Aab^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{a^2-b^2}}-\frac{2\left(Ba^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Aab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\cdots\right.}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\cdots\right.}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^
2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*b^2
*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(1/2*
f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*x + 1
/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) + 2*B*a
^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*
e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(a^2*b^2
- b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f
```

3.17 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=182

$$\frac{7ac^4(2A-B)\cos^3(e+fx)}{24f}+\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f}+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+f\right.}{40f}$$

```
[Out] (7*a*(2*A - B)*c^4*x)/16 + (7*a*(2*A - B)*c^4*Cos[e + f*x]^3)/(24*f) + (7*a
*(2*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a*B*c*Cos[e + f*x]^3*(c
 - c*Sin[e + f*x])^3)/(6*f) + (a*(2*A - B)*Cos[e + f*x]^3*(c^2 - c^2*Sin[e
+ f*x])^2)/(10*f) + (7*a*(2*A - B)*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))
/(40*f)
```

Rubi [A] time = 0.295074, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{7ac^4(2A-B)\cos^3(e+fx)}{24f}+\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f}+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+f\right.}{40f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```


```
[Out] (7*a*(2*A - B)*c^4*x)/16 + (7*a*(2*A - B)*c^4*Cos[e + f*x]^3)/(24*f) + (7*a
*(2*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) - (a*B*c*Cos[e + f*x]^3*(c
 - c*Sin[e + f*x])^3)/(6*f) + (a*(2*A - B)*Cos[e + f*x]^3*(c^2 - c^2*Sin[e
+ f*x])^2)/(10*f) + (7*a*(2*A - B)*Cos[e + f*x]^3*(c^4 - c^4*Sin[e + f*x]))
/(40*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{1}{2}(a(2A-B)c)$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{a(2A-B)\cos^3}{}$$

$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f} + \frac{a(2A-B)\cos^3}{}$$

$$= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} - \frac{aBc\cos^3(e+fx)(c-c\sin}{6f}$$

$$= \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} + \frac{7a(2A-B)c^4\cos(e+fx)}{16f}$$

$$= \frac{7}{16}a(2A-B)c^4x + \frac{7a(2A-B)c^4\cos^3(e+fx)}{24f} + \frac{7a(2A}{}$$

Mathematica [A] time = 0.934998, size = 131, normalized size = 0.72

$$\frac{ac^4(120(7A-5B)\cos(e+fx)+20(13A-7B)\cos(3(e+fx))+240A\sin(2(e+fx))-90A\sin(4(e+fx))-12A\cos(5(e}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,
x]
```

```
[Out] (a*c^4*(840*A*f*x - 420*B*f*x + 120*(7*A - 5*B)*Cos[e + f*x] + 20*(13*A - 7
*B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] + 36*B*Cos[5*(e + f*x)] + 240*
A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] - 90*A*Sin[4*(e + f*x)] + 105*B*
Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [B] time = 0.036, size = 342, normalized size = 1.9

$$\frac{1}{f}\left(-\frac{Ac^4a\cos(fx+e)}{5}\left(\frac{8}{3}+(\sin(fx+e))^4+\frac{4(\sin(fx+e))^2}{3}\right)-3Ac^4a\left(-1/4\left((\sin(fx+e))^3+3/2\sin(fx+e)\right)\text{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(-1/5*A*c^4*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*A*c^4*a*
(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*A*c^4*a*(
2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^4*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)+3*A*c^4*a*cos(f*x+e)+B*c^4*a*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3/5*B*c^4*a*(8/3+sin(f*x+e)^4+4/3*
sin(f*x+e)^2)*cos(f*x+e)+2*B*c^4*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-2/3*B*c^4*a*(2+sin(f*x+e)^2)*cos(f*x+e)-3*B*c^4*a*(-1
/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^4*a*(f*x+e)-B*c^4*a*cos(f*x+e))
```

Maxima [A] time = 0.977028, size = 454, normalized size = 2.49

$$-\frac{64\left(3\cos\left(fx+e\right)^{5}-10\cos\left(fx+e\right)^{3}+15\cos\left(fx+e\right)\right)Aac^{4}-640\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aac^{4}+90\left(12\,f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a*c^4
 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - sin(2*f*x +
 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a*c^4 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*
B*a*c^4 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2
*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^4 - 2880*
A*a*c^4*cos(f*x + e) + 960*B*a*c^4*cos(f*x + e))/f
```

Fricas [A] time = 1.48433, size = 302, normalized size = 1.66

$$-\frac{48\,(A-3\,B)ac^{4}\cos\left(fx+e\right)^{5}-320\,(A-B)ac^{4}\cos\left(fx+e\right)^{3}-105\,(2\,A-B)ac^{4}fx+5\left(8\,Bac^{4}\cos\left(fx+e\right)^{5}+2\,(18\,A\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] -1/240*(48*(A - 3*B)*a*c^4*cos(f*x + e)^5 - 320*(A - B)*a*c^4*cos(f*x + e)^
3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*cos(f*x + e)^5 + 2*(18*A - 25*B)
*a*c^4*cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 10.5282, size = 853, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)**
4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4*co
s(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos(e +
f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin(e + f
*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*cos(e +
 f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f*x)**6/1
6 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*x*sin(e +
 f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a*c**4*
x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)**2/2 + 5*B*
a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 - 3*B*a*c**4*x
*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 3*B*
a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)**3*cos(e +
f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a*c**4*
sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)
/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a*c**4*sin(e + f*
x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 8*B
*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)**3/(3*f) - B*a*c**4
*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c
)**4, True))
```

Giac [A] time = 1.20789, size = 248, normalized size = 1.36

$$-\frac{Bac^4\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{7}{16}\left(2\,Aac^4-Bac^4\right)x-\frac{\left(Aac^4-3\,Bac^4\right)\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{\left(13\,Aac^4-7\,Bac^4\right)\cos\left(3\,fx+3\,e\right)}{48\,f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/192*B*a*c^4*sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A*
a*c^4 - 3*B*a*c^4)*cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*cos(3
*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*cos(f*x + e)/f - 1/64*(6*A*a*c^
4 - 7*B*a*c^4)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*sin(2*f*x +
 2*e)/f
```

3.18 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=142

$$\frac{ac^3(5A-2B)\cos^3(e+fx)}{12f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}+\frac{ac^3(5A-2B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a$$

```
[Out] (a*(5*A - 2*B)*c^3*x)/8 + (a*(5*A - 2*B)*c^3*Cos[e + f*x]^3)/(12*f) + (a*(5
*A - 2*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(c -
 c*Sin[e + f*x])^2)/(5*f) + (a*(5*A - 2*B)*Cos[e + f*x]^3*(c^3 - c^3*Sin[e
+ f*x]))/(20*f)
```

Rubi [A] time = 0.249962, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{ac^3(5A-2B)\cos^3(e+fx)}{12f}+\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}+\frac{ac^3(5A-2B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```


```
[Out] (a*(5*A - 2*B)*c^3*x)/8 + (a*(5*A - 2*B)*c^3*Cos[e + f*x]^3)/(12*f) + (a*(5
*A - 2*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(c -
 c*Sin[e + f*x])^2)/(5*f) + (a*(5*A - 2*B)*Cos[e + f*x]^3*(c^3 - c^3*Sin[e
+ f*x]))/(20*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx$$
$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} + \frac{1}{5}(a(5A-2B)c$$
$$= -\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} + \frac{a(5A-2B)\cos$$
$$= \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} - \frac{aBc\cos^3(e+fx)(c-c\sin$$
$$= \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} + \frac{a(5A-2B)c^3\cos(e+fx)}{8f}$$
$$= \frac{1}{8}a(5A-2B)c^3x + \frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} + \frac{a(5A$$

Mathematica [A] time = 0.811702, size = 95, normalized size = 0.67

$$\frac{ac^3(15(-(A-2B)\sin(4(e+fx))+4fx(5A-2B)+8A\sin(2(e+fx)))+60(4A-3B)\cos(e+fx)+10(8A-5B)\cos(3(e}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,
x]
```

```
[Out] (a*c^3*(60*(4*A - 3*B)*Cos[e + f*x] + 10*(8*A - 5*B)*Cos[3*(e + f*x)] + 6*B
*Cos[5*(e + f*x)] + 15*(4*(5*A - 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A - 2*B
)*Sin[4*(e + f*x)])))/(480*f)
```

Maple [A] time = 0.033, size = 208, normalized size = 1.5

$$\frac{1}{f}\left(-Ac^3a\left(-\frac{\cos(fx+e)}{4}\left((\sin(fx+e))^3+\frac{3\sin(fx+e)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2Ac^3a\left(2+(\sin(fx+e))^2\right)\cos(fx+e)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-A*c^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
-2/3*A*c^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^3*a*cos(f*x+e)+1/5*B*c^3*a*(
8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*B*c^3*a*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*B*c^3*a*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+A*c^3*a*(f*x+e)-B*c^3*a*cos(f*x+e))
```

Maxima [A] time = 0.96582, size = 270, normalized size = 1.9

$$\frac{320\left(\cos\left(fx+e\right)^{3}-3\cos\left(fx+e\right)\right)Aac^{3}-15\left(12fx+12e+\sin\left(4fx+4e\right)-8\sin\left(2fx+2e\right)\right)Aac^{3}+480\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 + 32
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*x +
2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*co
s(f*x + e))/f
```

Fricas [A] time = 1.43166, size = 248, normalized size = 1.75

$$\frac{24Bac^{3}\cos\left(fx+e\right)^{5}+80(A-B)ac^{3}\cos\left(fx+e\right)^{3}+15(5A-2B)ac^{3}fx-15\left(2(A-2B)ac^{3}\cos\left(fx+e\right)^{3}-(5A-2\right.}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5*
A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a*c
^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 5.80988, size = 486, normalized size = 3.42

$$\begin{cases} -\frac{3Aac^3x\sin^4\left(e+fx\right)}{8} - \frac{3Aac^3x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - \frac{3Aac^3x\cos^4\left(e+fx\right)}{8} + Aac^3x + \frac{5Aac^3\sin^3\left(e+fx\right)\cos\left(e+fx\right)}{8f} - \frac{2Aac^3\sin^2\left(e+fx\right)\cos\left(e\right.}{f} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e + f
*x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**4/4 +
 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e + f*x)**
2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*a*c**3*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**3*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))
```

Giac [A] time = 1.19841, size = 196, normalized size = 1.38

$$\frac{Bac^3\cos\left(5\,fx+5\,e\right)}{80\,f} + \frac{Aac^3\sin\left(2\,fx+2\,e\right)}{4\,f} + \frac{1}{8}\left(5\,Aac^3-2\,Bac^3\right)x + \frac{\left(8\,Aac^3-5\,Bac^3\right)\cos\left(3\,fx+3\,e\right)}{48\,f} + \frac{\left(4\,Aac^3-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f + 1
/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*sin(
4*f*x + 4*e)/f
```

3.19 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=97

$$\frac{ac^2(A-B)\cos^3(e+fx)}{3f}+\frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ac^2x(4A-B)+\frac{aBc^2\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] (a*(4*A - B)*c^2*x)/8 + (a*(A - B)*c^2*Cos[e + f*x]^3)/(3*f) + (a*(4*A - B)
*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) + (a*B*c^2*Cos[e + f*x]^3*Sin[e + f*x
])/(4*f)
```

Rubi [A] time = 0.186352, antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{ac^2(4A-B)\cos^3(e+fx)}{12f}+\frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ac^2x(4A-B)-\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+f\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A - B)*c^2*x)/8 + (a*(4*A - B)*c^2*Cos[e + f*x]^3)/(12*f) + (a*(4*A -
 B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*Cos[e + f*x]^3*(c^2 - c^2*S
in[e + f*x]))/(4*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
```

```
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx$$

$$= -\frac{aB\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f} + \frac{1}{4}(a(4A-B)c$$

$$= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} - \frac{aB\cos^3(e+fx)\left(c^2-c^2\sin\right.}{4f}$$

$$= \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} + \frac{a(4A-B)c^2\cos(e+fx)\sin}{8f}$$

$$= \frac{1}{8}a(4A-B)c^2x + \frac{a(4A-B)c^2\cos^3(e+fx)}{12f} + \frac{a(4A-B}{}$$

Mathematica [A] time = 0.649647, size = 74, normalized size = 0.76

$$\frac{ac^2(3(8A\sin(2(e+fx))+16Afx+B\sin(4(e+fx))-4Bfx)+24(A-B)\cos(e+fx)+8(A-B)\cos(3(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (a*c^2*(24*(A - B)*Cos[e + f*x] + 8*(A - B)*Cos[3*(e + f*x)] + 3*(16*A*f*x
- 4*B*f*x + 8*A*Sin[2*(e + f*x)] + B*Sin[4*(e + f*x)])))/(96*f)
```

Maple [B] time = 0.029, size = 185, normalized size = 1.9

$$\frac{1}{f}\left(-\frac{Ac^2a\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}-Ac^2a\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Ac^2a\cos\left(fx+e\right)+Bc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*A*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)-A*c^2*a*(-1/2*sin(f*x+e)*cos(
f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*cos(f*x+e)+B*c^2*a*(-1/4*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/3*B*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)
-B*c^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*(f*x+e)-B*c^2*a
*cos(f*x+e))
```

Maxima [B] time = 0.966047, size = 242, normalized size = 2.49

$$\frac{32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aac^2-24\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aac^2+96\left(fx+e\right)Aac^2-32\left(\cos\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin(
2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos(f
*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*cos(f*
x + e) - 96*B*a*c^2*cos(f*x + e))/f
```

Fricas [A] time = 1.51681, size = 189, normalized size = 1.95

$$\frac{8\,(A-B)ac^2\cos\left(fx+e\right)^3+3\,(4\,A-B)ac^2fx+3\left(2\,Bac^2\cos\left(fx+e\right)^3+(4\,A-B)ac^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^2
*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 2.62768, size = 396, normalized size = 4.08

$$\begin{cases}-\frac{Aac^2x\sin^2\left(e+fx\right)}{2}-\frac{Aac^2x\cos^2\left(e+fx\right)}{2}+Aac^2x-\frac{Aac^2\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{Aac^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aac^2\cos^3\left(e+fx\right)}{3f}+\frac{Aac^2\cos(}{f} \\ x\,(A+B\sin\left(e\right))\,(a\sin\left(e\right)+a)\,(-c\sin\left(e\right)+c)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 + A
*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*x)
*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e + f
*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x)**4/
8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e + f*x)*c
os(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a*c**
2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

Giac [A] time = 1.17095, size = 154, normalized size = 1.59

$$\frac{Bac^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{Aac^2\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,Aac^2-Bac^2\right)x+\frac{\left(Aac^2-Bac^2\right)\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{\left(Aac^2-Bac^2\right)\text{co}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*A
*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(A*
a*c^2 - B*a*c^2)*cos(f*x + e)/f
```

3.20 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=49

$$\frac{aAc\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}$$

```
[Out] (a*A*c*x)/2 - (a*B*c*Cos[e + f*x]^3)/(3*f) + (a*A*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rubi [A] time = 0.0826031, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2967, 2669, 2635, 8}

$$\frac{aAc\sin(e+fx)\cos(e+fx)}{2f}+\frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a*A*c*x)/2 - (a*B*c*Cos[e + f*x]^3)/(3*f) + (a*A*c*Cos[e + f*x]*Sin[e + f*
x])/(2*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))\,dx\\ &= -\frac{aBc\cos^3(e+fx)}{3f}+(aAc)\int \cos^2(e+fx)\,dx\\ &= -\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}(\\ &= \frac{1}{2}aAcx-\frac{aBc\cos^3(e+fx)}{3f}+\frac{aAc\cos(e+fx)\sin(e+\end{aligned}$$

Mathematica [A] time = 0.15372, size = 48, normalized size = 0.98

$$-\frac{ac(-3A(\sin(2(e+fx))-2e+2fx)+3B\cos(e+fx)+B\cos(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] -(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Sin[2*(e
 + f*x)])))/(12*f)
```

Maple [A] time = 0.023, size = 74, normalized size = 1.5

$$\frac{1}{f}\left(\frac{Bac\left(2+\left(\sin\left(fx+e\right)\right)^{2}\right)\cos\left(fx+e\right)}{3}-Aac\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos\left(fx+e\right)+Aac\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*B*a*c*(2+sin(f*x+e)^2)*cos(f*x+e)-A*a*c*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)-B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

Maxima [A] time = 0.967706, size = 99, normalized size = 2.02

$$-\frac{3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aac-12\left(fx+e\right)Aac+4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)Bac+12\,Bac\cos\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(c
os(f*x + e)^3 - 3*cos(f*x + e))*B*a*c + 12*B*a*c*cos(f*x + e))/f
```

Fricas [A] time = 1.30224, size = 112, normalized size = 2.29

$$-\frac{2\,Bac\cos\left(fx+e\right)^{3}-3\,Aacfx-3\,Aac\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x +
 e))/f
```

Sympy [A] time = 1.22869, size = 138, normalized size = 2.82

$$\begin{cases}-\frac{Aacx\sin^{2}\left(e+fx\right)}{2}-\frac{Aacx\cos^{2}\left(e+fx\right)}{2}+Aacx+\frac{Aac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{Bac\sin^{2}\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{2Bac\cos^{3}\left(e+fx\right)}{3f}-\frac{Bac\cos\left(e+fx\right)}{f}\\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\left(-c\sin\left(e\right)+c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*x
 + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e + f*
x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))
```

Giac [A] time = 1.14198, size = 78, normalized size = 1.59

$$\frac{1}{2}Aacx - \frac{Bac\cos\left(3fx + 3e\right)}{12f} - \frac{Bac\cos\left(fx + e\right)}{4f} + \frac{Aac\sin\left(2fx + 2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1/
4*A*a*c*sin(2*f*x + 2*e)/f
```

3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax(A+2B)}{c}+\frac{aB\cos(e+fx)}{cf}$$

```
[Out] -((a*(A + 2*B)*x)/c) + (a*B*Cos[e + f*x])/(c*f) + (2*a*(A + B)*Cos[e + f*x]
)/(f*(c - c*Sin[e + f*x]))
```

Rubi [A] time = 0.169462, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2967, 2857, 2638}

$$\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}-\frac{ax(A+2B)}{c}+\frac{aB\cos(e+fx)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```


```
[Out] -((a*(A + 2*B)*x)/c) + (a*B*Cos[e + f*x])/(c*f) + (2*a*(A + B)*Cos[e + f*x]
)/(f*(c - c*Sin[e + f*x]))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} + \frac{a\int(-Ac-2Bc-Bc\sin(e+fx))\,dx}{c^2} \\
&= -\frac{a(A+2B)x}{c} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))} - \frac{(aB)\int\sin(e+fx)\,dx}{c} \\
&= -\frac{a(A+2B)x}{c} + \frac{aB\cos(e+fx)}{cf} + \frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}
\end{aligned}$$

Mathematica [B] time = 0.862014, size = 125, normalized size = 2.23

$$\frac{a(\sin(e+fx)+1)\left(\frac{4(A+B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+x(-(A+2B))-\frac{B\sin(e)\sin(fx)}{f}+\frac{B\cos(e)\cos(fx)}{f}\right)}{c\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a*(-((A + 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A
+ B)*Sin[(f*x)/2])/(f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])))*(1 + Sin[e + f*x]))/(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A] time = 0.1, size = 113, normalized size = 2.

$$-4\,\frac{aA}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-4\,\frac{Ba}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)}+2\,\frac{Ba}{cf\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}-2\,\frac{a\arctan\left(\tan\left(1/2\right.\right.}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -4/f*a/c/(tan(1/2*f*x+1/2*e)-1)*A-4/f*a/c/(tan(1/2*f*x+1/2*e)-1)*B+2/f*a/c*
B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*a/c*arctan(tan(1/2*f*x+1/2*e))*A-4/f*a/c*arc
tan(tan(1/2*f*x+1/2*e))*B
```

Maxima [B] time = 1.4559, size = 358, normalized size = 6.39

$$-\frac{2\left(Ba\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right)+Aa\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}-\frac{1}{c-\frac{c\sin(fx+e)}{\cos(fx+e)+1}}\right)+Ba\left(\frac{\arctan\left(\frac{\sin(}{\cos(}\right.}{c}\right.\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c -
 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] time = 1.40548, size = 289, normalized size = 5.16

$$-\frac{(A+2B)afx-Ba\cos\left(fx+e\right)^2-2(A+B)a+\left((A+2B)afx-(2A+3B)a\right)\cos\left(fx+e\right)-\left((A+2B)afx-Ba\cos\left(\right.\right.}{cf\cos\left(fx+e\right)-cf\sin\left(fx+e\right)+cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x - (
2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A + B
```

```
)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [A] time = 10.4835, size = 830, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(
e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)**2
/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
- c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 +
f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2)**3
- c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e/2 + f
*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 +
 c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 +
 f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*
f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2
+ c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f
*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*f*
x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2)
 - c*f) - 2*B*a*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2
+ f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*tan(e/2 + f*x/2)**2/(c*f*
tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f)
 - 4*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)
, True))
```

Giac [B] time = 1.19904, size = 167, normalized size = 2.98

$$-\frac{\frac{(Aa+2Ba)(fx+e)}{c}+\frac{2\left(2Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2Aa+3Ba\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(1
/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f*x
 + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f
```

3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{aBx}{c^2}$$

```
[Out] (a*B*x)/c^2 - (a*(A + 7*B)*Cos[e + f*x])/(3*c^2*f*(1 - Sin[e + f*x])) + (2*
a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2)
```

Rubi [A] time = 0.224578, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2735, 2648}

$$-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}+\frac{aBx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/c^2 - (a*(A + 7*B)*Cos[e + f*x])/(3*c^2*f*(1 - Sin[e + f*x])) + (2*
a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2} + \frac{a\int \frac{-Ac-4Bc-3Bc\sin(e+fx)}{c-c\sin(e+fx)}\,dx}{3c^2}$$

$$= \frac{aBx}{c^2} + \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2} - \frac{(a(A+7B))\int \frac{1}{c-c\sin(e+fx)}\,dx}{3c}$$

$$= \frac{aBx}{c^2} + \frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2} - \frac{a(A+7B)\cos(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)}$$

Mathematica [B] time = 0.607064, size = 160, normalized size = 2.22

$$-\frac{a\left(-6(A+3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)+9Bfx\sin\left(e+\frac{fx}{2}\right)+3Bfx\sin\left(e+\frac{3fx}{2}\right)+14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\,c\right.}{6c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] -(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*
f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin
[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*c^2*
f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

Maple [B] time = 0.102, size = 160, normalized size = 2.2

$$-2\,{\frac {aA}{f{c}^{2} \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) }}+2\,{\frac {Ba}{f{c}^{2} \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) }}-{\frac {8\,aA}{3\,f{c}^{2}} \left(\tan \left({\frac {fx}{2}}+{\frac {e}{2}} \right) -1 \right) ^{-3}}-{\frac {8\,Ba}{3\,f{c}^{2}} \left(\tan \left({\frac {fx}{2}}+{\frac {e}{2}} \right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -2/f*a/c^2/(tan(1/2*f*x+1/2*e)-1)*A+2/f*a/c^2/(tan(1/2*f*x+1/2*e)-1)*B-8/3/
f*a/c^2/(tan(1/2*f*x+1/2*e)-1)^3*A-8/3/f*a/c^2/(tan(1/2*f*x+1/2*e)-1)^3*B-4
/f*a/c^2/(tan(1/2*f*x+1/2*e)-1)^2*A-4/f*a/c^2/(tan(1/2*f*x+1/2*e)-1)^2*B+2/
f*a/c^2*B*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.48382, size = 616, normalized size = 8.56

$$\frac{2\left(Ba\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-4}{c^2-\frac{3c^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3c^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{c^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{3\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{c^2}\right)-\frac{Aa\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-2\right)}{c^2-\frac{3c^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3c^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{c^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{\cdots}{c^2-\frac{3c^2\sin(f\cdots}{\cos(fx+e\cdots}}\cdots}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a*(3
*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.42143, size = 402, normalized size = 5.58

$$-\frac{6\,Bafx - \left(3\,Bafx + (A + 7\,B)a\right)\cos\left(fx + e\right)^{2} + 2\,(A + B)a + \left(3\,Bafx + (A - 5\,B)a\right)\cos\left(fx + e\right) - \left(6\,Bafx - 2\,(A + B\right.}{3\left(c^{2}f\cos\left(fx + e\right)^{2} - c^{2}f\cos\left(fx + e\right) - 2\,c^{2}f + \left(c^{2}f\cos\left(fx + e\right) + 2\,c^{2}f\right)\mathrm{si}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a +
(3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*a*
f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*
f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [A] time = 19.9417, size = 711, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-2*A*a*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**
2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 6*A*a*tan
(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2
+ 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*
c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/
2 + f*x/2) - 3*c**2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 +
f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) + 9*B*a*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3
*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e
/2 + f*x/2) - 3*c**2*f) + 2*B*a*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x
/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*
f) - 18*B*a*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*B*a/(3*c**2*f*ta
n(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2
) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)*
```

```
*2, True))
```

Giac [A] time = 1.14274, size = 124, normalized size = 1.72

$$\frac{\frac{3\,(fx+e)Ba}{c^2} - \frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 3\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 12\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + Aa - 5\,Ba\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2*
f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2*f
*x + 1/2*e) - 1)^3))/f
```

3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=104

$$-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}-\frac{ac(A+11B)\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}$$

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) - (a*(A + 11*B)*c*C
os[e + f*x])/(15*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(A - 4*B)*Cos[e + f*x])
/(15*f*(c^3 - c^3*Sin[e + f*x]))
```

Rubi [A] time = 0.237488, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2750, 2648}

$$-\frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}-\frac{ac(A+11B)\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) - (a*(A + 11*B)*c*C
os[e + f*x])/(15*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(A - 4*B)*Cos[e + f*x])
/(15*f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
```

```
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} + \frac{a\int \frac{-Ac-6Bc-5Bc\sin(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{5c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2} - \frac{(a(A-4B))\int \frac{1}{c-c\sin}$$

$$= \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)\cos(e+fx)}{15cf(c-c\sin(e+fx))^2} - \frac{a(A-4B)\cos(e+}{15f\left(c^3-c^3\sin(e+\right.}$$

Mathematica [A] time = 0.683176, size = 147, normalized size = 1.41

$$\frac{a\left(15(A-B)\cos\left(e+\frac{fx}{2}\right)-5(A-B)\cos\left(e+\frac{3fx}{2}\right)+A\sin\left(2e+\frac{5fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)+15B\sin\left(2e+\frac{3fx}{2}\right)-4B\sin\left(2e\right.}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(f
*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*x
)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.108, size = 115, normalized size = 1.1

$$2\,{\frac {a}{f{c}^{3}}} \left(-1/5\,{\frac {8\,A+8\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{5}}}-1/3\,{\frac {14\,A+10\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{3}}}-1/2\,{\frac {6\,A+2\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{2}}}-{\frac {A}{\tan \left(1/2\,fx+e/2 \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f*a/c^3*(-1/5*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(14*A+10*B)/(tan(1/2
*f*x+1/2*e)-1)^3-1/2*(6*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-A/(tan(1/2*f*x+1/2*
e)-1)-1/4*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

Maxima [B] time = 1.06084, size = 995, normalized size = 9.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 3*A*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/
(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3
*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)
```

```
/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*a*(5*sin(f*x + e)/(cos
(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A] time = 1.35311, size = 466, normalized size = 4.48

$$-\frac{(A-4B)a\cos(fx+e)^3-(2A+7B)a\cos(fx+e)^2+3(A+B)a\cos(fx+e)+6(A+B)a+\left((A-4B)a\cos(fx+\right.}{15\left(c^3f\cos(fx+e)^3+3c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)-4c^3f-\left(c^3f\cos(fx+e)^2-2c^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A + B
)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*a*
cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*c
os(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*
c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [A] time = 29.8805, size = 1035, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*
c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*ta
n(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan(e/
2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**
4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c*
*3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c**3*f*
```

```
tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 1
5*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**
3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e
/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a/(15*c**3*f
*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 +
 f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75
*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e
/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)*
*4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f*x/2)/(15*c**3*f*ta
n(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*
x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*
c**3*f) + 2*B*a/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)
**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*
c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e
) + a)/(-c*sin(e) + c)**3, True))
```

Giac [A] time = 1.16847, size = 188, normalized size = 1.81

$$-\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,Ba\tan\left(\cdots\right.\right.}{15\,c^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*a*tan(1/2*f*x + 1/2*e)^4 - 15*A*a*tan(1/2*f*x + 1/2*e)^3 + 15*B
*a*tan(1/2*f*x + 1/2*e)^3 + 25*A*a*tan(1/2*f*x + 1/2*e)^2 + 5*B*a*tan(1/2*f
*x + 1/2*e)^2 - 5*A*a*tan(1/2*f*x + 1/2*e) + 5*B*a*tan(1/2*f*x + 1/2*e) + 4
*A*a - B*a)/(c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=142

$$-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}+\frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}$$

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) - (a*(A + 15*B)*Cos
[e + f*x])/(35*c*f*(c - c*Sin[e + f*x])^3) - (a*(2*A - 5*B)*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^
4 - c^4*Sin[e + f*x]))
```

Rubi [A] time = 0.285445, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2857, 2750, 2650, 2648}

$$-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}-\frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2}-\frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3}+\frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) - (a*(A + 15*B)*Cos
[e + f*x])/(35*c*f*(c - c*Sin[e + f*x])^3) - (a*(2*A - 5*B)*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^
4 - c^4*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
```

```
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} + \frac{a\int \frac{-Ac-8Bc-7Bc\sin(e+fx)}{(c-c\sin(e+fx))^3}\,dx}{7c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{(a(2A-5B))\int \frac{\ldots}{(c-c\,\mathrm{si}\ldots}}{35c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2A-5B)\cos(e+\ldots}{105f\left(c^2-c^2\sin(e+\ldots\right.}$$

$$= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} - \frac{a(2A-5B)\cos(e+\ldots}{105f\left(c^2-c^2\sin(e+\ldots\right.}$$

Mathematica [A] time = 0.825032, size = 174, normalized size = 1.23

$$\frac{a\left(35(4A-B)\cos\left(e+\frac{fx}{2}\right)+14A\sin\left(2e+\frac{5fx}{2}\right)-42A\cos\left(e+\frac{3fx}{2}\right)+2A\cos\left(3e+\frac{7fx}{2}\right)+70A\sin\left(\frac{fx}{2}\right)+105B\sin\left(2\right.\right.}{420c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e +
 (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f*x
)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin[2*
e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^7)
```

Maple [A] time = 0.119, size = 159, normalized size = 1.1

$$2\,\frac{a}{fc^4}\left(-1/6\,\frac{48\,A+48\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^6}-1/4\,\frac{56\,A+40\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^4}-1/5\,\frac{68\,A+60\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-1/2\,\frac{8\,A+}{\left(\tan\left(1/2\,fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a/c^4*(-1/6*(48*A+48*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(56*A+40*B)/(tan(1
/2*f*x+1/2*e)-1)^4-1/5*(68*A+60*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(8*A+2*B)/(
tan(1/2*f*x+1/2*e)-1)^2-1/3*(28*A+14*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/7*(16*A+
16*B)/(tan(1/2*f*x+1/2*e)-1)^7-A/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] time = 1.09437, size = 1458, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] 2/105*(A*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + B*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(
c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7) - 3*A*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 210*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x + e)/(cos
(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*B*a*(14*sin(f*
x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7))/f
```

Fricas [A] time = 1.3599, size = 637, normalized size = 4.49

$$\frac{(2A-5B)a\cos\left(fx+e\right)^4+4(2A-5B)a\cos\left(fx+e\right)^3-3(3A+10B)a\cos\left(fx+e\right)^2+15(A+B)a\cos\left(fx+e\right)+30}{105\left(c^4f\cos\left(fx+e\right)^4-3c^4f\cos\left(fx+e\right)^3-8c^4f\cos\left(fx+e\right)^2+4c^4f\cos\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(3
```

```
*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a - ((
2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B)*a
*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f
*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f +
 (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*
c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19253, size = 252, normalized size = 1.77

$$-\frac{2\left(105\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+105\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-35\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -2/105*(105*A*a*tan(1/2*f*x + 1/2*e)^6 - 210*A*a*tan(1/2*f*x + 1/2*e)^5 + 1
05*B*a*tan(1/2*f*x + 1/2*e)^5 + 455*A*a*tan(1/2*f*x + 1/2*e)^4 - 35*B*a*tan
(1/2*f*x + 1/2*e)^4 - 350*A*a*tan(1/2*f*x + 1/2*e)^3 + 140*B*a*tan(1/2*f*x
+ 1/2*e)^3 + 273*A*a*tan(1/2*f*x + 1/2*e)^2 - 56*A*a*tan(1/2*f*x + 1/2*e) +
 35*B*a*tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(tan(1/2*f*x + 1/2*e)
 - 1)^7)
```

3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5} dx$

Optimal. Leaf size=176

$$-\frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}-\frac{2ac(A-2B)\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{ac(A-2B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4}+\frac{2a(}{9f}$$

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(9*f*(c - c*Sin[e + f*x])^5) - (a*(A + 19*B)*Cos
[e + f*x])/(63*c*f*(c - c*Sin[e + f*x])^4) - (a*(A - 2*B)*c*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a*(A - 2*B)*c*Cos[e + f*x])/(315*f*(
c^3 - c^3*Sin[e + f*x])^2) - (2*a*(A - 2*B)*Cos[e + f*x])/(315*f*(c^5 - c^5
*Sin[e + f*x]))
```

Rubi [A] time = 0.307073, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2857, 2750, 2650, 2648}

$$-\frac{2a(A-2B)\cos(e+fx)}{315f\left(c^5-c^5\sin(e+fx)\right)}-\frac{2ac(A-2B)\cos(e+fx)}{315f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{ac(A-2B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4}+\frac{2a(}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (2*a*(A + B)*Cos[e + f*x])/(9*f*(c - c*Sin[e + f*x])^5) - (a*(A + 19*B)*Cos
[e + f*x])/(63*c*f*(c - c*Sin[e + f*x])^4) - (a*(A - 2*B)*c*Cos[e + f*x])/(
105*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a*(A - 2*B)*c*Cos[e + f*x])/(315*f*(
c^3 - c^3*Sin[e + f*x])^2) - (2*a*(A - 2*B)*Cos[e + f*x])/(315*f*(c^5 - c^5
*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} + \frac{a\int \frac{-Ac-10Bc-9Bc\sin(e+fx)}{(c-c\sin(e+fx))^4}\,dx}{9c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{(a(A-2B))\int \frac{}{(c-c\sin}}{21c^2}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)\cos(e+}{105c^2f(c-c\sin(e+}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)\cos(e+}{105c^2f(c-c\sin(e+}$$

$$= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)\cos(e+}{105c^2f(c-c\sin(e+}$$

Mathematica [A] time = 0.824741, size = 200, normalized size = 1.14

$$\frac{a\left(-42(2A+B)\cos\left(e+\frac{3fx}{2}\right)+36A\sin\left(2e+\frac{5fx}{2}\right)-A\sin\left(4e+\frac{9fx}{2}\right)+315A\cos\left(e+\frac{fx}{2}\right)+9A\cos\left(3e+\frac{7fx}{2}\right)+189A\right.}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] (a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e
+ (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin[(
f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*Sin
[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))/(12
60*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

Maple [A] time = 0.136, size = 203, normalized size = 1.2

$$2\,\frac{a}{fc^5}\left(-1/8\,\frac{128\,A+128\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^8}-1/2\,\frac{10\,A+2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-1/5\,\frac{236\,A+168\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-1/3\,\frac{46\,A+}{\left(\tan\left(1/2\,fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a/c^5*(-1/8*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/2*(10*A+2*B)/(tan(
1/2*f*x+1/2*e)-1)^2-1/5*(236*A+168*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(46*A+18
*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/6*(296*A+248*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/9
*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/7*(248*A+232*B)/(tan(1/2*f*x+1/2*e)
-1)^7-1/4*(128*A+72*B)/(tan(1/2*f*x+1/2*e)-1)^4-A/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] time = 1.155, size = 1924, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="maxima")
```

```
[Out] -2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9
*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) - 5*A*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14
7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*
c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a*(45*sin(f*x + e)/(cos(f*x + e
) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin
```

```
(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x +
e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*
c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 14*B*a
*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^
5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [A] time = 1.4781, size = 794, normalized size = 4.51

$$-\frac{2\,(A-2\,B)a\cos\left(fx+e\right)^{5}-8\,(A-2\,B)a\cos\left(fx+e\right)^{4}-25\,(A-2\,B)a\cos\left(fx+e\right)^{3}+5\,(4\,A+13\,B)a\cos\left(fx+e\right)^{2}-3}{315\left(c^{5}f\cos\left(fx+e\right)^{5}+5\,c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{3}-20\,c^{5}f\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="fricas")
```

```
[Out] -1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(A
 - 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*c
os(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B)*a
*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x + e)
 - 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)
^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e
) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*co
s(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20943, size = 360, normalized size = 2.05

$$-\frac{2\left(315\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-945\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+315\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+2625\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-315\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="giac")
```

```
[Out] -2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 + 3
15*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a*t
an(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/2*f
*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x + 1/2
*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)^3 +
1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207*A*a*t
an(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)/(c^5*f
*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

3.26 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=229

$$\frac{3a^2c^5(8A-3B)\cos^5(e+fx)}{80f}+\frac{3a^2c^5(8A-3B)\sin(e+fx)\cos^3(e+fx)}{64f}+\frac{a^2c^3(8A-3B)\cos^5(e+fx)(c-c\sin(e+fx))}{56f}$$

```
[Out] (9*a^2*(8*A - 3*B)*c^5*x)/128 + (3*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]^5)/(80*
f) + (9*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (3*a^2*(8*
A - 3*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64*f) + (a^2*(8*A - 3*B)*c^3*Cos
[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(56*f) - (a^2*B*c^2*Cos[e + f*x]^5*(c -
 c*Sin[e + f*x])^3)/(8*f) + (3*a^2*(8*A - 3*B)*Cos[e + f*x]^5*(c^5 - c^5*Si
n[e + f*x]))/(112*f)
```

Rubi [A] time = 0.367915, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{3a^2c^5(8A-3B)\cos^5(e+fx)}{80f}+\frac{3a^2c^5(8A-3B)\sin(e+fx)\cos^3(e+fx)}{64f}+\frac{a^2c^3(8A-3B)\cos^5(e+fx)(c-c\sin(e+fx))}{56f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```


```
[Out] (9*a^2*(8*A - 3*B)*c^5*x)/128 + (3*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]^5)/(80*
f) + (9*a^2*(8*A - 3*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (3*a^2*(8*
A - 3*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(64*f) + (a^2*(8*A - 3*B)*c^3*Cos
[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(56*f) - (a^2*B*c^2*Cos[e + f*x]^5*(c -
 c*Sin[e + f*x])^3)/(8*f) + (3*a^2*(8*A - 3*B)*Cos[e + f*x]^5*(c^5 - c^5*Si
n[e + f*x]))/(112*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+$$

$$= -\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f} + \frac{1}{8}\left(a^2(8A-\right.$$

$$= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} - \frac{a^2Bc}{}$$

$$= \frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f} - \frac{a^2Bc}{}$$

$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{a^2(8A-3B)c^3\cos^5(e}{5}$$

$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{3a^2(8A-3B)c^5\cos^3(e}{64f}$$

$$= \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} + \frac{9a^2(8A-3B)c^5\cos(e}{128f}$$

$$= \frac{9}{128}a^2(8A-3B)c^5x + \frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f} +$$

Mathematica [A] time = 1.93804, size = 219, normalized size = 0.96

$$\frac{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5(2520(8A-3B)(e+fx)+560(19A-3B)\sin(2(e+fx))-280(2A-7B)\sin(4(e+}{35840f\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x)
+ 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 112*
(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A - 3
*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*Sin[6
*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [B] time = 0.037, size = 569, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(A*a^2*c^5*(f*x+e)-B*a^2*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*
sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+B*a^2*c^5*(8/3+sin(f*x+e)^4+4/3*sin
(f*x+e)^2)*cos(f*x+e)+5*B*a^2*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f
*x+e)+3/8*f*x+3/8*e)-3/7*B*a^2*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*
sin(f*x+e)^2)*cos(f*x+e)+3*A*a^2*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+1
5/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/5*A*a^2*c^5*(8/3+sin(f*x+e)^4
+4/3*sin(f*x+e)^2)*cos(f*x+e)-5*A*a^2*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)-5/3*A*a^2*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^
2*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x
+e))*cos(f*x+e)+35/128*f*x+35/128*e)+1/7*A*a^2*c^5*(16/5+sin(f*x+e)^6+6/5*s
in(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-1/3*B*a^2*c^5*(2+sin(f*x+e)^2)*cos
(f*x+e)+3*A*a^2*c^5*cos(f*x+e)-3*B*a^2*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e)+A*a^2*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^5*c
os(f*x+e))
```

<u>Maxima</u> [B] time = 1.02262, size = 771, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2
*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*
sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*
c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f*x + e)
^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(f*x + e)
^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^2*
c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*s
in(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^
```

5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x + e))/f

Fricas [A] time = 1.68156, size = 381, normalized size = 1.66

$$-\frac{640\,(A-3\,B)a^2c^5\cos\left(fx+e\right)^7-3584\,(A-B)a^2c^5\cos\left(fx+e\right)^5-315\,(8\,A-3\,B)a^2c^5fx+35\left(16\,Ba^2c^5\cos\left(fx+e\right)^7\right.}{4480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")

[Out] -1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f*
x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7 +
8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x + e)^
3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f

Sympy [A] time = 40.2389, size = 1586, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)

[Out] Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*x*co
s(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*cos(e
 + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f -
 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*sin(e +
 f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5
*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**5*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*A*a**2*c**
5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)
5/(16*f) + 15*A*a2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*c**
5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*x)**7/(35*f) +

```
 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e + f*x)**3/(3*f
) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e + f*x)**8/128 - 3
5*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a**2*c**5*x*sin(e
+ f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 15*B*
a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a**2*c**5*x*sin(e + f
*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 15*B*a**2*
c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 15*B*a**2*c**5*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 - 3*B*a**2*c**5*x*sin(e + f*x)**2/2 - 35*B*a**2*c**5*x*
cos(e + f*x)**8/128 - 5*B*a**2*c**5*x*cos(e + f*x)**6/16 + 15*B*a**2*c**5*x
*cos(e + f*x)**4/8 - 3*B*a**2*c**5*x*cos(e + f*x)**2/2 + 93*B*a**2*c**5*sin
(e + f*x)**7*cos(e + f*x)/(128*f) - 3*B*a**2*c**5*sin(e + f*x)**6*cos(e + f
*x)/f + 511*B*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 11*B*a**2
*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 6*B*a**2*c**5*sin(e + f*x)**4*c
os(e + f*x)**3/f + 5*B*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*B*a**
2*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**2*c**5*sin(e + f*x)
**3*cos(e + f*x)**3/(6*f) - 25*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*
f) - 24*B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 20*B*a**2*c**5*
sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*c**5*sin(e + f*x)**2*cos(e +
 f*x)/f + 35*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 5*B*a**2*c*
*5*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 15*B*a**2*c**5*sin(e + f*x)*cos(e
+ f*x)**3/(8*f) + 3*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) - 48*B*a**2
*c**5*cos(e + f*x)**7/(35*f) + 8*B*a**2*c**5*cos(e + f*x)**5/(3*f) - 2*B*a*
*2*c**5*cos(e + f*x)**3/(3*f) - B*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**5, True))
```

Giac [A] time = 1.24666, size = 375, normalized size = 1.64

$$-\frac{Ba^2c^5\sin(8fx+8e)}{1024f}+\frac{9}{128}(8Aa^2c^5-3Ba^2c^5)x-\frac{(Aa^2c^5-3Ba^2c^5)\cos(7fx+7e)}{448f}+\frac{(11Aa^2c^5-Ba^2c^5)\cos(5f}{320f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x
- 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^5
- B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(3*f
*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*(A*a
^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*c^5)*
sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*e)/f
```

3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=189

$$\frac{a^2c^4(7A-2B)\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos^3(e+fx)}{24f}$$

```
[Out] (a^2*(7*A - 2*B)*c^4*x)/16 + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^5)/(30*f) +
(a^2*(7*A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(7*A - 2*B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^2 - c^2*S
in[e + f*x])^2)/(7*f) + (a^2*(7*A - 2*B)*Cos[e + f*x]^5*(c^4 - c^4*Sin[e +
f*x]))/(42*f)
```

Rubi [A] time = 0.296036, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^2c^4(7A-2B)\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}+\frac{a^2c^4(7A-2B)\sin(e+fx)\cos^3(e+fx)}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```


```
[Out] (a^2*(7*A - 2*B)*c^4*x)/16 + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^5)/(30*f) +
(a^2*(7*A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(7*A - 2*B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^2 - c^2*S
in[e + f*x])^2)/(7*f) + (a^2*(7*A - 2*B)*Cos[e + f*x]^5*(c^4 - c^4*Sin[e +
f*x]))/(42*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+$$

$$= -\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f}+\frac{1}{7}\left(a^2(7A-\right.$$

$$= -\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f}+\frac{a^2(7A-2B}{}$$

$$= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f}-\frac{a^2B\cos^5(e+fx)\left(c^2-c\right.}{7f}$$

$$= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)c^4\cos^3(e+}{24f}$$

$$= \frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f}+\frac{a^2(7A-2B)c^4\cos(e+}{16f}$$

$$= \frac{1}{16}a^2(7A-2B)c^4x+\frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f}+\frac{a^2}{}$$

Mathematica [A] time = 1.51981, size = 163, normalized size = 0.86

$$\frac{a^2c^4(105(16A-11B)\cos(e+fx)+105(8A-5B)\cos(3(e+fx))+1785A\sin(2(e+fx))+105A\sin(4(e+fx))-35A\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] (a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*C
os[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] - 6
3*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] - 21
0*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)] - 35
*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)]))/(6720*f)
```

Maple [B] time = 0.035, size = 463, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(A*a^2*c^4*(f*x+e)+1/5*B*a^2*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*co
s(f*x+e)+4*B*a^2*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)+2/5*A*a^2*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-A*a^2*
c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-4/3*A*a^2
*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-1/7*B*a^2*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f
*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-2*B*a^2*c^4*(-1/6*(sin(f*x+e)^5+5/4*si
n(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+A*a^2*c^4*(-1/6*(si
n(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-2*
B*a^2*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+1/3*B*a^2*c^4*(2+sin(f
*x+e)^2)*cos(f*x+e)+2*A*a^2*c^4*cos(f*x+e)-A*a^2*c^4*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-B*a^2*c^4*cos(f*x+e))
```

Maxima [B] time = 1.00143, size = 621, normalized size = 3.29

$$\frac{896\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)Aa^2c^4+8960\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^2c^4+35\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*
c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x +
2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2*c^
4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^4 -
 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^2*c^4
 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f
*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x
 + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4 - 70*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2
*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)
)*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 + 13440*A*a^2
*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

Fricas [A] time = 1.59265, size = 323, normalized size = 1.71

$$\frac{240\,Ba^2c^4\cos\left(fx+e\right)^7+672\,(A-B)a^2c^4\cos\left(fx+e\right)^5+105\,(7\,A-2\,B)a^2c^4fx-35\left(8\,(A-2\,B)a^2c^4\cos\left(fx+e\right)^5-2\right.}{1680\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5 +
 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B)*a^2*c^4*cos(f*x + e)^5 - 2*(
7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [A] time = 23.3628, size = 1210, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*c*
*4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*cos(e
+ f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e + f*
x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f
) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8
*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a**2*c**4*s
in(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/
(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**4*sin
(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(15*f) - 8*A*
a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f - 5*B*a**2*c
**4*x*sin(e + f*x)**6/8 - 15*B*a**2*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/
8 + 3*B*a**2*c**4*x*sin(e + f*x)**4/2 - 15*B*a**2*c**4*x*sin(e + f*x)**2*co
s(e + f*x)**4/8 + 3*B*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2 - B*a**2*
c**4*x*sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8 + 3*B*a**2*c**4*
x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a**2*c**4*sin(e + f
```

```
*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(8*f) -
 2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B*a**2*c**4*sin(e + f*x)
**4*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) -
5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(2*f) - 8*B*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**5/(5*f) + 4*B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3
/(3*f) + B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e +
 f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(2
*f) + B*a**2*c**4*sin(e + f*x)*cos(e + f*x)/f - 16*B*a**2*c**4*cos(e + f*x)
**7/(35*f) + 8*B*a**2*c**4*cos(e + f*x)**5/(15*f) + 2*B*a**2*c**4*cos(e + f
*x)**3/(3*f) - B*a**2*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**4, True))
```

Giac [A] time = 1.20985, size = 329, normalized size = 1.74

$$\frac{Ba^2c^4\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{1}{16}\left(7\,Aa^2c^4-2\,Ba^2c^4\right)x+\frac{\left(8\,Aa^2c^4-3\,Ba^2c^4\right)\cos\left(5\,fx+5\,e\right)}{320\,f}+\frac{\left(8\,Aa^2c^4-5\,Ba^2c^4\right)\cos\left(3\,f\right.}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x + 1
/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 - 5
*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(f*x
 + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*a^2*
c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c^4)*s
in(2*f*x + 2*e)/f
```

3.28 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=147

$$\frac{a^2c^3(6A-B)\cos^5(e+fx)}{30f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2c^3x(6$$

```
[Out] (a^2*(6*A - B)*c^3*x)/16 + (a^2*(6*A - B)*c^3*Cos[e + f*x]^5)/(30*f) + (a^2
*(6*A - B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(6*A - B)*c^3*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^3 - c^3*Sin[e + f
*x]))/(6*f)
```

Rubi [A] time = 0.216463, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{a^2c^3(6A-B)\cos^5(e+fx)}{30f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^2c^3(6A-B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^2c^3x(6$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*(6*A - B)*c^3*x)/16 + (a^2*(6*A - B)*c^3*Cos[e + f*x]^5)/(30*f) + (a^2
*(6*A - B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(6*A - B)*c^3*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^3 - c^3*Sin[e + f
*x]))/(6*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e$$

$$= -\frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}+\frac{1}{6}\left(a^2(6A\right.$$

$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}-\frac{a^2B\cos^5(e+fx)\left(c^3-\right.}{6f}$$

$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos^3(e+}{24f}$$

$$= \frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(6A-B)c^3\cos(e+f}{16f}$$

$$= \frac{1}{16}a^2(6A-B)c^3x+\frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f}+\frac{a^2(}{}$$

Mathematica [A] time = 1.05085, size = 137, normalized size = 0.93

$$\frac{a^2c^3(120(A-B)\cos(e+fx)+60(A-B)\cos(3(e+fx))+240A\sin(2(e+fx))+30A\sin(4(e+fx))+12A\cos(5(e+fx}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*x
] + 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [B] time = 0.03, size = 365, normalized size = 2.5

$$\frac{1}{f}\left(\frac{Aa^2c^3\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+Aa^2c^3\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+\right.}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(1/5*A*a^2*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*a^2*c^3
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*A*a^2*c^
3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*A*a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f
*x+1/2*e)-B*a^2*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*c
os(f*x+e)+5/16*f*x+5/16*e)-1/5*B*a^2*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2
)*cos(f*x+e)+2*B*a^2*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e)+2/3*B*a^2*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a^2*c^3*cos(f*x+e)-
B*a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*a^2*c^3*(f*x+e)-B*a^
2*c^3*cos(f*x+e))
```

Maxima [B] time = 0.988055, size = 486, normalized size = 3.31

$$64\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)Aa^2c^3+640\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^2c^3+30\left(12f\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c^
3 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^3 - 640*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
 + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^2*c^3 + 960*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e))/f
```

Fricas [A] time = 1.50883, size = 257, normalized size = 1.75

$$\frac{48\,(A-B)a^2c^3\cos\left(fx+e\right)^5+15\,(6\,A-B)a^2c^3fx+5\left(8\,Ba^2c^3\cos\left(fx+e\right)^5+2\,(6\,A-B)a^2c^3\cos\left(fx+e\right)^3+3\,(6\,A-}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8*
B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A - B)
*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 16.9081, size = 910, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*cos
```

```
(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*c**
3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e + f*
x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a**2*c
**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3*cos(e
+ f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**3*cos(e
+ f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15*B*a**2*c*
*3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c**3*x*cos(e
+ f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3*x*cos(e + f*
x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**2*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c**
3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*sin(e + f*x)**2*cos
(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*
c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**3*cos(e + f
*x)**3/(3*f) - B*a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**3, True))
```

Giac [A] time = 1.16322, size = 281, normalized size = 1.91

$$\frac{Ba^2c^3\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(6\,Aa^2c^3-Ba^2c^3\right)x+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{\left(Aa^2c^3-Ba^2c^3\right)\cos\left(3\,fx+3\,e\right)}{16\,f}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/192*B*a^2*c^3*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^2*c^3 - B*a^2*c^3)*x + 1/8
0*(A*a^2*c^3 - B*a^2*c^3)*cos(5*f*x + 5*e)/f + 1/16*(A*a^2*c^3 - B*a^2*c^3)
*cos(3*f*x + 3*e)/f + 1/8*(A*a^2*c^3 - B*a^2*c^3)*cos(f*x + e)/f + 1/64*(2*
A*a^2*c^3 + B*a^2*c^3)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^2*c^3 - B*a^2*c^3)
*sin(2*f*x + 2*e)/f
```

3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=89

$$\frac{a^2Ac^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2Ac^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}$$

```
[Out] (3*a^2*A*c^2*x)/8 - (a^2*B*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^2*A*c^2*Cos[e +
 f*x]*Sin[e + f*x])/(8*f) + (a^2*A*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rubi [A] time = 0.136876, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2669, 2635, 8}

$$\frac{a^2Ac^2\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3a^2Ac^2\sin(e+fx)\cos(e+fx)}{8f}+\frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (3*a^2*A*c^2*x)/8 - (a^2*B*c^2*Cos[e + f*x]^5)/(5*f) + (3*a^2*A*c^2*Cos[e +
 f*x]*Sin[e + f*x])/(8*f) + (a^2*A*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx &= \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))\,dx\\ &= -\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\left(a^2Ac^2\right)\int \cos^4(e+fx)\,dx\\ &= -\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}\\ &= -\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f}\\ &= \frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)}{8f}\end{aligned}$$

Mathematica [A] time = 0.143001, size = 54, normalized size = 0.61

$$\frac{a^2c^2\left(5A(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx)))-32B\cos^5(e+fx)\right)}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + S
in[4*(e + f*x)])))/(160*f)
```

Maple [B] time = 0.026, size = 166, normalized size = 1.9

$$\frac{1}{f}\left(-\frac{Ba^2c^2\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+Aa^2c^2\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(f\right.}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*B*a^2*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*a^2*c^
2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*B*a^2*c
^2*(2+sin(f*x+e)^2)*cos(f*x+e)-2*A*a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e)-B*a^2*c^2*cos(f*x+e)+A*a^2*c^2*(f*x+e))
```

Maxima [B] time = 0.96787, size = 221, normalized size = 2.48

$$\frac{15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)Aa^2c^2-240\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c^2+480\left(fx+e\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^2
 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c^2
 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^2 -
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*cos(f*x + e
))/f
```

Fricas [A] time = 1.46277, size = 176, normalized size = 1.98

$$-\frac{8\,Ba^2c^2\cos\left(fx+e\right)^5-15\,Aa^2c^2fx-5\left(2\,Aa^2c^2\cos\left(fx+e\right)^3+3\,Aa^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(f
*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 4.94064, size = 372, normalized size = 4.18

$$\begin{cases} \frac{3Aa^2c^2x\sin^4\left(e+fx\right)}{8} + \frac{3Aa^2c^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} - Aa^2c^2x\sin^2\left(e+fx\right) + \frac{3Aa^2c^2x\cos^4\left(e+fx\right)}{8} - Aa^2c^2x\cos^2\left(e+fx\right) + Aa^2c^2 \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*cos
(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**2*c
**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos(e +
 f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2*sin(
e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)**3/
(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2*cos(e
+ f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e
+ f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**2
, True))
```

Giac [A] time = 1.21036, size = 159, normalized size = 1.79

$$\frac{3}{8}Aa^2c^2x - \frac{Ba^2c^2\cos\left(5fx+5e\right)}{80f} - \frac{Ba^2c^2\cos\left(3fx+3e\right)}{16f} - \frac{Ba^2c^2\cos\left(fx+e\right)}{8f} + \frac{Aa^2c^2\sin\left(4fx+4e\right)}{32f} + \frac{Aa^2c^2\sin\left(2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3*
f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x + 4*
```

```
e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f
```

3.30 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=98

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f}+\frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx(4A+B)-\frac{Bc\cos^3(e+fx)\left(a^2\sin(e+fx)+\right.}{4f}$$

```
[Out] (a^2*(4*A + B)*c*x)/8 - (a^2*(4*A + B)*c*Cos[e + f*x]^3)/(12*f) + (a^2*(4*A
 + B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (B*c*Cos[e + f*x]^3*(a^2 + a^2*S
in[e + f*x]))/(4*f)
```

Rubi [A] time = 0.148919, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2860, 2669, 2635, 8}

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f}+\frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}a^2cx(4A+B)-\frac{Bc\cos^3(e+fx)\left(a^2\sin(e+fx)+\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(4*A + B)*c*x)/8 - (a^2*(4*A + B)*c*Cos[e + f*x]^3)/(12*f) + (a^2*(4*A
 + B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (B*c*Cos[e + f*x]^3*(a^2 + a^2*S
in[e + f*x]))/(4*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
```

```
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))(A+B\sin(e+$$

$$= -\frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f} + \frac{1}{4}(a(4A+B$$

$$= -\frac{a^2(4A+B)c\cos^3(e+fx)}{12f} - \frac{Bc\cos^3(e+fx)\left(a^2+a^2\right.}{4f}$$

$$= -\frac{a^2(4A+B)c\cos^3(e+fx)}{12f} + \frac{a^2(4A+B)c\cos(e+fx}{8f}$$

$$= \frac{1}{8}a^2(4A+B)cx - \frac{a^2(4A+B)c\cos^3(e+fx)}{12f} + \frac{a^2(4A}{}$$

Mathematica [A] time = 0.786035, size = 67, normalized size = 0.68

$$-\frac{a^2c(24(A+B)\cos(e+fx)+8(A+B)\cos(3(e+fx))-12fx(4A+B)-24A\sin(2(e+fx))+3B\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] -(a^2*c*(-12*(4*A + B)*f*x + 24*(A + B)*Cos[e + f*x] + 8*(A + B)*Cos[3*(e +
 f*x)] - 24*A*Sin[2*(e + f*x)] + 3*B*Sin[4*(e + f*x)]))/(96*f)
```

Maple [B] time = 0.027, size = 186, normalized size = 1.9

$$\frac{1}{f}\left(\frac{Aa^2c\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}-Aa^2c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Ba^2c\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(1/3*A*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-A*a^2*c*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-B*a^2*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)
+3/8*f*x+3/8*e)+1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-A*a^2*c*cos(f*x+e)+
B*a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*a^2*c*(f*x+e)-B*a^2*c*
cos(f*x+e))
```

Maxima [A] time = 0.971595, size = 242, normalized size = 2.47

$$-\frac{32\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aa^2c+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c-96\left(fx+e\right)Aa^2c+32\left(\cos\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*cos(f
*x + e) + 96*B*a^2*c*cos(f*x + e))/f
```

Fricas [A] time = 1.40412, size = 190, normalized size = 1.94

$$-\frac{8\,(A+B)a^2c\cos\left(fx+e\right)^3-3\,(4\,A+B)a^2cfx+3\left(2\,Ba^2c\cos\left(fx+e\right)^3-(4\,A+B)a^2c\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2*
c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 1.79887, size = 396, normalized size = 4.04

$$\begin{cases}-\frac{Aa^2cx\sin^2\left(e+fx\right)}{2}-\frac{Aa^2cx\cos^2\left(e+fx\right)}{2}+Aa^2cx+\frac{Aa^2c\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}+\frac{Aa^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{2Aa^2c\cos^3\left(e+fx\right)}{3f}-\frac{Aa^2c\cos\ldots}{\ldots} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\left(-c\sin\left(e\right)+c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*x)
*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e + f
*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*cos(e
 + f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x)**4/
8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a**2*
c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

Giac [A] time = 1.16806, size = 150, normalized size = 1.53

$$-\frac{Ba^2c\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{Aa^2c\sin\left(2\,fx+2\,e\right)}{4\,f}+\frac{1}{8}\left(4\,Aa^2c+Ba^2c\right)x-\frac{\left(Aa^2c+Ba^2c\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(Aa^2c+Ba^2c\ldots\right.}{4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4*
A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*(A
*a^2*c + B*a^2*c)*cos(f*x + e)/f
```

3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{3a^2x(2A+3B)}{2c}$$

```
[Out] (-3*a^2*(2*A + 3*B)*x)/(2*c) + (3*a^2*(2*A + 3*B)*Cos[e + f*x])/(2*c*f) + (
a^2*(A + B)*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) + (a^2*(2*A + 3*
B)*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x]))
```

Rubi [A] time = 0.288545, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2679, 2682, 8}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}-\frac{3a^2x(2A+3B)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-3*a^2*(2*A + 3*B)*x)/(2*c) + (3*a^2*(2*A + 3*B)*Cos[e + f*x])/(2*c*f) + (
a^2*(A + B)*c^2*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) + (a^2*(2*A + 3*
B)*Cos[e + f*x]^3)/(2*f*(c - c*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} - \left(a^2(2A+3B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} + \frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))} - \frac{1}{2}\left(3a^2(2A+3B\right.$$

$$= \frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3} + \frac{a^2(2A+3B)\text{co}}{2f(c-c\sin(}$$

$$= -\frac{3a^2(2A+3B)x}{2c} + \frac{3a^2(2A+3B)\cos(e+fx)}{2cf} + \frac{a^2(A+B)c^2\cos^5(e+f}{f(c-c\sin(e+fx))^3}$$

Mathematica [A] time = 1.23508, size = 191, normalized size = 1.63

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A+3B)(e+fx)-4(A+3B)\cos(e+fx)\right.}{4cf(\sin(e+fx)-1)\left(\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e + f
*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f
*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x) -
4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

Maple [B] time = 0.113, size = 299, normalized size = 2.6

$$-8\,\frac{Aa^2}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-8\,\frac{Ba^2}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-\frac{Ba^2}{cf}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}+2\,\frac{}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -8/f*a^2/c/(tan(1/2*f*x+1/2*e)-1)*A-8/f*a^2/c/(tan(1/2*f*x+1/2*e)-1)*B-1/f*
a^2/c/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B+2/f*a^2/c/(1+tan(1/
2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+6/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)
^2*tan(1/2*f*x+1/2*e)^2*B+1/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*
x+1/2*e)+2/f*a^2/c/(1+tan(1/2*f*x+1/2*e)^2)^2*A+6/f*a^2/c/(1+tan(1/2*f*x+1/
2*e)^2)^2*B-9/f*a^2/c*arctan(tan(1/2*f*x+1/2*e))*B-6/f*a^2/c*arctan(tan(1/2
*f*x+1/2*e))*A
```

Maxima [B] time = 1.48282, size = 842, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -(2*A*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c) + 4*B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) +
1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + B*a^2*((sin(f*x + e
)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c -
 c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/c) + 4*A*a^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c
- 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*a^2*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*a
^2/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.39811, size = 423, normalized size = 3.62

$$\frac{Ba^2\cos\left(fx+e\right)^3-3\,(2\,A+3\,B)a^2fx+2\,(A+3\,B)a^2\cos\left(fx+e\right)^2+8\,(A+B)a^2-\left(3\,(2\,A+3\,B)a^2fx-(10\,A+13\,B)a\right.}{2\left(cf\cos\left(fx+e\right)-cf\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*x
 + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos(f
*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^2*c
os(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x
+ e) + c*f)
```

Sympy [A] time = 9.73573, size = 2365, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*a**2*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2
*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/
2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x*tan(e/2 + f*x/2)**4/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 12*
A*a**2*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/
2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c
*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*A*a**2*f*x*tan
(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
 - 2*c*f) + 6*A*a**2*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)
**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
 + f*x/2) - 2*c*f) - 16*A*a**2*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)*
*5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2
+ f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 28*A*a**2*tan(e/2 + f*x/2)*
*3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 +
 f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) -
4*A*a**2*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f
*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*ta
n(e/2 + f*x/2) - 2*c*f) - 12*A*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2
)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/
2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 4*A*a**2/(2*c*f*tan(e/2 +
 f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*
tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/
2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*
f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2)
 - 2*c*f) + 9*B*a**2*f*x*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*f*x*tan(e/2 + f*x/2)**3/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*
B*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
 f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 +
```

```
f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*t
an(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 9*B*a**2*f*x/(2*c*f*
tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*
tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f
*x/2) - 2*c*f) - 22*B*a**2*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 -
 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*B*a**2*tan(e/2 + f*x/2)**2/(2
*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/
2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 8*B*a
**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 +
f*x/2) - 2*c*f) - 10*B*a**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*
x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan
(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*
sin(e) + c), True))
```

Giac [A] time = 1.17209, size = 220, normalized size = 1.88

$$-\frac{\frac{3\left(2\,Aa^{2}+3\,Ba^{2}\right)\left(fx+e\right)}{c}+\frac{16\left(Aa^{2}+Ba^{2}\right)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,Aa^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-6\,Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Ba^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Aa^{2}-6\,Ba^{2}\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)^{2}c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(tan(1/2*f*
x + 1/2*e) - 1)) + 2*(B*a^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*tan(1/2*f*x +
1/2*e)^2 - 6*B*a^2*tan(1/2*f*x + 1/2*e)^2 - B*a^2*tan(1/2*f*x + 1/2*e) - 2*
A*a^2 - 6*B*a^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{a^2x(A+4B)}{c^2}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

```
[Out] (a^2*(A + 4*B)*x)/c^2 - (a^2*(A + 4*B)*Cos[e + f*x])/(c^2*f) + (a^2*(A + B)
*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - (2*a^2*(A + 4*B)*Cos[e
+ f*x]^3)/(3*f*(c - c*Sin[e + f*x])^2)
```

Rubi [A] time = 0.283956, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$-\frac{a^2(A+4B)\cos(e+fx)}{c^2f}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}+\frac{a^2x(A+4B)}{c^2}-\frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(A + 4*B)*x)/c^2 - (a^2*(A + 4*B)*Cos[e + f*x])/(c^2*f) + (a^2*(A + B)
*c^2*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - (2*a^2*(A + 4*B)*Cos[e
+ f*x]^3)/(3*f*(c - c*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{1}{3}\left(a^2(A+4B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2} + \frac{\left(a^2(A+4B)\right)\int}{c}$$

$$= -\frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a^2(A+4B)\cos}{3f(c-c\sin(e}$$

$$= \frac{a^2(A+4B)x}{c^2} - \frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a}{}$$

Mathematica [B] time = 0.606948, size = 238, normalized size = 2.18

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(A+4B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A + B
)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*
Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4*(c - c*Sin[e + f*x])^2)
```

Maple [A] time = 0.116, size = 198, normalized size = 1.8

$$-\frac{16\,Aa^2}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-3}-\frac{16\,Ba^2}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-3}-8\,\frac{Aa^2}{fc^2\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-8\,\frac{Ba^2}{fc^2\left(\tan\left(1/2\,fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -16/3/f*a^2/c^2/(tan(1/2*f*x+1/2*e)-1)^3*A-16/3/f*a^2/c^2/(tan(1/2*f*x+1/2*
e)-1)^3*B-8/f*a^2/c^2/(tan(1/2*f*x+1/2*e)-1)^2*A-8/f*a^2/c^2/(tan(1/2*f*x+1
/2*e)-1)^2*B+8/f*a^2/c^2*B/(tan(1/2*f*x+1/2*e)-1)-2/f*a^2/c^2*B/(1+tan(1/2*
f*x+1/2*e)^2)+2/f*a^2/c^2*arctan(tan(1/2*f*x+1/2*e))*A+8/f*a^2/c^2*arctan(t
an(1/2*f*x+1/2*e))*B
```

Maxima [B] time = 1.53119, size = 1133, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + A*a
^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e)/(cos(f*x
 + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
 + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c
^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a^2*(3*sin(f*x + e)/(cos(f*x + e
) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

<u>Fricas</u> [B] time = 1.43296, size = 568, normalized size = 5.21

$$-\frac{3\,Ba^2\cos\left(fx+e\right)^3+6\,(A+4\,B)a^2fx+4\,(A+B)a^2-\left(3\,(A+4\,B)a^2fx+(8\,A+23\,B)a^2\right)\cos\left(fx+e\right)^2+\left(3\,(A+4\,B)\right.}{3\left(c^2f\cos\left(fx+e\right)^2-c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A
+ 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x -
2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*x +
 e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(f*x
+ e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f +
```

```
(c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [A] time = 26.4778, size = 2474, normalized size = 22.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*A*a**2*f*x*t
an(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2
)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c*
*2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c*
*2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2
 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
3*c**2*f) - 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5
 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*A*a**2*f*x
*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)
**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**
2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*A*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)*
*5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2
*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*A*a**2*
tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/
2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c
**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f
*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f
*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c*
*2*f) - 24*A*a**2*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2/(3*c**2*f*tan(
e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)
**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f)
 + 12*B*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2
*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 36*B*a**2*f*x*tan(e/2
 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) + 48*B*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*t
```

```
an(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x
/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 48*B*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 36*B*a**2*f*x*tan(
e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) - 12*B*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 -
 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*B*a**2*tan(
e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**
4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*
f*tan(e/2 + f*x/2) - 3*c**2*f) - 78*B*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*ta
n(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*
f) + 74*B*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 90*B*a**2*tan(e/2 + f*x
/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*
f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f) + 38*B*a**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan
(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))
```

Giac [A] time = 1.19942, size = 182, normalized size = 1.67

$$\frac{\frac{3\left(Aa^2+4\,Ba^2\right)(fx+e)}{c^2}-\frac{6\,Ba^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2}+\frac{8\left(3\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2+4\,Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*e)
 - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1/2*
e) - 1)^3))/f
```

3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=112

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^2Bx}{c^3}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

```
[Out] -((a^2*B*x)/c^3) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x
])^5) - (2*a^2*B*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^2*B*Co
s[e + f*x])/(f*(c^3 - c^3*Sin[e + f*x]))
```

Rubi [A] time = 0.277027, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2680, 8}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}-\frac{a^2Bx}{c^3}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^2*B*x)/c^3) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x
])^5) - (2*a^2*B*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^3) + (2*a^2*B*Co
s[e + f*x])/(f*(c^3 - c^3*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
```

```
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\left(a^2Bc\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}\,dx\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{\left(a^2B\right)\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))}}{c}\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+f\right.}\\ &= -\frac{a^2Bx}{c^3}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\,\mathrm{c}}{f\left(c^3-c^3\right.}\end{aligned}$$

Mathematica [B] time = 0.695633, size = 278, normalized size = 2.48

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+2(3A+43B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 -
 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin[(e
 + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e +
 f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e +
f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4*(c - c*Sin[e + f*x])^3)
```

Maple [B] time = 0.127, size = 249, normalized size = 2.2

$$-2\,{\frac{A{a}^{2}}{f{c}^{3}\left(\tan\left(1/2\,fx+e/2\right)-1\right)}}-2\,{\frac{B{a}^{2}}{f{c}^{3}\left(\tan\left(1/2\,fx+e/2\right)-1\right)}}-{\frac{32\,A{a}^{2}}{5\,f{c}^{3}}\left(\tan\left({\frac{fx}{2}}+{\frac{e}{2}}\right)-1\right)^{-5}}-{\frac{32\,B{a}^{2}}{5\,f{c}^{3}}\left(\tan\left({\frac{fx}{2}}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] -2/f*a^2/c^3/(tan(1/2*f*x+1/2*e)-1)*A-2/f*a^2/c^3/(tan(1/2*f*x+1/2*e)-1)*B-
32/5/f*a^2/c^3/(tan(1/2*f*x+1/2*e)-1)^5*A-32/5/f*a^2/c^3/(tan(1/2*f*x+1/2*e
)-1)^5*B-16/f*a^2/c^3/(tan(1/2*f*x+1/2*e)-1)^3*A-32/3/f*a^2/c^3/(tan(1/2*f*
x+1/2*e)-1)^3*B-16/f*a^2/c^3/(tan(1/2*f*x+1/2*e)-1)^4*A-16/f*a^2/c^3/(tan(1
/2*f*x+1/2*e)-1)^4*B-8/f*a^2/c^3*A/(tan(1/2*f*x+1/2*e)-1)^2-2/f*a^2/c^3*B*a
rctan(tan(1/2*f*x+1/2*e))
```

Maxima [B] time = 1.57944, size = 1538, normalized size = 13.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(B*a^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)
```

```
^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^3
) + A*a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 6*A*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) -
3*B*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*A*a^2*(5*sin(f*x + e)
/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*
c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*a^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
)/f
```

Fricas [B] time = 1.42677, size = 655, normalized size = 5.85

$$\frac{60\,Ba^2fx - \left(15\,Ba^2fx - (3\,A + 43\,B)a^2\right)\cos\left(fx+e\right)^3 - 12\,(A+B)a^2 - \left(45\,Ba^2fx - (9\,A - 11\,B)a^2\right)\cos\left(fx+e\right)^2 + 6\left(5\right.}{15\left(c^3f\cos\left(fx+e\right)^3 + 3\,c^3f\cos\left(fx+e\right)^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12*
(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2
```

```
*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (15*
B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*B)*a
^2)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e
)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21705, size = 215, normalized size = 1.92

$$-\frac{\frac{15\left(fx+e\right)Ba^2}{c^3}+\frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+170\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-100\,Ba^2\,\mathrm{t}\cdots}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 15*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*tan(
1/2*f*x + 1/2*e)^2 + 170*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*tan(1/2*f
*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=75

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c(A-6B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*(A - 6
*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.228854, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 2671}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2c(A-6B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + (a^2*(A - 6
*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^5)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{1}{7}\left(a^2(A-6B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}\,dx\\ &= \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}\end{aligned}$$

Mathematica [B] time = 0.912054, size = 191, normalized size = 2.55

$$-\frac{a^2\left(-35(A+4B)\cos\left(\frac{1}{2}(e+fx)\right)+7(2A+13B)\cos\left(\frac{3}{2}(e+fx)\right)-70A\sin\left(\frac{1}{2}(e+fx)\right)-35A\sin\left(\frac{3}{2}(e+fx)\right)+7A\sin\right.}{140c^4f\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] -(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f*x))/2]
 + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*(e + f*x
))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(3*(e + f*
x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7*B*Sin[(5*
(e + f*x))/2]))/(140*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

Maple [B] time = 0.131, size = 161, normalized size = 2.2

$$2\,\frac{a^2}{fc^4}\left(-1/6\,\frac{96\,A+96\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^6}-1/3\,\frac{42\,A+18\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^3}-1/4\,\frac{96\,A+64\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^4}-1/7\,\frac{32\,A+}{\left(\tan\left(1/2\,fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f*a^2/c^4*(-1/6*(96*A+96*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/3*(42*A+18*B)/(tan
(1/2*f*x+1/2*e)-1)^3-1/4*(96*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/7*(32*A+32*
B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(10*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-A/(tan(
1/2*f*x+1/2*e)-1)-1/5*(128*A+112*B)/(tan(1/2*f*x+1/2*e)-1)^5)
```

Maxima [B] time = 1.1733, size = 2121, normalized size = 28.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(2*A*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/
(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7) + B*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7) - 3*A*a^2*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2
10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*sin(f*x +
 e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^
4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 4*A*a^2*
(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e
```

```
) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7) - 8*B*a^2*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 4
2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 6*B*a^2*(7*
sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c^4*sin(f*x + e)/(cos(
f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7))/f
```

Fricas [B] time = 1.41181, size = 652, normalized size = 8.69

$$-\frac{(A-6B)a^2\cos\left(fx+e\right)^4+(4A+11B)a^2\cos\left(fx+e\right)^3+(13A+27B)a^2\cos\left(fx+e\right)^2-10(A+B)a^2\cos\left(fx+e\right)\cdots}{35\left(c^4f\cos\left(fx+e\right)^4-3c^4f\cos\left(fx+e\right)^3-8c^4f\cos\left(fx+e\right)^2+4c^4f\cos\left(fx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (13
*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*a^
2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10*(A
 + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2)*sin(f*x + e))/(c^4*f*cos(f*x + e)^
4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e)
+ 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*
x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24011, size = 309, normalized size = 4.12

$$-\frac{2\left(35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-35\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+35\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+140\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+35\,Ba\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 +
35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B*a
^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2*f*x
 + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2*e) +
 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=115

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{a^2c(2A-7B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (a^2*(2*A -
 7*B)*c*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (a^2*(2*A - 7*B)*Co
s[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.285661, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}+\frac{a^2c(2A-7B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (a^2*(2*A -
 7*B)*c*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (a^2*(2*A - 7*B)*Co
s[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^5)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{1}{9}\left(a^2(2A-7B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{1}{63}\left(a^2(2A-7\right. \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{a^2(2A-7B)\,c}{315f(c-c\sin}
\end{aligned}$$

Mathematica [B] time = 1.1964, size = 261, normalized size = 2.27

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(315(2A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-63(4A+11B)\cos\left(\frac{3}{2}(e+fx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(315*(2*A
+ 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 315*B*Cos[
(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f*x))/2] +
882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e + f*x))/2]
 + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*B*Sin[(5*(e
+ f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/2]))/(2520*c^
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)
```

Maple [A] time = 0.153, size = 205, normalized size = 1.8

$$2\,{\frac{{a}^{2}}{f{c}^{5}} \left(-1/3\,{\frac{64\,A+22\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{3}}}-1/9\,{\frac{64\,A+64\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{9}}}-1/8\,{\frac{256\,A+256\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{8}}}-1/6\,{\frac{544\,A+}{ \left(\tan \left(1/2\,fx \right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^2/c^5*(-1/3*(64*A+22*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/9*(64*A+64*B)/(tan
(1/2*f*x+1/2*e)-1)^9-1/8*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/6*(544*A+
448*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(200*A+104*B)/(tan(1/2*f*x+1/2*e)-1)^4-
1/7*(480*A+448*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(12*A+2*B)/(tan(1/2*f*x+1/2*
e)-1)^2-A/(tan(1/2*f*x+1/2*e)-1)-1/5*(404*A+276*B)/(tan(1/2*f*x+1/2*e)-1)^5
)
```

Maxima [B] time = 1.25865, size = 2817, normalized size = 24.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
```

4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) - 10*A*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
 10*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) + 14*A*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/
(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^
9) + 28*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) +
 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(c

```
os(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(
cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [B] time = 1.45711, size = 833, normalized size = 7.24

$$\frac{(2A-7B)a^2\cos\left(fx+e\right)^5-4(2A-7B)a^2\cos\left(fx+e\right)^4-5(5A+14B)a^2\cos\left(fx+e\right)^3-5(17A+35B)a^2\cos\left(fx\right.}{315\left(c^5f\cos\left(fx+e\right)^5+5c^5f\cos\left(fx+e\right)^4-8c^5f\cos\left(fx+e\right)^3-20c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 -
5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 + 70
*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x + e)
^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2 + 7
0*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e))/(c^5*f*cos(f*x
+ e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
 + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f
)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23672, size = 406, normalized size = 3.53

$$-\frac{2\left(315\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-630\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+315\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+2310\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/315*(315*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*tan(1/2*f*x + 1/2*e)^7
 + 315*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*tan(1/2*f*x + 1/2*e)^6 + 1
05*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 945*B
*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 63*B*a^2*
tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2*tan(
1/2*f*x + 1/2*e)^3 + 1062*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*tan(1/2*f
*x + 1/2*e)^2 - 108*A*a^2*tan(1/2*f*x + 1/2*e) + 63*B*a^2*tan(1/2*f*x + 1/2
*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=156

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{a^2c(3A-8B)\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*(3*A
- 8*B)*c*Cos[e + f*x]^5)/(99*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(3*A - 8*B)
*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(3*A - 8*B)*Cos[e
+ f*x]^5)/(3465*c*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.374481, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{a^2c(3A-8B)\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*(3*A
- 8*B)*c*Cos[e + f*x]^5)/(99*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(3*A - 8*B)
*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(3*A - 8*B)*Cos[e
+ f*x]^5)/(3465*c*f*(c - c*Sin[e + f*x])^5)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{1}{11}\left(a^2(3A-8B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}\,dx \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{1}{99}\left(2a^2(3A-\right. \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A-8B)}{693f(c-c\,\mathrm{si}} \\
&= \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8} + \frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A-8B)}{693f(c-c\,\mathrm{si}}
\end{aligned}
$$

Mathematica [A] time = 1.54175, size = 285, normalized size = 1.83

$$
\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(231(27A+28B)\cos\left(\frac{1}{2}(e+fx)\right)-2475(A+2B)\cos\left(\frac{3}{2}(e+fx\right.\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A
+ 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*Cos
[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))/2]
 + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(e +
f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310*B*Si
n[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f*x))/2
] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c^6*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)
```

Maple [A] time = 0.153, size = 249, normalized size = 1.6

$$2\,{\frac{{a}^{2}}{f{c}^{6}}}\left(-1/3\,{\frac{90\,A+26\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{3}}}-1/6\,{\frac{1752\,A+1208\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{6}}}-1/9\,{\frac{1536\,A+1472\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{9}}}-1/4\,{\frac{352\,A+}{\left(\tan\left(1/2\,fx\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^2/c^6*(-1/3*(90*A+26*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/6*(1752*A+1208*B)/
(tan(1/2*f*x+1/2*e)-1)^6-1/9*(1536*A+1472*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/4*(
352*A+152*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/8*(2304*A+2048*B)/(tan(1/2*f*x+1/2*
e)-1)^8-1/7*(2376*A+1896*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/11*(128*A+128*B)/(ta
n(1/2*f*x+1/2*e)-1)^11-1/10*(640*A+640*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/2*(14
*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-A/(tan(1/2*f*x+1/2*e)-1)-1/5*(932*A+528*B)
/(tan(1/2*f*x+1/2*e)-1)^5)
```

Maxima [B] time = 1.37671, size = 3515, normalized size = 22.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*A*a^2*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^2*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*B*a^
2*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
```

```
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*A*a^2*(253*sin
(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e
) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*
c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11) + 8*B*a^2*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(
cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 -
 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11))/f
```

Fricas [B] time = 1.46612, size = 1027, normalized size = 6.58

$$-\frac{2\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^6+12\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^5-25\,(3\,A-8\,B)a^2\cos\left(fx+e\right)^4-35\,(6\,A+17\,B)a^2\cos}{3465\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+e\right)^5-18\,c^6f\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e)
^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)^3
 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) + 126
0*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*cos(
```

```
f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*cos(f*
x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x + e))/(
c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 2
0*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) -
32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*
x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(
f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23176, size = 504, normalized size = 3.23

$$-\frac{2\left(3465\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-10395\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+3465\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+41580\,Aa^2\tan\left(\frac{1}{2}\,fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/3465*(3465*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 10395*A*a^2*tan(1/2*f*x + 1/2
*e)^9 + 3465*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*tan(1/2*f*x + 1/2*e
)^8 - 1155*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*tan(1/2*f*x + 1/2*e)^
7 + 16170*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*tan(1/2*f*x + 1/2*e)^
6 - 6006*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*tan(1/2*f*x + 1/2*e)^5
+ 22176*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*tan(1/2*f*x + 1/2*e)^4 -
 3960*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 8
910*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 110
*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*tan(1/2*f*x + 1/2*e) + 671*B*a^2
```

```
*tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(tan(1/2*f*x + 1/2*e)
- 1)^11)
```

3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=197

$$\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}+$$

```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + (a^2*(4*A
- 9*B)*c*Cos[e + f*x]^5)/(143*f*(c - c*Sin[e + f*x])^8) + (a^2*(4*A - 9*B)*
Cos[e + f*x]^5)/(429*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(4*A - 9*B)*Cos[e +
 f*x]^5)/(3003*c*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(4*A - 9*B)*Cos[e + f*x
]^5)/(15015*c^2*f*(c - c*Sin[e + f*x])^5)
```

Rubi [A] time = 0.464564, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}+\frac{a^2c^2(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}+$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```



```
[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + (a^2*(4*A
- 9*B)*c*Cos[e + f*x]^5)/(143*f*(c - c*Sin[e + f*x])^8) + (a^2*(4*A - 9*B)*
Cos[e + f*x]^5)/(429*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(4*A - 9*B)*Cos[e +
 f*x]^5)/(3003*c*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(4*A - 9*B)*Cos[e + f*x
]^5)/(15015*c^2*f*(c - c*Sin[e + f*x])^5)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{1}{13}\left(a^2(4A-9B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{1}{143}\left(3a^2(4A\right.$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4A-9B)}{429f(c-c\,\mathrm{s}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4A-9B)}{429f(c-c\,\mathrm{s}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9} + \frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8} + \frac{a^2(4A-9B)}{429f(c-c\,\mathrm{s}}$$

Mathematica [A] time = 3.59157, size = 313, normalized size = 1.59

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6006(8A+7B)\cos\left(\frac{1}{2}(e+fx)\right)-1716(11A+19B)\cos\left(\frac{3}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6006*(8*A
 + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/2] - 15015*
B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*Cos[(7*(e + f
*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x))/2] + 54912*
A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(3*(e + f*x))/2
] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/2] - 8580*B*Sin
[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[(9*(e + f*x))/2]
 - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))/(240240*c^7*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)
```

Maple [A] time = 0.185, size = 293, normalized size = 1.5

$$2\frac{a^2}{fc^7}\left(-1/10\frac{8320\,A+7680\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{10}}-1/3\frac{120\,A+30\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{3}}-1/13\frac{256\,A+256\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{13}}-1/7\frac{7744\,A}{\left(\tan\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
```

```
[Out] 2/f*a^2/c^7*(-1/10*(8320*A+7680*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/3*(120*A+30*
B)/(tan(1/2*f*x+1/2*e)-1)^3-1/13*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/
7*(7744*A+5368*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/8*(10560*A+8256*B)/(tan(1/2*f*
x+1/2*e)-1)^8-1/11*(4480*A+4352*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/2*(16*A+2*B)
/(tan(1/2*f*x+1/2*e)-1)^2-1/6*(4320*A+2568*B)/(tan(1/2*f*x+1/2*e)-1)^6-A/(t
an(1/2*f*x+1/2*e)-1)-1/9*(10896*A+9360*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/4*(560
*A+208*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/12*(1536*A+1536*B)/(tan(1/2*f*x+1/2*e)
-1)^12-1/5*(1816*A+884*B)/(tan(1/2*f*x+1/2*e)-1)^5)
```

Maxima [B] time = 1.52729, size = 4212, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

```
[Out] -2/45045*(2*A*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 4*B*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) +
 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
 + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^2*(3796*sin(f*x + e)/(cos(f*x + e)
 + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) +
```

1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin
(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(
f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 70*A*a^2*(6
11*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 2
7027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^
11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1
) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos
(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(
f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^2*(611*sin(f*x + e)/(cos(f*x +
e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(co
s(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) +
 1)^13) - 462*B*a^2*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e)

```
 + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e)
+ 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^1
3/(cos(f*x + e) + 1)^13))/f
```

Fricas [B] time = 1.42552, size = 1204, normalized size = 6.11

$$\frac{2\,(4\,A-9\,B)a^2\cos\left(fx+e\right)^7-12\,(4\,A-9\,B)a^2\cos\left(fx+e\right)^6-49\,(4\,A-9\,B)a^2\cos\left(fx+e\right)^5+70\,(4\,A-9\,B)a^2\cos\left(f\right.}{15015\left(c^7f\cos\left(fx+e\right)^7+7\,c^7f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] 1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e)
^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^4
+ 105*(7*A + 20*B)*a^2*cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*cos(f*x + e)^
2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a^2*c
os(f*x + e)^6 + 14*(4*A - 9*B)*a^2*cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2*cos(
f*x + e)^4 - 105*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 105*(3*A + 29*B)*a^2*cos(
f*x + e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2)*sin(f*x + e)
)/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^5
- 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos(f*x + e
)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*c^7*f*co
s(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 + 80*c^7*f
*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29137, size = 601, normalized size = 3.05

$$-\frac{2\left(15015\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-60060\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+15015\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+270270\,Aa^2\tan\left(\frac{1}{2}\,f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x + 1
/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x +
 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*f*x
 + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1/2*f
*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan(1/2*
f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*tan(1/
2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a^2*tan(
1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A*a^2*tan
(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*A*a^2*tan
(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*A*a^2*tan(
1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*a^2*tan(1/2
*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 - 213*B*a^2)/(
c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

3.38 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$

Optimal. Leaf size=265

$$\frac{11a^3c^6(10A-3B)\cos^7(e+fx)}{560f}+\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f}+\frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-\right.}{720f}$$

```
[Out] (11*a^3*(10*A - 3*B)*c^6*x)/256 + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^7)/
(560*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]*Sin[e + f*x])/(256*f) + (11
*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(384*f) + (11*a^3*(10*A
- 3*B)*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(480*f) - (a^3*B*Cos[e + f*x]^7*(c^
2 - c^2*Sin[e + f*x])^3)/(10*f) + (a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^3 - c
^3*Sin[e + f*x])^2)/(90*f) + (11*a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^6 - c^6
*Sin[e + f*x]))/(720*f)
```

Rubi [A] time = 0.390784, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{11a^3c^6(10A-3B)\cos^7(e+fx)}{560f}+\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f}+\frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-\right.}{720f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (11*a^3*(10*A - 3*B)*c^6*x)/256 + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^7)/
(560*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]*Sin[e + f*x])/(256*f) + (11
*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(384*f) + (11*a^3*(10*A
- 3*B)*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(480*f) - (a^3*B*Cos[e + f*x]^7*(c^
2 - c^2*Sin[e + f*x])^3)/(10*f) + (a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^3 - c
^3*Sin[e + f*x])^2)/(90*f) + (11*a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^6 - c^6
*Sin[e + f*x]))/(720*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^6 \, dx = \left(a^3 c^3\right) \int \cos^6(e + fx)(A + B\sin(e + fx))(c - c\sin(e$$

$$= -\frac{a^3 B \cos^7(e + fx)\left(c^2 - c^2\sin(e + fx)\right)^3}{10f} + \frac{1}{10}\left(a^3(10$$

$$= -\frac{a^3 B \cos^7(e + fx)\left(c^2 - c^2\sin(e + fx)\right)^3}{10f} + \frac{a^3(10A -$$

$$= -\frac{a^3 B \cos^7(e + fx)\left(c^2 - c^2\sin(e + fx)\right)^3}{10f} + \frac{a^3(10A -$$

$$= \frac{11a^3(10A - 3B)c^6\cos^7(e + fx)}{560f} - \frac{a^3 B \cos^7(e + fx)\left(c\right.}{1}$$

$$= \frac{11a^3(10A - 3B)c^6\cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B)c^6}{}$$

$$= \frac{11a^3(10A - 3B)c^6\cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B)c^6}{}$$

$$= \frac{11a^3(10A - 3B)c^6\cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B)c^6}{}$$

$$= \frac{11}{256}a^3(10A - 3B)c^6 x + \frac{11a^3(10A - 3B)c^6\cos^7(e + f}{560f}$$

Mathematica [A] time = 4.2867, size = 255, normalized size = 0.96

$$\frac{(a\sin(e + fx) + a)^3 (c - c\sin(e + fx))^6 (27720(10A - 3B)(e + fx) + 1260(144A - 25B)\sin(2(e + fx)) + 2520(6A + 7B)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
6,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*x
) + 5040*(33*A - 19*B)*Cos[e + f*x] + 3360*(29*A - 15*B)*Cos[3*(e + f*x)] +
 10080*(3*A - B)*Cos[5*(e + f*x)] + 360*(9*A + 5*B)*Cos[7*(e + f*x)] - 280*
(A - 3*B)*Cos[9*(e + f*x)] + 1260*(144*A - 25*B)*Sin[2*(e + f*x)] + 2520*(6
*A + 7*B)*Sin[4*(e + f*x)] - 210*(32*A - 51*B)*Sin[6*(e + f*x)] - 315*(6*A
- 5*B)*Sin[8*(e + f*x)] - 126*B*Sin[10*(e + f*x)]))/(645120*f*(Cos[(e + f*x
```

```
)/2] - Sin[(e + f*x)/2])^12*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.148, size = 651, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x)
```

```
[Out] 1/f*(A*a^3*c^6*(f*x+e)-8/3*A*a^3*c^6*(2+sin(f*x+e)^2)*cos(f*x+e)+8*B*a^3*c^
6*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*A*a^3*c^6
*cos(f*x+e)-3*B*a^3*c^6*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^
6*cos(f*x+e)-1/9*A*a^3*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(
f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)-3*A*a^3*c^6*(-1/8*(sin(f*x+e)^7+7/6
*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35
/128*e)-8/7*B*a^3*c^6*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)
*cos(f*x+e)+1/3*B*a^3*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f
*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)+6/5*A*a^3*c^6*(8/3+sin(f*x+e)^4+4/3*
sin(f*x+e)^2)*cos(f*x+e)-6*A*a^3*c^6*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)+B*a^3*c^6*(-1/10*(sin(f*x+e)^9+9/8*sin(f*x+e)^7+21/
16*sin(f*x+e)^5+105/64*sin(f*x+e)^3+315/128*sin(f*x+e))*cos(f*x+e)+63/256*f
*x+63/256*e)+8*A*a^3*c^6*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+
e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*B*a^3*c^6*(8/3+sin(f*x+e)^4+4/3*sin(f*x
+e)^2)*cos(f*x+e)-6*B*a^3*c^6*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin
(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e))
```

Maxima [B] time = 1.04242, size = 892, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -1/645120*(2048*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^
5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*A*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c^6 - 1720320*(cos(f*x +
```

```
e)^3 - 3*cos(f*x + e))*A*a^3*c^6 + 630*(128*sin(2*f*x + 2*e)^3 + 840*f*x +
840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A
*a^3*c^6 - 26880*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e)
 - 48*sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + sin(4*f*x + 4*e
) - 8*sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 6144*(35*c
os(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^5 - 420*cos(f*x + e)^
3 + 315*cos(f*x + e))*B*a^3*c^6 - 147456*(5*cos(f*x + e)^7 - 21*cos(f*x + e
)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^6 - 258048*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^6 + 63*(32*sin(2*f*x +
2*e)^5 - 640*sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*sin(8*f*x + 8*e) -
 600*sin(4*f*x + 4*e) + 2560*sin(2*f*x + 2*e))*B*a^3*c^6 + 20160*(4*sin(2*f
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a
^3*c^6 - 161280*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a
^3*c^6 + 483840*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^6 - 1935360*A*a^3*
c^6*cos(f*x + e) + 645120*B*a^3*c^6*cos(f*x + e))/f
```

Fricas [A] time = 1.75442, size = 455, normalized size = 1.72

$$-\frac{8960\,(A-3\,B)a^3c^6\cos\left(fx+e\right)^9-46080\,(A-B)a^3c^6\cos\left(fx+e\right)^7-3465\,(10\,A-3\,B)a^3c^6fx+21\left(384\,Ba^3c^6\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] -1/80640*(8960*(A - 3*B)*a^3*c^6*cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*cos
(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*cos(f*x + e
)^9 + 48*(30*A - 41*B)*a^3*c^6*cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*cos
(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*cos(f*x + e)^3 - 165*(10*A - 3*B)*a^
3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 83.288, size = 1948, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)
```

```
[Out] Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(e
 + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A*a
**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e + f*
x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**3*c*
*6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x)**2*
cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105
*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6/2 - 9*
A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e + f*x)*
*8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) -
8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*c**6*sin(e
 + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)**5*cos(e +
 f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) + 6*A*a*
*3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e + f*x)**3*co
s(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**3/(3*f
) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A*a**3*c**6*sin(
e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e + f*x)**2*cos(e +
f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f + 105*A*a**3*c**6*
sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(e + f*x)*cos(e + f
*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 128*A*a**
3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(e + f*x)**5/(5*f) - 16*
A*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*A*a**3*c**6*cos(e + f*x)/f + 63*B*a**
3*c**6*x*sin(e + f*x)**10/256 + 315*B*a**3*c**6*x*sin(e + f*x)**8*cos(e + f
*x)**2/256 + 315*B*a**3*c**6*x*sin(e + f*x)**6*cos(e + f*x)**4/128 - 15*B*a
**3*c**6*x*sin(e + f*x)**6/8 + 315*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*
x)**6/128 - 45*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*B*a**3*c
**6*x*sin(e + f*x)**4 + 315*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**8/2
56 - 45*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/8 + 6*B*a**3*c**6*x*s
in(e + f*x)**2*cos(e + f*x)**2 - 3*B*a**3*c**6*x*sin(e + f*x)**2/2 + 63*B*a
**3*c**6*x*cos(e + f*x)**10/256 - 15*B*a**3*c**6*x*cos(e + f*x)**6/8 + 3*B*
a**3*c**6*x*cos(e + f*x)**4 - 3*B*a**3*c**6*x*cos(e + f*x)**2/2 - 193*B*a**
3*c**6*sin(e + f*x)**9*cos(e + f*x)/(256*f) + 3*B*a**3*c**6*sin(e + f*x)**8
*cos(e + f*x)/f - 237*B*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)**3/(128*f) +
 8*B*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/f - 8*B*a**3*c**6*sin(e + f*
x)**6*cos(e + f*x)/f - 21*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**5/(10*f
) + 33*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 48*B*a**3*c**6*sin(
e + f*x)**4*cos(e + f*x)**5/(5*f) - 16*B*a**3*c**6*sin(e + f*x)**4*cos(e +
f*x)**3/f + 6*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f - 147*B*a**3*c**6*
sin(e + f*x)**3*cos(e + f*x)**7/(128*f) + 5*B*a**3*c**6*sin(e + f*x)**3*cos
(e + f*x)**3/f - 5*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/f + 192*B*a**3*
c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 64*B*a**3*c**6*sin(e + f*x)**
2*cos(e + f*x)**5/(5*f) + 8*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**3/f -
 63*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**9/(256*f) + 15*B*a**3*c**6*sin(e
 + f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
f + 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)/(2*f) + 128*B*a**3*c**6*cos(e +
 f*x)**9/(105*f) - 128*B*a**3*c**6*cos(e + f*x)**7/(35*f) + 16*B*a**3*c**6*
```

```
cos(e + f*x)**5/(5*f) - B*a**3*c**6*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*si
n(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))
```

Giac [A] time = 1.48789, size = 468, normalized size = 1.77

$$-\frac{Ba^3c^6\sin\left(10\,fx+10\,e\right)}{5120\,f}+\frac{11}{256}\left(10\,Aa^3c^6-3\,Ba^3c^6\right)x-\frac{\left(Aa^3c^6-3\,Ba^3c^6\right)\cos\left(9\,fx+9\,e\right)}{2304\,f}+\frac{\left(9\,Aa^3c^6+5\,Ba^3c^6\right)\cos}{1792\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/5120*B*a^3*c^6*sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^6
)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*cos(9*f*x + 9*e)/f + 1/1792*(9*A*a^3
*c^6 + 5*B*a^3*c^6)*cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*cos
(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*cos(3*f*x + 3*e)/f +
1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*cos(f*x + e)/f - 1/2048*(6*A*a^3*c^6 -
5*B*a^3*c^6)*sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6)*sin(
6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*sin(4*f*x + 4*e)/f + 1/5
12*(144*A*a^3*c^6 - 25*B*a^3*c^6)*sin(2*f*x + 2*e)/f
```

3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal. Leaf size=222

$$\frac{a^3c^5(9A-2B)\cos^7(e+fx)}{56f}+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}+\frac{a^3c^5(9A-2B)\sin(e+fx)\cos^5(e+fx)}{48f}$$

```
[Out] (5*a^3*(9*A - 2*B)*c^5*x)/128 + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^7)/(56*f)
 + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(9*A
- 2*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(9*A - 2*B)*c^5*Cos[
e + f*x]^5*Sin[e + f*x])/(48*f) - (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e +
f*x])^2)/(9*f) + (a^3*(9*A - 2*B)*Cos[e + f*x]^7*(c^5 - c^5*Sin[e + f*x]))/
(72*f)
```

Rubi [A] time = 0.322109, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^3c^5(9A-2B)\cos^7(e+fx)}{56f}+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}+\frac{a^3c^5(9A-2B)\sin(e+fx)\cos^5(e+fx)}{48f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```


```
[Out] (5*a^3*(9*A - 2*B)*c^5*x)/128 + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^7)/(56*f)
 + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(9*A
- 2*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(9*A - 2*B)*c^5*Cos[
e + f*x]^5*Sin[e + f*x])/(48*f) - (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e +
f*x])^2)/(9*f) + (a^3*(9*A - 2*B)*Cos[e + f*x]^7*(c^5 - c^5*Sin[e + f*x]))/
(72*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{1}{9}\left(a^3(9A-\right.$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f} + \frac{a^3(9A-2B}{}$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} - \frac{a^3Bc^3\cos^7(e+fx)(c-}{9f}$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{a^3(9A-2B)c^5\cos^5(e+}{48f}$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A-2B)c^5\cos^3(e}{192f}$$

$$= \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5a^3(9A-2B)c^5\cos(e+}{128f}$$

$$= \frac{5}{128}a^3(9A-2B)c^5x + \frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f} + \frac{5}{}$$

Mathematica [A] time = 2.52293, size = 232, normalized size = 1.05

$$\frac{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5(2520(9A-2B)(e+fx)+2016(8A-B)\sin(2(e+fx))+504(5A+2B)\sin(4(e+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
5,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x)
+ 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 100
8*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[9*(
e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e + f*
x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6)
```

Maple [B] time = 0.036, size = 611, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

```
[Out] 1/f*(A*a^3*c^5*(f*x+e)-2/7*B*a^3*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/
5*sin(f*x+e)^2)*cos(f*x+e)-6*B*a^3*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3
+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/9*B*a^3*c^5*(128/35+sin(f*x
+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)+2*
B*a^3*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin
(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+6/5*A*a^3*c^5*(8/3+sin(f*x+e)^4+4/
3*sin(f*x+e)^2)*cos(f*x+e)-A*a^3*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+3
5/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*A*a
^3*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*A
*a^3*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)+2*A*a^3*c^5*cos(f*x+e)-2*B*a^3*c^5*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2*A*a^3*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^3*c^5*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*A*a^3*c^5*(-1/2
*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2/3*B*a^3*c^5*(2+sin(f*x+e)^2)*cos(f*
x+e)-B*a^3*c^5*cos(f*x+e))
```

Maxima [B] time = 1.03487, size = 833, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] 1/322560*(18432*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^3*c^5 + 129024*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^3*c^5 + 645120*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^3*c^5 - 105*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e)
 + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*sin(2*f
*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a
^3*c^5 - 161280*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*(f*x +
e)*A*a^3*c^5 + 1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x +
 e)^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^5 + 18432*(5*cos(f*x
```

```
 + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^
5 - 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^5 + 210*(128*sin(2*f*x
 + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 7
68*sin(2*f*x + 2*e))*B*a^3*c^5 - 10080*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*
e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^5 + 60480*(12*f*x + 1
2*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^5 - 161280*(2*f*x + 2*
e - sin(2*f*x + 2*e))*B*a^3*c^5 + 645120*A*a^3*c^5*cos(f*x + e) - 322560*B*
a^3*c^5*cos(f*x + e))/f
```

Fricas [A] time = 1.82007, size = 381, normalized size = 1.72

$$\frac{896\,Ba^3c^5\cos\left(fx+e\right)^9+2304\,(A-B)a^3c^5\cos\left(fx+e\right)^7+315\,(9\,A-2\,B)a^3c^5fx-21\left(48\,(A-2\,B)a^3c^5\cos\left(fx+e\right)^7-\right.}{8064\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7
+ 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 - 8
*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^3
 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 52.2156, size = 1753, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

```
[Out] Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e +
 f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a**
3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f*x)
**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32
 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*sin(e
 + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*cos(e +
 f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a**3*c**5
```

```
*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)**6*cos(e
 + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*A*
a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(e + f*x)**
4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*
a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c**5*sin(e + f
*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f - 6*A*a**3*c**5*
sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**
7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) + A*a**3*c**5*
sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)**7/(35*f) + 16*A*
a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f*x)**3/f + 2*A*a**
3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8/64 + 35*B*a**3*c**
5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c**5*x*sin(e + f*x)**6/8
 + 105*B*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/32 - 45*B*a**3*c**5*x*
sin(e + f*x)**4*cos(e + f*x)**2/8 + 9*B*a**3*c**5*x*sin(e + f*x)**4/4 + 35*
B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/16 - 45*B*a**3*c**5*x*sin(e +
 f*x)**2*cos(e + f*x)**4/8 + 9*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**
2/2 - B*a**3*c**5*x*sin(e + f*x)**2 + 35*B*a**3*c**5*x*cos(e + f*x)**8/64 -
 15*B*a**3*c**5*x*cos(e + f*x)**6/8 + 9*B*a**3*c**5*x*cos(e + f*x)**4/4 - B
*a**3*c**5*x*cos(e + f*x)**2 + B*a**3*c**5*sin(e + f*x)**8*cos(e + f*x)/f -
 93*B*a**3*c**5*sin(e + f*x)**7*cos(e + f*x)/(64*f) + 8*B*a**3*c**5*sin(e +
 f*x)**6*cos(e + f*x)**3/(3*f) - 2*B*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)
/f - 511*B*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(192*f) + 33*B*a**3*c*
*5*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 16*B*a**3*c**5*sin(e + f*x)**4*cos(
e + f*x)**5/(5*f) - 4*B*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f - 385*B
*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(192*f) + 5*B*a**3*c**5*sin(e +
f*x)**3*cos(e + f*x)**3/f - 15*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)/(4*
f) + 64*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 16*B*a**3*c**5
*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**5*sin(e + f*x)**2*cos(
e + f*x)/f - 35*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**7/(64*f) + 15*B*a**3
*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) - 9*B*a**3*c**5*sin(e + f*x)*cos(e
 + f*x)**3/(4*f) + B*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f + 128*B*a**3*c**
5*cos(e + f*x)**9/(315*f) - 32*B*a**3*c**5*cos(e + f*x)**7/(35*f) + 4*B*a**
3*c**5*cos(e + f*x)**3/(3*f) - B*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(A
 + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**5, True))
```

Giac [A] time = 1.34458, size = 406, normalized size = 1.83

$$\frac{Ba^3c^5\cos\left(9\,fx+9\,e\right)}{2304\,f}+\frac{Ba^3c^5\sin\left(6\,fx+6\,e\right)}{96\,f}+\frac{5}{128}\left(9\,Aa^3c^5-2\,Ba^3c^5\right)x+\frac{\left(8\,Aa^3c^5-Ba^3c^5\right)\cos\left(7\,fx+7\,e\right)}{1792\,f}+\frac{(2\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] 1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f + 5
/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos(7
*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/192*(
18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 - 13*
B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x + 8*
e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8*A*a^3
*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f
```

3.40 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal. Leaf size=181

$$\frac{a^3c^4(8A-B)\cos^7(e+fx)}{56f}+\frac{a^3c^4(8A-B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^4(8A-B)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5a^3c^4(}{}$$

```
[Out] (5*a^3*(8*A - B)*c^4*x)/128 + (a^3*(8*A - B)*c^4*Cos[e + f*x]^7)/(56*f) + (
5*a^3*(8*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(8*A - B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(8*A - B)*c^4*Cos[e + f*x]^5
*Sin[e + f*x])/(48*f) - (a^3*B*Cos[e + f*x]^7*(c^4 - c^4*Sin[e + f*x]))/(8*
f)
```

Rubi [A] time = 0.233604, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{a^3c^4(8A-B)\cos^7(e+fx)}{56f}+\frac{a^3c^4(8A-B)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5a^3c^4(8A-B)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5a^3c^4(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```



```
[Out] (5*a^3*(8*A - B)*c^4*x)/128 + (a^3*(8*A - B)*c^4*Cos[e + f*x]^7)/(56*f) + (
5*a^3*(8*A - B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(8*A - B)*c
^4*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(8*A - B)*c^4*Cos[e + f*x]^5
*Sin[e + f*x])/(48*f) - (a^3*B*Cos[e + f*x]^7*(c^4 - c^4*Sin[e + f*x]))/(8*
f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx = \left(a^3c^3\right)\int \cos^6(e + fx)(A + B\sin(e + fx))(c - c\sin(e$$

$$= -\frac{a^3B\cos^7(e + fx)\left(c^4 - c^4\sin(e + fx)\right)}{8f} + \frac{1}{8}\left(a^3(8A\right.$$

$$= \frac{a^3(8A - B)c^4\cos^7(e + fx)}{56f} - \frac{a^3B\cos^7(e + fx)\left(c^4 - \right.}{8f}$$

$$= \frac{a^3(8A - B)c^4\cos^7(e + fx)}{56f} + \frac{a^3(8A - B)c^4\cos^5(e +}{48f}$$

$$= \frac{a^3(8A - B)c^4\cos^7(e + fx)}{56f} + \frac{5a^3(8A - B)c^4\cos^3(e +}{192f}$$

$$= \frac{a^3(8A - B)c^4\cos^7(e + fx)}{56f} + \frac{5a^3(8A - B)c^4\cos(e +}{128f}$$

$$= \frac{5}{128}a^3(8A - B)c^4x + \frac{a^3(8A - B)c^4\cos^7(e + fx)}{56f} + \frac{5a}{}$$

Mathematica [A] time = 1.88535, size = 209, normalized size = 1.15

$$\frac{(a\sin(e + fx) + a)^3(c - c\sin(e + fx))^4(840(8A - B)(e + fx) + 336(15A - B)\sin(2(e + fx)) + 168(6A + B)\sin(4(e + f}{21504f\left(\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) + 1
680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Cos[
5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f*x)
] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*B*Si
n[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.032, size = 568, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

```
[Out] 1/f*(A*a^3*c^4*(f*x+e)+B*a^3*c^4*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24
*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+1/7*B*a^3*c
^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-3*B*a^3
*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*
f*x+5/16*e)-3/5*B*a^3*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*
a^3*c^4*cos(f*x+e)-B*a^3*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/7
*A*a^3*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)
-A*a^3*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)
+5/16*f*x+5/16*e)+3/5*A*a^3*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x
+e)+3*A*a^3*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)+B*a^3*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-3*A*a^3*c^4*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-A*a^3*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^4*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-B*a^3*c^4*cos(f*x
+e))
```

<u>Maxima</u> [B] time = 1.01738, size = 771, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3
*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c
^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^
5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f*x + e)^
5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f*x + e)^3
 - 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
 + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*
c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
```

```
^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*x + e))/f
```

Fricas [A] time = 1.59575, size = 315, normalized size = 1.74

$$\frac{384\,(A-B)a^3c^4\cos\left(fx+e\right)^7+105\,(8\,A-B)a^3c^4fx+7\left(48\,Ba^3c^4\cos\left(fx+e\right)^7+8\,(8\,A-B)a^3c^4\cos\left(fx+e\right)^5+10\,(\right.}{2688\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7*
(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8*A
 - B)*a^3*c^4*cos(f*x + e)^3 + 15*(8*A - B)*a^3*c^4*cos(f*x + e))*sin(f*x +
 e))/f
```

Sympy [A] time = 36.1194, size = 1579, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

```
[Out] Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*cos(e
 + f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f +
 11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*sin(e +
 f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
 5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c**4*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)*
*5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A*a**3*c**4*
sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**5
/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**3*c**4*
sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)**7/(35*f) + 8
```

```
*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f*x)**3/f + A*a*
*3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/128 + 35*B*a**3*c
**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*x*sin(e + f*x)**6
/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 45*B*a**3*c**4
*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*sin(e + f*x)**4/8 +
 35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 45*B*a**3*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 - B*a**3*c**4*x*sin(e + f*x)**2/2 + 35*B*a**3*c**4*x*cos(e + f*x)*
*8/128 - 15*B*a**3*c**4*x*cos(e + f*x)**6/16 + 9*B*a**3*c**4*x*cos(e + f*x)
**4/8 - B*a**3*c**4*x*cos(e + f*x)**2/2 - 93*B*a**3*c**4*sin(e + f*x)**7*co
s(e + f*x)/(128*f) + B*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f - 511*B*a**
3*c**4*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 33*B*a**3*c**4*sin(e + f*x
)**5*cos(e + f*x)/(16*f) + 2*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
- 3*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 385*B*a**3*c**4*sin(e + f*
x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)*
*3/(2*f) - 15*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*B*a**3*c**
4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**3/f + 3*B*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 35*B*a**3*c
**4*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 15*B*a**3*c**4*sin(e + f*x)*cos(
e + f*x)**5/(16*f) - 9*B*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + B*a
**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*B*a**3*c**4*cos(e + f*x)**7/(
35*f) - 8*B*a**3*c**4*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**4*cos(e + f*x)**3
/f - B*a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3*(-c*sin(e) + c)**4, True))
```

Giac [A] time = 1.29101, size = 369, normalized size = 2.04

$$\frac{Ba^3c^4\sin\left(8\,fx+8\,e\right)}{1024\,f}+\frac{5}{128}\left(8\,Aa^3c^4-Ba^3c^4\right)x+\frac{\left(Aa^3c^4-Ba^3c^4\right)\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{\left(Aa^3c^4-Ba^3c^4\right)\cos\left(5\,fx+5\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x + 1
/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3*c
^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f +
5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*c^4)
*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)/f +
1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f
```

3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=117

$$\frac{a^3Ac^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3Ac^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3Ac^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3Ac^3x-\frac{a^3}{}$$

```
[Out] (5*a^3*A*c^3*x)/16 - (a^3*B*c^3*Cos[e + f*x]^7)/(7*f) + (5*a^3*A*c^3*Cos[e
+ f*x]*Sin[e + f*x])/(16*f) + (5*a^3*A*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(24
*f) + (a^3*A*c^3*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rubi [A] time = 0.147686, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2669, 2635, 8}

$$\frac{a^3Ac^3\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5a^3Ac^3\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{5a^3Ac^3\sin(e+fx)\cos(e+fx)}{16f}+\frac{5}{16}a^3Ac^3x-\frac{a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (5*a^3*A*c^3*x)/16 - (a^3*B*c^3*Cos[e + f*x]^7)/(7*f) + (5*a^3*A*c^3*Cos[e
+ f*x]*Sin[e + f*x])/(16*f) + (5*a^3*A*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(24
*f) + (a^3*A*c^3*Cos[e + f*x]^5*Sin[e + f*x])/(6*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))\,dx\\ &= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\left(a^3Ac^3\right)\int \cos^6(e+fx)\,dx\\ &= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx}{6f}\\ &= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos^3(e+fx)\sin(e+f}{24f}\\ &= -\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos(e+fx)\sin(e+fx}{16f}\\ &= \frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos(e+fx)}{16f}\end{aligned}$$

Mathematica [A] time = 0.225704, size = 64, normalized size = 0.55

$$\frac{a^3c^3\left(7A(45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))+60e+60fx)-192B\cos^7(e+fx)\right)}{1344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] (a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)]
+ 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)
```

Maple [B] time = 0.024, size = 263, normalized size = 2.3

$$\frac{1}{f}\left(\frac{Ba^3c^3\cos\left(fx+e\right)}{7}\left(\frac{16}{5}+\left(\sin\left(fx+e\right)\right)^6+\frac{6\left(\sin\left(fx+e\right)\right)^4}{5}+\frac{8\left(\sin\left(fx+e\right)\right)^2}{5}\right)-Aa^3c^3\left(-\frac{\cos\left(fx+e\right)}{6}\left(\left(\sin\left(f\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(1/7*B*a^3*c^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*co
s(f*x+e)-A*a^3*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*co
s(f*x+e)+5/16*f*x+5/16*e)-3/5*B*a^3*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)
*cos(f*x+e)+3*A*a^3*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*
f*x+3/8*e)+B*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*A*a^3*c^3*(-1/2*sin(f*x+
e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^3*cos(f*x+e)+A*a^3*c^3*(f*x+e))
```

Maxima [B] time = 0.992634, size = 356, normalized size = 3.04

$$-\frac{35\left(4\sin\left(2fx+2e\right)^3+60fx+60e+9\sin\left(4fx+4e\right)-48\sin\left(2fx+2e\right)\right)Aa^3c^3-630\left(12fx+12e+\sin\left(4fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^3
 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 3
5*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5 - 10
*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f
```

Fricas [A] time = 1.45975, size = 221, normalized size = 1.89

$$-\frac{48Ba^3c^3\cos\left(fx+e\right)^7-105Aa^3c^3fx-7\left(8Aa^3c^3\cos\left(fx+e\right)^5+10Aa^3c^3\cos\left(fx+e\right)^3+15Aa^3c^3\cos\left(fx+e\right)\right)\mathrm{si}}{336f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*co
s(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*sin
(f*x + e))/f
```

Sympy [A] time = 15.9223, size = 682, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*cos(e
 + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/
(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c*
*3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*cos(e +
f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**
3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)**6*cos(e
+ f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)
**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f + 3*B*a**3*c**3
*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)**7/(35*f) - 8
*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f*x)**3/f - B*a*
*3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*
sin(e) + c)**3, True))
```

Giac [A] time = 1.30246, size = 219, normalized size = 1.87

$$\frac{5}{16}Aa^3c^3x - \frac{Ba^3c^3\cos\left(7fx+7e\right)}{448f} - \frac{Ba^3c^3\cos\left(5fx+5e\right)}{64f} - \frac{3Ba^3c^3\cos\left(3fx+3e\right)}{64f} - \frac{5Ba^3c^3\cos\left(fx+e\right)}{64f} + \frac{Aa^3c^3 s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos(
5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f*x
 + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x + 4
*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f
```

3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{a^3c^2(6A+B)\cos^5(e+fx)}{30f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^3c^2x$$

```
[Out] (a^3*(6*A + B)*c^2*x)/16 - (a^3*(6*A + B)*c^2*Cos[e + f*x]^5)/(30*f) + (a^3
*(6*A + B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^3*(6*A + B)*c^2*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (B*c^2*Cos[e + f*x]^5*(a^3 + a^3*Sin[e + f
*x]))/(6*f)
```

Rubi [A] time = 0.199824, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2669, 2635, 8}

$$-\frac{a^3c^2(6A+B)\cos^5(e+fx)}{30f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{a^3c^2(6A+B)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}a^3c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(6*A + B)*c^2*x)/16 - (a^3*(6*A + B)*c^2*Cos[e + f*x]^5)/(30*f) + (a^3
*(6*A + B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^3*(6*A + B)*c^2*Cos[e
 + f*x]^3*Sin[e + f*x])/(24*f) - (B*c^2*Cos[e + f*x]^5*(a^3 + a^3*Sin[e + f
*x]))/(6*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(a+a\sin(e+fx))(A+B\sin(e$$

$$= -\frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f}+\frac{1}{6}\left(a^2(6A\right.$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}-\frac{Bc^2\cos^5(e+fx)\left(a^3\right.}{6f}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(6A+B)c^2\cos^3(e}{24f}$$

$$= -\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(6A+B)c^2\cos(e+}{16f}$$

$$= \frac{1}{16}a^3(6A+B)c^2x-\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}+\frac{a^3(}{}$$

Mathematica [A] time = 1.03372, size = 133, normalized size = 0.96

$$\frac{a^3c^2(-120(A+B)\cos(e+fx)-60(A+B)\cos(3(e+fx))+240A\sin(2(e+fx))+30A\sin(4(e+fx))-12A\cos(5(e+fx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] (a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*x
] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Maple [B] time = 0.03, size = 364, normalized size = 2.6

$$\frac{1}{f}\left(-\frac{Aa^3c^2\cos\left(fx+e\right)}{5}\left(\frac{8}{3}+\left(\sin\left(fx+e\right)\right)^4+\frac{4\left(\sin\left(fx+e\right)\right)^2}{3}\right)+Aa^3c^2\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx\right.}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/5*A*a^3*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*a^3*c^
2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*A*a^3*c
^2*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)
^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/5*B*a^3*c^2*(8/3+sin(f*x+
e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*B*a^3*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*A*a^3*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)+2/3*B*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-A*a^3*c^2*cos(f*x+e)
+B*a^3*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*a^3*c^2*(f*x+e)-B*a
^3*c^2*cos(f*x+e))
```

Maxima [B] time = 0.992559, size = 486, normalized size = 3.52

$$-\frac{64\left(3\cos\left(fx+e\right)^5-10\cos\left(fx+e\right)^3+15\cos\left(fx+e\right)\right)Aa^3c^2+640\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^3c^2-30\left(12\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c
^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 - 30*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2 + 480*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a^3*c^2 - 960*(f*x + e)*A*a^3*c^2 + 64*(3*cos(f*x + e)^5 - 1
0*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2 + 640*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a^3*c^2 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*
x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^2 + 60*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e)
)*B*a^3*c^2 + 960*A*a^3*c^2*cos(f*x + e) + 960*B*a^3*c^2*cos(f*x + e))/f
```

Fricas [A] time = 1.50649, size = 258, normalized size = 1.87

$$-\frac{48\,(A+B)a^3c^2\cos\left(fx+e\right)^5 - 15\,(6\,A+B)a^3c^2fx + 5\left(8\,Ba^3c^2\cos\left(fx+e\right)^5 - 2\,(6\,A+B)a^3c^2\cos\left(fx+e\right)^3 - 3\,(6\,A\right.}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(8
*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A + B
)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 11.6746, size = 910, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*cos
```

```
(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*c**
2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e + f*
x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a**3*c
**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e + f*x
)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2*cos(e
+ f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**2*cos(e
+ f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15*B*a**3*c*
*2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c**2*x*cos(e
+ f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2*x*cos(e + f*
x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**3*c**2*
sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**3*c**
2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*sin(e + f*x)**2*cos
(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*B*a**3*
c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*B*a**3*c**2*cos(e + f
*x)**3/(3*f) - B*a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**3*(-c*sin(e) + c)**2, True))
```

Giac [A] time = 1.17512, size = 275, normalized size = 1.99

$$-\frac{Ba^3c^2\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(6\,Aa^3c^2+Ba^3c^2\right)x-\frac{\left(Aa^3c^2+Ba^3c^2\right)\cos\left(5\,fx+5\,e\right)}{80\,f}-\frac{\left(Aa^3c^2+Ba^3c^2\right)\cos\left(3\,fx+3\,e\right)}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*c^2*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^3*c^2 + B*a^3*c^2)*x - 1/
80*(A*a^3*c^2 + B*a^3*c^2)*cos(5*f*x + 5*e)/f - 1/16*(A*a^3*c^2 + B*a^3*c^2
)*cos(3*f*x + 3*e)/f - 1/8*(A*a^3*c^2 + B*a^3*c^2)*cos(f*x + e)/f + 1/64*(2
*A*a^3*c^2 - B*a^3*c^2)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^3*c^2 + B*a^3*c^2
)*sin(2*f*x + 2*e)/f
```

3.43 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^3c(5A+2B)\cos^3(e+fx)}{12f}-\frac{c(5A+2B)\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{20f}+\frac{a^3c(5A+2B)\sin(e+fx)\cos(e+fx)}{8f}$$

```
[Out] (a^3*(5*A + 2*B)*c*x)/8 - (a^3*(5*A + 2*B)*c*Cos[e + f*x]^3)/(12*f) + (a^3*
(5*A + 2*B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(a +
 a*Sin[e + f*x])^2)/(5*f) - ((5*A + 2*B)*c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e
+ f*x]))/(20*f)
```

Rubi [A] time = 0.222197, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$-\frac{a^3c(5A+2B)\cos^3(e+fx)}{12f}-\frac{c(5A+2B)\cos^3(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{20f}+\frac{a^3c(5A+2B)\sin(e+fx)\cos(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^3*(5*A + 2*B)*c*x)/8 - (a^3*(5*A + 2*B)*c*Cos[e + f*x]^3)/(12*f) + (a^3*
(5*A + 2*B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(a +
 a*Sin[e + f*x])^2)/(5*f) - ((5*A + 2*B)*c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e
+ f*x]))/(20*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = (ac)\int \cos^2(e+fx)(a+a\sin(e+fx))^2(A+B\sin(e+$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} + \frac{1}{5}(a(5A+2B$$

$$= -\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f} - \frac{(5A+2B)c\,c$$

$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} - \frac{aBc\cos^3(e+fx)(a+a}{5f}$$

$$= -\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} + \frac{a^3(5A+2B)c\cos(e+}{8f}$$

$$= \frac{1}{8}a^3(5A+2B)cx - \frac{a^3(5A+2B)c\cos^3(e+fx)}{12f} + \frac{a^3(5$$

Mathematica [A] time = 0.824205, size = 95, normalized size = 0.68

$$\frac{a^3c(15(-(A+2B)\sin(4(e+fx))+4fx(5A+2B)+8A\sin(2(e+fx)))-60(4A+3B)\cos(e+fx)-10(8A+5B)\cos(3}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] (a^3*c*(-60*(4*A + 3*B)*Cos[e + f*x] - 10*(8*A + 5*B)*Cos[3*(e + f*x)] + 6*
B*Cos[5*(e + f*x)] + 15*(4*(5*A + 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A + 2*
B)*Sin[4*(e + f*x)])))/(480*f)
```

Maple [A] time = 0.031, size = 208, normalized size = 1.5

$$\frac{1}{f}\left(-Aa^3c\left(-\frac{\cos(fx+e)}{4}\left(\left(\sin(fx+e)\right)^3+\frac{3\sin(fx+e)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2Aa^3c\left(2+\left(\sin(fx+e)\right)^2\right)\cos(fx+e)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] 1/f*(-A*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
+2/3*A*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+1/5*B*a^3*c*(8/3+sin(f*x+e)^4+4/3*
sin(f*x+e)^2)*cos(f*x+e)-2*B*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-2*A*a^3*c*cos(f*x+e)+2*B*a^3*c*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+A*a^3*c*(f*x+e)-B*a^3*c*cos(f*x+e))
```

Maxima [A] time = 0.968768, size = 270, normalized size = 1.93

$$-\frac{320\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)Aa^{3}c+15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)Aa^{3}c-480\left(fx+e\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c - 3
2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*(12
*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f*x +
 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^3*c*c
os(f*x + e))/f
```

Fricas [A] time = 1.34974, size = 248, normalized size = 1.77

$$\frac{24\,Ba^{3}c\cos\left(fx+e\right)^{5}-80\,(A+B)a^{3}c\cos\left(fx+e\right)^{3}+15\,(5\,A+2\,B)a^{3}cfx-15\left(2\,(A+2\,B)a^{3}c\cos\left(fx+e\right)^{3}-(5\,A+2}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5*
A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a^3
*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 6.0621, size = 486, normalized size = 3.47

$$\begin{cases} -\frac{3Aa^3cx\sin^4(e+fx)}{8} - \frac{3Aa^3cx\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3Aa^3cx\cos^4(e+fx)}{8} + Aa^3cx + \frac{5Aa^3c\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{2Aa^3c\sin^2(e+fx)\cos \\ x(A+B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3*c
*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e + f*
x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e + f
*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**4/4 -
 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e + f*x)**
2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*a**3*c*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(4
*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*c*sin(e +
 f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*
*3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B
*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))
```

Giac [A] time = 1.134, size = 196, normalized size = 1.4

$$\frac{Ba^3c\cos(5fx+5e)}{80f} + \frac{Aa^3c\sin(2fx+2e)}{4f} + \frac{1}{8}\left(5Aa^3c + 2Ba^3c\right)x - \frac{\left(8Aa^3c + 5Ba^3c\right)\cos(3fx+3e)}{48f} - \frac{(4Aa^3c +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*A
*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f - 1
/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*sin(
4*f*x + 4*e)/f
```

3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=156

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\sin(e+fx)\cos(}{2cf}$$

```
[Out] (-5*a^3*(3*A + 4*B)*x)/(2*c) + (5*a^3*(3*A + 4*B)*Cos[e + f*x]^3)/(3*c*f) -
 (5*a^3*(3*A + 4*B)*Cos[e + f*x]*Sin[e + f*x])/(2*c*f) + (a^3*(A + B)*c^3*C
os[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) + (2*a^3*(3*A + 4*B)*c^3*Cos[e +
f*x]^5)/(f*(c^2 - c^2*Sin[e + f*x])^2)
```

Rubi [A] time = 0.310391, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}-\frac{5a^3(3A+4B)\sin(e+fx)\cos(}{2cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (-5*a^3*(3*A + 4*B)*x)/(2*c) + (5*a^3*(3*A + 4*B)*Cos[e + f*x]^3)/(3*c*f) -
 (5*a^3*(3*A + 4*B)*Cos[e + f*x]*Sin[e + f*x])/(2*c*f) + (a^3*(A + B)*c^3*C
os[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) + (2*a^3*(3*A + 4*B)*c^3*Cos[e +
f*x]^5)/(f*(c^2 - c^2*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} - \left(a^3(3A+4B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)c\cos^5(e+fx)}{f(c-c\sin(e+fx))^2} - \left(5a^3(3A+4\right.$$
$$= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3(3A+4B)}{f(c-c\sin}$$
$$= \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf} + \frac{a}{}$$
$$= -\frac{5a^3(3A+4B)x}{2c} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\cos(e+}{2cf}$$

Mathematica [A] time = 1.50778, size = 223, normalized size = 1.43

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A+4B)(e+fx)-3(A+4B)\sin(2(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3*(
e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 + 5*e
 + 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

Maple [B] time = 0.121, size = 449, normalized size = 2.9

$$-16\,\frac{Aa^3}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)} - 16\,\frac{Ba^3}{cf\left(\tan\left(1/2\,fx+e/2\right)-1\right)} - \frac{Aa^3}{cf}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-3} - 4\,\frac{}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -16/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)*A-16/f*a^3/c/(tan(1/2*f*x+1/2*e)-1)*B-1/
f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A-4/f*a^3/c/(1+tan(
1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B+8/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^
2)^3*tan(1/2*f*x+1/2*e)^4*A+14/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f
*x+1/2*e)^4*B+16/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*A+
32/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B+1/f*a^3/c/(1+t
an(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A+4/f*a^3/c/(1+tan(1/2*f*x+1/2*e)
^2)^3*tan(1/2*f*x+1/2*e)*B+8/f*a^3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*A+46/3/f*a^
3/c/(1+tan(1/2*f*x+1/2*e)^2)^3*B-15/f*a^3/c*arctan(tan(1/2*f*x+1/2*e))*A-20
/f*a^3/c*arctan(tan(1/2*f*x+1/2*e))*B
```

Maxima [B] time = 1.54042, size = 1538, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*a^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a
^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((sin(f*x + e)/(c
os(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*s
in(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
```

```
2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/c) + 9*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
 + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 6*B*a^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*a^3/(c - c*sin(f*x + e)/(cos(f*x +
 e) + 1)))/f
```

Fricas [A] time = 1.50131, size = 533, normalized size = 3.42

$$-\frac{2\,Ba^3\cos\left(fx+e\right)^4-(3\,A+10\,B)a^3\cos\left(fx+e\right)^3+15\,(3\,A+4\,B)a^3fx-24\,(A+2\,B)a^3\cos\left(fx+e\right)^2-48\,(A+B)a^3 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*a^3*cos(f*x + e)^4 - (3*A + 10*B)*a^3*cos(f*x + e)^3 + 15*(3*A +
4*B)*a^3*f*x - 24*(A + 2*B)*a^3*cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3*A
 + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*cos(f*x + e) - (2*B*a^3*cos(f*x + e)^3
 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*cos(f*x + e)^2 - 3*(7*A + 12*B)
*a^3*cos(f*x + e) + 48*(A + B)*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*s
in(f*x + e) + c*f)
```

Sympy [A] time = 32.4657, size = 4255, normalized size = 27.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f
*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*
tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 + f*x/2)
**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x
*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6
 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2
 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f)
 - 135*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*ta
n(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4
 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
+ f*x/2) - 6*c*f) + 135*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x
/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*ta
n(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*tan(e/2 + f*x/2)/(6*c*f*
tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**
5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/
2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x/(6*c*f*tan(
e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 102*A*a**3*tan(e/2 + f*x/2)**
6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 +
 f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c
*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 54*A*a**3*tan(e/
2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c
*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/
2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 336*
A*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x
/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*t
an(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
6*c*f) + 96*A*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 378*A*a**3*tan(e/2 + f*x/2)**2/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 42*A*a**3*tan(e/2 + f*x/2)/(6*c*f*tan(e/2
 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*
c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x
/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 144*A*a**3/(6*c*f*tan(e/2 + f*x/
```

```
2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan
(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
+ 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*
f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)
**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(
e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 60*B*a**3*f*x*tan(e/2 +
 f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*
tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)*
*3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 180*B*a
**3*f*x*tan(e/2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*
x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*
tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) -
 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6
*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*
x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*t
an(e/2 + f*x/2) - 6*c*f) - 180*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/
2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18
*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)
**2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*
tan(e/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 1
8*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f
*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 6
0*B*a**3*f*x/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*
f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 120*B
*a**3*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/
2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*ta
n(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6
*c*f) + 108*B*a**3*tan(e/2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
 + f*x/2) - 6*c*f) - 372*B*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 192*B*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan
(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
 f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 456*B*a**3*tan(e/2 + f*x/2)*
*2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
+ f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*
c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 68*B*a**3*tan(e
/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f
```

```
*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)
**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 188*B*
a**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/
2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 1
8*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f), Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c), True))
```

Giac [A] time = 1.17808, size = 316, normalized size = 2.03

$$-\frac{\frac{15\left(3\,Aa^3+4\,Ba^3\right)(fx+e)}{c}+\frac{96\left(Aa^3+Ba^3\right)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+12\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-\cdots}{\left(\tan\left(\cdots\right.\right.}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(tan(1/2*f
*x + 1/2*e) - 1)) + 2*(3*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*tan(1/2*f*
x + 1/2*e)^5 - 24*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*tan(1/2*f*x + 1/2
*e)^4 - 48*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*a^3*tan(1/2*f*x + 1/2*e) - 12*B*a^3*tan(1/2*f*x + 1/2*e) - 24*A*a^3 -
46*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f
```

3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2} dx$

Optimal. Leaf size=163

$$-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{5a^3x(2A+5B)}{2c^2}-\frac{2a^3c(2A+5B}{3f(c-c\,\mathrm{si}}$$

```
[Out] (5*a^3*(2*A + 5*B)*x)/(2*c^2) - (5*a^3*(2*A + 5*B)*Cos[e + f*x])/(2*c^2*f)
+ (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - (2*a^3*(2
*A + 5*B)*c*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^3) - (5*a^3*(2*A + 5*
B)*Cos[e + f*x]^3)/(6*f*(c^2 - c^2*Sin[e + f*x]))
```

Rubi [A] time = 0.34774, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$-\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}+\frac{5a^3x(2A+5B)}{2c^2}-\frac{2a^3c(2A+5B}{3f(c-c\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```


```
[Out] (5*a^3*(2*A + 5*B)*x)/(2*c^2) - (5*a^3*(2*A + 5*B)*Cos[e + f*x])/(2*c^2*f)
+ (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - (2*a^3*(2
*A + 5*B)*c*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^3) - (5*a^3*(2*A + 5*
B)*Cos[e + f*x]^3)/(6*f*(c^2 - c^2*Sin[e + f*x]))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{1}{3}\left(a^3(2A+5B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3} + \frac{1}{3}\left(5a^3(2A+\right.$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{5a^3(2A+5B}{6f\left(c^2-c^2\right.}$$
$$= -\frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)}{3f(c-c\,\mathrm{si}}$$
$$= \frac{5a^3(2A+5B)x}{2c^2} - \frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}$$

Mathematica [A] time = 0.848578, size = 280, normalized size = 1.72

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+30(2A+5B)(e+fx)\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

Maple [B] time = 0.131, size = 399, normalized size = 2.5

$$8\,\frac{Aa^3}{fc^2\left(\tan\left(1/2\,fx+e/2\right)-1\right)} + 24\,\frac{Ba^3}{fc^2\left(\tan\left(1/2\,fx+e/2\right)-1\right)} - \frac{32\,Aa^3}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{-3} - \frac{32\,Ba^3}{3\,fc^2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 8/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)*A+24/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)*B-
32/3/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)^3*A-32/3/f*a^3/c^2/(tan(1/2*f*x+1/2*e
)-1)^3*B-16/f*a^3/c^2/(tan(1/2*f*x+1/2*e)-1)^2*A-16/f*a^3/c^2/(tan(1/2*f*x+
1/2*e)-1)^2*B+1/f*a^3/c^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B
-2/f*a^3/c^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A-10/f*a^3/c^2
/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B-1/f*a^3/c^2/(1+tan(1/2*f
*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)-2/f*a^3/c^2/(1+tan(1/2*f*x+1/2*e)^2)^2*
A-10/f*a^3/c^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B+25/f*a^3/c^2*arctan(tan(1/2*f*x
+1/2*e))*B+10/f*a^3/c^2*arctan(tan(1/2*f*x+1/2*e))*A
```

Maxima [B] time = 1.61027, size = 1871, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*a^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) - 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 7*c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*c^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/c^2) + 4*A*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/c^2) + 12*B*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x
```

```
+ e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^
2) + 6*A*a^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 6*B*a^3*((9*sin(f*x + e
)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*
c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c^2) - 2*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e
) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3) + 6*A*a^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 -
 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*a^3*(3*sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3))/f
```

Fricas [A] time = 1.42961, size = 687, normalized size = 4.21

$$\frac{3\,Ba^3\cos\left(fx+e\right)^4 - 6\,(A+4\,B)a^3\cos\left(fx+e\right)^3 - 30\,(2\,A+5\,B)a^3fx - 16\,(A+B)a^3 + \left(15\,(2\,A+5\,B)a^3fx + (62\,A+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5*
B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^3)
*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x +
e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a^3*c
os(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 79*B)*
a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e)
 - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23941, size = 315, normalized size = 1.93

$$\frac{\frac{15\left(2\,Aa^3+5\,Ba^3\right)\left(fx+e\right)}{c^2}+\frac{6\left(Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-10\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Aa^3-10\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c^2}+\frac{16\left(3\,Aa^3\tan\left(\cdots\right.\right.}{}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a^3
*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*e)^2
 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A*a^3
+ 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=153

$$\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}+\frac{2a^3c^3(A+6B)\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{a^3x(A+6B)}{c^3}-\frac{2a^3c(A+6B)\cos^5(\ldots}{15f(c-c\sin(e+\ldots}$$

```
[Out] -((a^3*(A + 6*B)*x)/c^3) + (a^3*(A + 6*B)*Cos[e + f*x])/(c^3*f) + (a^3*(A +
 B)*c^3*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - (2*a^3*(A + 6*B)*c*C
os[e + f*x]^5)/(15*f*(c - c*Sin[e + f*x])^4) + (2*a^3*(A + 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(c^3 - c^3*Sin[e + f*x])^2)
```

Rubi [A] time = 0.34216, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}+\frac{2a^3c^3(A+6B)\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}-\frac{a^3x(A+6B)}{c^3}-\frac{2a^3c(A+6B)\cos^5(\ldots}{15f(c-c\sin(e+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -((a^3*(A + 6*B)*x)/c^3) + (a^3*(A + 6*B)*Cos[e + f*x])/(c^3*f) + (a^3*(A +
 B)*c^3*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - (2*a^3*(A + 6*B)*c*C
os[e + f*x]^5)/(15*f*(c - c*Sin[e + f*x])^4) + (2*a^3*(A + 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(c^3 - c^3*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{1}{5}\left(a^3(A+6B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4} + \frac{1}{3}\left(a^3(A+6B)\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4} + \frac{2a^3(A+6B)}{3cf(c-c\,\mathrm{si}}$$

$$= \frac{a^3(A+6B)\cos(e+fx)}{c^3f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\,\mathrm{co}}{15f(c-c\sin(}$$

$$= -\frac{a^3(A+6B)x}{c^3} + \frac{a^3(A+6B)\cos(e+fx)}{c^3f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}$$

Mathematica [B] time = 1.06856, size = 316, normalized size = 2.07

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-15(A+6B)(e+fx)\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B*Co
s[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e + f*
x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*
x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*
(c - c*Sin[e + f*x])^3)
```

Maple [B] time = 0.138, size = 323, normalized size = 2.1

$$-4\,\frac{Aa^3}{fc^3\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-12\,\frac{Ba^3}{fc^3\left(\tan\left(1/2\,fx+e/2\right)-1\right)}-8\,\frac{Aa^3}{fc^3\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}+8\,\frac{Ba^3}{fc^3\left(\tan\left(1/2\,fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] -4/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)*A-12/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)*B
-8/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^2*A+8/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^
2*B-64/5/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^5*A-64/5/f*a^3/c^3/(tan(1/2*f*x+1
/2*e)-1)^5*B-80/3/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^3*A-16/f*a^3/c^3/(tan(1/
2*f*x+1/2*e)-1)^3*B-32/f*a^3/c^3/(tan(1/2*f*x+1/2*e)-1)^4*A-32/f*a^3/c^3/(t
an(1/2*f*x+1/2*e)-1)^4*B+2/f*a^3/c^3*B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*a^3/c^3
*arctan(tan(1/2*f*x+1/2*e))*A-12/f*a^3/c^3*arctan(tan(1/2*f*x+1/2*e))*B
```

Maxima [B] time = 1.637, size = 2275, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*a^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*c^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1
1*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/c^3) + A*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c
^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) -
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/c^3) + A*a^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 9*A*a^3*(5*sin(f*x + e)/(cos(
f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c
^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5) - 3*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) +
```

```
6*A*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5) + 6*B*a^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.45899, size = 821, normalized size = 5.37

$$15\,Ba^3\cos\left(fx+e\right)^4+60\,(A+6\,B)a^3fx-24\,(A+B)a^3-\left(15\,(A+6\,B)a^3fx-(46\,A+231\,B)a^3\right)\cos\left(fx+e\right)^3-\left(45\,(A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*a^3*cos(f*x + e)^4 + 60*(A + 6*B)*a^3*f*x - 24*(A + B)*a^3 - (15
*(A + 6*B)*a^3*f*x - (46*A + 231*B)*a^3)*cos(f*x + e)^3 - (45*(A + 6*B)*a^3
*f*x + 2*(A + 66*B)*a^3)*cos(f*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(6*A +
 31*B)*a^3)*cos(f*x + e) - (15*B*a^3*cos(f*x + e)^3 + 60*(A + 6*B)*a^3*f*x
+ 24*(A + B)*a^3 - (15*(A + 6*B)*a^3*f*x + 2*(23*A + 108*B)*a^3)*cos(f*x +
e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(4*A + 29*B)*a^3)*cos(f*x + e))*sin(f*x +
 e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e)
- 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x
 + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23816, size = 305, normalized size = 1.99

$$\frac{\frac{30\,Ba^{3}}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)c^{3}}-\frac{15\,\left(Aa^{3}+6\,Ba^{3}\right)\left(fx+e\right)}{c^{3}}-\frac{4\left(15\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+45\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-30\,Aa^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-210\,Ba^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}\cdots}{c^{3}\left(\mathrm{t}\cdots\right.}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^3) - 15*(A*a^3 + 6*B*a^3)*(f
*x + e)/c^3 - 4*(15*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*a^3*tan(1/2*f*x + 1
/2*e)^4 - 30*A*a^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*a^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 420*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 5
0*A*a^3*tan(1/2*f*x + 1/2*e) - 270*B*a^3*tan(1/2*f*x + 1/2*e) + 13*A*a^3 +
63*B*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=151

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}+\frac{a^3Bx}{c^4}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

```
[Out] (a^3*B*x)/c^4 + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^
7) - (2*a^3*B*c*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) + (2*a^3*B*c^2
*Cos[e + f*x]^3)/(3*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a^3*B*Cos[e + f*x])/
(f*(c^4 - c^4*Sin[e + f*x]))
```

Rubi [A] time = 0.329975, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2680, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}+\frac{a^3Bx}{c^4}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

```
[Out] (a^3*B*x)/c^4 + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^
7) - (2*a^3*B*c*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) + (2*a^3*B*c^2
*Cos[e + f*x]^3)/(3*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a^3*B*Cos[e + f*x])/
(f*(c^4 - c^4*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \left(a^3Bc^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \left(a^3B\right)\int \frac{\cos^4(}{(c-c\sin}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2a^3B\cos^3(e+}{3cf(c-c\sin(e+}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2a^3B\cos^3(e+}{3cf(c-c\sin(e+}$$
$$= \frac{a^3Bx}{c^4} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7} - \frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} + \frac{2a^3B\,c}{3cf(c-c}$$

Mathematica [B] time = 1.15233, size = 356, normalized size = 2.36

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(240(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-2(15A+337B)\sin\left(\frac{1}{2}(e+fx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^4,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e +
f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x)/2]
 - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2
] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/
2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)
/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(
c - c*Sin[e + f*x])^4)
```

Maple [B] time = 0.142, size = 374, normalized size = 2.5

$$-2\,{\frac {A{a}^{3}}{f{c}^{4} \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) }}+2\,{\frac {B{a}^{3}}{f{c}^{4} \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) }}-12\,{\frac {A{a}^{3}}{f{c}^{4} \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{2}}}-4\,{\frac {B{a}^{3}}{f{c}^{4} \left(\tan \left(1/2\,fx+ \right. \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] -2/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)*A+2/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)*B-
12/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^2*A-4/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^
2*B-40/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^3*A-40/3/f*a^3/c^4/(tan(1/2*f*x+1/2
*e)-1)^3*B-128/7/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^7*A-128/7/f*a^3/c^4/(tan(
1/2*f*x+1/2*e)-1)^7*B-80/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^4*A-48/f*a^3/c^4/
(tan(1/2*f*x+1/2*e)-1)^4*B-64/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^6*A-64/f*a^3
/c^4/(tan(1/2*f*x+1/2*e)-1)^6*B-96/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^5*A-416
/5/f*a^3/c^4/(tan(1/2*f*x+1/2*e)-1)^5*B+2/f*a^3/c^4*B*arctan(tan(1/2*f*x+1/
2*e))
```

Maxima [B] time = 1.71883, size = 2859, normalized size = 18.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] 2/105*(5*B*a^3*((203*sin(f*x + e)/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f
*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^4) + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1)
 - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*
c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x + e)/(cos(f*x + e)
 + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^3*(49*sin(f*x + e)/(cos(f
*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7) - 12*A*a^3*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x
+ e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*
c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 12*B*a^3*(14*sin(f*x
+ e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2
)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(
```

```
f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7) + 6*A*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + 18*B*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*
c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7))/f
```

Fricas [B] time = 1.56404, size = 891, normalized size = 5.9

$$\frac{840\,Ba^3fx + \left(105\,Ba^3fx + (15\,A + 337\,B)a^3\right)\cos\left(fx+e\right)^4 + 120\,(A+B)a^3 - \left(315\,Ba^3fx + (45\,A - 613\,B)a^3\right)\cos\left(fx+\right.}{105\left(c^4 f\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] 1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4
+ 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 - 2
4*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (A -
 13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^3*f*
x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*B)*a^
3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*sin(f*x
 + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e
)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(
f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20714, size = 288, normalized size = 1.91

$$\frac{\frac{105\left(fx+e\right)Ba^3}{c^4}-\frac{2\left(105\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-105\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+840\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+525\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1925\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+392}{c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] 1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105*
B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a^3
*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3*ta
n(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*tan(1/
2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*a^3)/
(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=77

$$\frac{a^3c^2(A-8B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}$$

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*(A - 8
*B)*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.235635, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 2671}

$$\frac{a^3c^2(A-8B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*(A - 8
*B)*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^7)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx\\ &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{1}{9}\left(a^3(A-8B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}\,dx\\ &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}\end{aligned}$$

Mathematica [B] time = 2.44707, size = 283, normalized size = 3.68

$$-\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(315(A-B)\cos\left(\frac{1}{2}(e+fx)\right)-189(A-B)\cos\left(\frac{3}{2}(e+fx)\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^5,x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(315*(A -
B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(5*(e + f
*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9*B*Cos[(7
*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2] + 105*A*Si
n[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e + f*x))/2]
 - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*Sin[(9*(e + f
*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(504*c^5*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^6*(-1 + Sin[e + f*x])^5)
```

Maple [B] time = 0.152, size = 205, normalized size = 2.7

$$2\,\frac{a^3}{fc^5}\left(-1/3\,\frac{86\,A+26\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^3}-1/7\,\frac{928\,A+864\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^7}-1/8\,\frac{512\,A+512\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^8}-1/2\,\frac{14\,A+}{\left(\tan\left(1/2\,fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a^3/c^5*(-1/3*(86*A+26*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/7*(928*A+864*B)/(t
an(1/2*f*x+1/2*e)-1)^7-1/8*(512*A+512*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/2*(14*A
+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(680*A+440*B)/(tan(1/2*f*x+1/2*e)-1)^5-1
/4*(304*A+144*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/6*(992*A+800*B)/(tan(1/2*f*x+1/
2*e)-1)^6-1/9*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)^9-A/(tan(1/2*f*x+1/2*e)-
1))
```

Maxima [B] time = 1.33655, size = 3646, normalized size = 47.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f
*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8
4*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x +
e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
```

```
126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
 10*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) - 30*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c
^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 8*B*a^3*(9*sin(f*x + e)/(cos(f*x + e
) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 - 126*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)/(c^5 - 9*c^5*s
in(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*
x + e) + 1)^9) + 42*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(co
s(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(
```

```
f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)
+ 42*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f
```

Fricas [B] time = 1.46076, size = 824, normalized size = 10.7

$$-\frac{(A-8\,B)a^3\cos\left(fx+e\right)^5-(4\,A+31\,B)a^3\cos\left(fx+e\right)^4+(19\,A+37\,B)a^3\cos\left(fx+e\right)^3+4\,(13\,A+22\,B)a^3\cos\left(fx+\right.}{63\left(c^5f\cos\left(fx+e\right)^5+5\,c^5f\cos\left(fx+e\right)^4-8\,c^5f\cos\left(fx+e\right)^3-20\,c^5f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (19
*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(A
+ B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 + (5
*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*(A +
 B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5
+ 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2
 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*
x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f
*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

[Out] Timed out

Giac [B] time = 1.2649, size = 406, normalized size = 5.27

$$-\frac{2\left(63\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-63\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+63\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+483\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+105\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*B*
a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^3*t
an(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*tan(1/
2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/2*f*x
 + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x + 1/2
*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e) + 8*A*a
^3 - B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=118

$$\frac{a^3c^2(2A-9B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3c(2A-9B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (a^3*(2*A
- 9*B)*c^2*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (a^3*(2*A - 9*B)
*c*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.289233, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(2A-9B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3c(2A-9B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (a^3*(2*A
- 9*B)*c^2*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (a^3*(2*A - 9*B)
*c*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^7)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{1}{11}\left(a^3(2A-9B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{1}{99}\left(a^3(2A\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9} + \frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8} + \frac{a^3(2A-9B}{693f(c-c}$$

Mathematica [B] time = 2.81248, size = 313, normalized size = 2.65

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(462(11A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-594(5A+2B)\cos\left(\frac{3}{2}(e+fx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^6,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A
+ 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Cos[
(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/2]
+ 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*(e +
 f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A*Sin[
(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*x))/2]
 - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*Sin[(9*(
e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)
```

Maple [B] time = 0.164, size = 249, normalized size = 2.1

$$2\,\frac{a^3}{fc^6}\left(-1/6\,\frac{2960\,A+1968\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^6}-1/3\,\frac{116\,A+30\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^3}-1/2\,\frac{16\,A+2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-1/7\,\frac{4272\,A+3}{\left(\tan\left(1/2\,fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f*a^3/c^6*(-1/6*(2960*A+1968*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/3*(116*A+30*B)
/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(16*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/7*(4272
*A+3344*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/4*(504*A+200*B)/(tan(1/2*f*x+1/2*e)-1
)^4-1/10*(1280*A+1280*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/11*(256*A+256*B)/(tan(
1/2*f*x+1/2*e)-1)^11-1/5*(1460*A+780*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/8*(4352*
A+3840*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/9*(3008*A+2880*B)/(tan(1/2*f*x+1/2*e)-
1)^9-A/(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] time = 1.49558, size = 4577, normalized size = 38.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/3465*(5*A*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e) + 1
```

)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/(cos
(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 -
146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^
6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5
5*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x
 + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9*A*a^3*(671*s
in(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33
0*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^3*(6
71*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34
65*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*A*a^
3*(341*sin(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 5115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
1)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*
c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*B*a^3*(341*sin

(f*x + e)/(cos(f*x + e) + 1) - 1705*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
115*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 9471*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 3465*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 31)/(c^6 -
11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4
62*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*
x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 12*A*a^3*(253*sin(f*x + e)
/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2640*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11) + 12*B*a^3*(253*sin(f*x + e)/(cos(f*x + e) + 1) - 1265*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2640*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 5313*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*
sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 48*B*a^3*(11*sin(f*x + e)/(cos(f*
x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 165*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 231*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)
/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55*c^
6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11))/f

Fricas [B] time = 1.5571, size = 1014, normalized size = 8.59

$$\frac{(2A-9B)a^3\cos\left(fx+e\right)^6+6(2A-9B)a^3\cos\left(fx+e\right)^5-(25A+234B)a^3\cos\left(fx+e\right)^4+7(23A+45B)a^3\cos\left(f\right.}{693\left(c^6f\cos\left(fx+e\right)^6-5c^6f\cos\left(fx+e\right)^5-18c^6f\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 -
(25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 + 28
*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(A +
 B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x + e)^
4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x + e)^2
+ 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e))/(c^6*f*cos(
f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*co
s(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f +
 (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 -
 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23906, size = 504, normalized size = 4.27

$$-\frac{2\left(693\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-1386\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+693\,Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+8085\,Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -2/693*(693*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*A*a^3*tan(1/2*f*x + 1/2*e)
^9 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*tan(1/2*f*x + 1/2*e)^8 +
 693*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 41
58*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 1386
*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 5544*B
*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 1188*B*a
^3*tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 2178*B*a^3*
tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 198*B*a^3*tan(
1/2*f*x + 1/2*e)^2 - 176*A*a^3*tan(1/2*f*x + 1/2*e) + 99*B*a^3*tan(1/2*f*x
+ 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(tan(1/2*f*x + 1/2*e) - 1)^11)
```

3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal. Leaf size=156

$$\frac{a^3c^2(3A-10B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}+\frac{2a^3c(3A-10B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}$$

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (a^3*(3*A
 - 10*B)*c^2*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(3*A -
 10*B)*c*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(3*A - 10
*B)*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.37521, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(3A-10B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}+\frac{2a^3c(3A-10B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]
```



```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (a^3*(3*A
 - 10*B)*c^2*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(3*A -
 10*B)*c*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(3*A - 10
*B)*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^7)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}\,dx$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{1}{13}\left(a^3(3A-10B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{1}{143}\left(2a^3(3A\right.$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3(3A-10}{1287f(c-}$$
$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}} + \frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9} + \frac{2a^3(3A-10}{1287f(c-}$$

Mathematica [B] time = 5.08478, size = 339, normalized size = 2.17

$$-\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6006(9A+5B)\cos\left(\frac{1}{2}(e+fx)\right)-7722(4A+3B)\cos\left(\frac{3}{2}(e+f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^7,x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(6006*(9*A
 + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2] - 9009*A*C
os[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[(7*(e + f*x)
)/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2] + 130*B*Cos
[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(e + f*x)/2] +
27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2] - 6864*A*Sin[(5
*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[(7*(e + f*x))/2]
 - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2] + 3*A*Sin[(13*(e
 + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(144144*c^7*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)
```

Maple [A] time = 0.184, size = 293, normalized size = 1.9

$$2\,{\frac {{a}^{3}}{f{c}^{7}}\left(-1/3\,{\frac {150\,A+34\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{3}}}-1/13\,{\frac {512\,A+512\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{13}}}-1/12\,{\frac {3072\,A+3072\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{12}}}-1/10\,{\frac {1600\ldots}{ \left(\tan \left(1/\ldots \right.\right.}} \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
```

```
[Out] 2/f*a^3/c^7*(-1/3*(150*A+34*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/13*(512*A+512*B)/
(tan(1/2*f*x+1/2*e)-1)^13-1/12*(3072*A+3072*B)/(tan(1/2*f*x+1/2*e)-1)^12-1/
10*(16000*A+14720*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/7*(13112*A+8840*B)/(tan(1/
2*f*x+1/2*e)-1)^7-1/11*(8832*A+8576*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/6*(6888*
A+3928*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(768*A+264*B)/(tan(1/2*f*x+1/2*e)-1)
^4-1/5*(2700*A+1240*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(18*A+2*B)/(tan(1/2*f*x
+1/2*e)-1)^2-1/9*(20256*A+17248*B)/(tan(1/2*f*x+1/2*e)-1)^9-A/(tan(1/2*f*x+
1/2*e)-1)-1/8*(18816*A+14464*B)/(tan(1/2*f*x+1/2*e)-1)^8)
```

Maxima [B] time = 1.67179, size = 5505, normalized size = 35.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="maxima")
```

```
[Out] -2/45045*(6*A*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1873
30*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7507
5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*
c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x
+ e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^
11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(co
s(f*x + e) + 1)^13) + 6*B*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) +
 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(
cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) +
 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x
 + e)^13/(cos(f*x + e) + 1)^13) + 15*A*a^3*(3796*sin(f*x + e)/(cos(f*x + e)
 + 1) - 22776*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 77506*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 193765*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 339768*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 - 453024*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 444444*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 333333*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 180180*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 72072*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 18018*sin(f*x + e)^11/(cos(f*x + e) +
1)^11 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*sin
(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e
```

)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 105*A*a^3*(611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^3*(611*sin(f*x + e)/(cos(f*x + e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) + 8*B*a^3*(559*sin(f*x + e)/(cos(f*x + e) + 1) - 3354*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12298*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 30745*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 37323*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 49764*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24024*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 18018*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 43)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) + 1)^13) -

```
462*A*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 858*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3
51*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x
 + e) + 1)^13) - 1386*B*a^3*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5
20*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 - 858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1)
+ 78*c^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 28
6*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*
x + e)^13/(cos(f*x + e) + 1)^13))/f
```

<u>Fricas</u> [B] time = 1.50546, size = 1215, normalized size = 7.79

$$-\frac{2\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^7-12\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^6-49\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^5+7\,(30\,A+329\,B)a^3\,c}{9009\left(c^7f\cos\left(fx+e\right)^7+7\,c^7f\cos\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="fricas")
```

```
[Out] -1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x +
e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x +
e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f*x
```

```
+ e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 10*B)
*a^3*cos(f*x + e)^6 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 35*(3*A - 10*B)*
a^3*cos(f*x + e)^4 - 63*(5*A + 31*B)*a^3*cos(f*x + e)^3 - 252*(8*A + 21*B)*
a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3)*sin(
f*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x
 + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*cos
(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 - 6*
c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3 +
80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30108, size = 601, normalized size = 3.85

$$-\frac{2\left(9009\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-27027\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+9009\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+153153\,Aa^3\tan\left(\frac{1}{2}\,f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorit
hm="giac")
```

```
[Out] -2/9009*(9009*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 27027*A*a^3*tan(1/2*f*x + 1/2
*e)^11 + 9009*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 153153*A*a^3*tan(1/2*f*x + 1/
2*e)^10 + 3003*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 297297*A*a^3*tan(1/2*f*x + 1
/2*e)^9 + 69069*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*tan(1/2*f*x + 1
/2*e)^8 - 9009*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*tan(1/2*f*x + 1/
2*e)^7 + 150150*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*tan(1/2*f*x + 1
/2*e)^6 - 16302*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*tan(1/2*f*x + 1
```

```
/2*e)^5 + 115830*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*tan(1/2*f*x +
1/2*e)^4 - 286*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*tan(1/2*f*x + 1/
2*e)^3 + 30745*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*tan(1/2*f*x + 1/2
*e)^2 + 1443*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*tan(1/2*f*x + 1/2*e)
 + 1261*B*a^3*tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7*f*(tan(1/2*
f*x + 1/2*e) - 1)^13)
```

3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$

Optimal. Leaf size=197

$$\frac{a^3c^2(4A-11B)\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}$$

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (a^3*(4*A
 - 11*B)*c^2*Cos[e + f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (a^3*(4*A -
11*B)*c*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(4*A - 11*B
)*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(4*A - 11*B)*Cos
[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rubi [A] time = 0.44408, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(4A-11B)\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}}+\frac{a^3c^3(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}+\frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x]
```

```
[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (a^3*(4*A
 - 11*B)*c^2*Cos[e + f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (a^3*(4*A -
11*B)*c*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(4*A - 11*B
)*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(4*A - 11*B)*Cos
[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{1}{15}\left(a^3(4A-11B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{1}{65}\left(a^3(4A-\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4A-11B}{715f(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4A-11B}{715f(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4A-11B}{715f(c-c}$$

Mathematica [A] time = 6.64529, size = 378, normalized size = 1.92

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(437580A\sin\left(\frac{1}{2}(e+fx)\right)+240240A\sin\left(\frac{3}{2}(e+fx)\right)-60060\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^8,x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(463320*A*Cos
[(e + f*x)/2] + 302445*B*Cos[(e + f*x)/2] - 260260*A*Cos[(3*(e + f*x))/2] -
 230230*B*Cos[(3*(e + f*x))/2] - 72072*A*Cos[(5*(e + f*x))/2] - 117117*B*Co
s[(5*(e + f*x))/2] + 5460*A*Cos[(7*(e + f*x))/2] + 30030*B*Cos[(7*(e + f*x)
)/2] - 420*A*Cos[(11*(e + f*x))/2] + 1155*B*Cos[(11*(e + f*x))/2] + 4*A*Cos
[(15*(e + f*x))/2] - 11*B*Cos[(15*(e + f*x))/2] + 437580*A*Sin[(e + f*x)/2]
 + 373230*B*Sin[(e + f*x)/2] + 240240*A*Sin[(3*(e + f*x))/2] + 285285*B*Sin
[(3*(e + f*x))/2] - 60060*A*Sin[(5*(e + f*x))/2] - 150150*B*Sin[(5*(e + f*x
))/2] - 45045*B*Sin[(7*(e + f*x))/2] - 1820*A*Sin[(9*(e + f*x))/2] + 5005*B
*Sin[(9*(e + f*x))/2] + 60*A*Sin[(13*(e + f*x))/2] - 165*B*Sin[(13*(e + f*x
))/2]))/(1441440*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f
*x])^8)
```

Maple [A] time = 0.201, size = 337, normalized size = 1.7

$$2\,\frac{a^3}{fc^8}\left(-1/10\,\frac{94144\,A+78144\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{10}}-1/3\,\frac{188\,A+38\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{3}}-1/4\,\frac{1104\,A+336\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{4}}-1/14\,\frac{7168}{\left(\tan\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x)
```

```
[Out] 2/f*a^3/c^8*(-1/10*(94144*A+78144*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/3*(188*A+3
8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/4*(1104*A+336*B)/(tan(1/2*f*x+1/2*e)-1)^4-1
/14*(7168*A+7168*B)/(tan(1/2*f*x+1/2*e)-1)^14-1/13*(24320*A+23808*B)/(tan(1
/2*f*x+1/2*e)-1)^13-1/12*(52736*A+49664*B)/(tan(1/2*f*x+1/2*e)-1)^12-1/7*(3
2288*A+19176*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/5*(4536*A+1836*B)/(tan(1/2*f*x+1
/2*e)-1)^5-1/8*(58816*A+40000*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/11*(81344*A+725
12*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/15*(1024*A+1024*B)/(tan(1/2*f*x+1/2*e)-1)
```

```
^15-1/2*(20*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/6*(13824*A+6936*B)/(tan(1/2*f
*x+1/2*e)-1)^6-1/9*(84112*A+63856*B)/(tan(1/2*f*x+1/2*e)-1)^9-A/(tan(1/2*f*
x+1/2*e)-1))
```

Maxima [B] time = 1.89065, size = 6433, normalized size = 32.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="maxima")
```

```
[Out] 2/45045*(3*A*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891
345*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 2867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^10/(cos
(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 195195*s
in(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 643
5*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e)
 + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x
+ e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^1
4 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + B*a^3*(17715*sin(f*x + e)/
(cos(f*x + e) + 1) - 78960*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 342160*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891345*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 1960959*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3043040*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 3912480*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3687255
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2867865*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 - 1585584*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 720720*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 195195*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
+ 45045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x
 + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 45
5*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin
```

(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 7*A*a^3*(7845*sin(f*x + e)/(cos(f*x + e) + 1) - 54915*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 222950*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 668850*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1444443*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2407405*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3063060*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3063060*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2357355*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1414413*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 630630*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 210210*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 6435*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 952)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 12*A*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) - 12*B*a^3*(1740*sin(f*x + e)/(cos(f*x + e) + 1) - 12180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 37765*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 113295*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 204204*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 340340*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 373230*sin(f*x + e)^7/(c

os(f*x + e) + 1)^7 - 373230*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 240240*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 144144*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 45045*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 15015*sin(f*x + e)^12
/(cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1)
 + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
+ 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^11
/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 10
5*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f
*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + 6*A*a^3*(675
*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 20475*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 46410*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
 122265*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 95095*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*sin(f*x
+ e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e
) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30
03*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) +
 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x +
 e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^1
2 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/
(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + 18*B*a
^3*(675*sin(f*x + e)/(cos(f*x + e) + 1) - 4725*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 20475*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 46410*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 102102*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 130130*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 167310*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 - 122265*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 95095*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9 - 33033*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 15015*s
in(f*x + e)^11/(cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*sin(f*x + e)/(cos(
f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435
*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*si
n(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e)
 + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x +
 e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) -

```
 48*B*a^3*(60*sin(f*x + e)/(cos(f*x + e) + 1) - 420*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 1820*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5460*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 9009*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15015*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12870*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 12870*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 5005*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 - 3003*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 4)/(c^8 - 1
5*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 105*c^8*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1365*c^8*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 5005*c^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6435*c^8*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 6435*c^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3003*c^8*sin(f*x + e)^10/(cos(f*x +
e) + 1)^10 - 1365*c^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f
*x + e)^12/(cos(f*x + e) + 1)^12 - 105*c^8*sin(f*x + e)^13/(cos(f*x + e) +
1)^13 + 15*c^8*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15))/f
```

Fricas [B] time = 1.6306, size = 1400, normalized size = 7.11

$$\frac{2\,(4\,A-11\,B)a^{3}\cos\left(fx+e\right)^{8}+16\,(4\,A-11\,B)a^{3}\cos\left(fx+e\right)^{7}-49\,(4\,A-11\,B)a^{3}\cos\left(fx+e\right)^{6}-168\,(4\,A-11\,B)a^{3}\,\mathrm{c}}{45045\left(c^{8}f\cos\left(fx+e\right)^{8}-7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="fricas")
```

```
[Out] 1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x +
e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x +
e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f*x
+ e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f*x +
 e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A - 11
*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*A - 11
*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3*A + 8*
B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A + B)*a^3)
*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32*c^8*f*co
s(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4 - 112*c^8
*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x + e) + 128*
c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8*f*cos(f*x +
 e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 192*c^8*f*cos(f
```

```
*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26361, size = 698, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorit
hm="giac")
```

```
[Out] -2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x +
1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f*x
 + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1/2*
f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*tan(
1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*A*a^3
*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 14124825*A
*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 1317888
0*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 110
26015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/2*e)^6 -
6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x + 1/2*e)^5
+ 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*x + 1/2*e)^
4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*f*x + 1/2*e)
^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*f*x + 1/2*e)^
2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x + 1/2*e) + 42
43*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=190

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}-\frac{2a^2c^4(4A-5B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{35c^4(4A-5B)\cos^3(e+fx)}{12af}-\frac{7c^4(4A-5B)\cos^5(e+fx)}{4f(a\sin(e+fx)+a)}$$

```
[Out] (-35*(4*A - 5*B)*c^4*x)/(8*a) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]^3)/(12*a*f
) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(8*a*f) - (a^4*(A - B)*c
^4*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5) - (2*a^2*(4*A - 5*B)*c^4*Cos[
e + f*x]^7)/(f*(a + a*Sin[e + f*x])^3) - (7*(4*A - 5*B)*c^4*Cos[e + f*x]^5)
/(4*f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.362638, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2967, 2859, 2680, 2679, 2682, 2635, 8}

$$-\frac{a^4c^4(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}-\frac{2a^2c^4(4A-5B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{35c^4(4A-5B)\cos^3(e+fx)}{12af}-\frac{7c^4(4A-5B)\cos^5(e+fx)}{4f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]),x]
```


```
[Out] (-35*(4*A - 5*B)*c^4*x)/(8*a) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]^3)/(12*a*f
) - (35*(4*A - 5*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(8*a*f) - (a^4*(A - B)*c
^4*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5) - (2*a^2*(4*A - 5*B)*c^4*Cos[
e + f*x]^7)/(f*(a + a*Sin[e + f*x])^3) - (7*(4*A - 5*B)*c^4*Cos[e + f*x]^5)
/(4*f*(a + a*Sin[e + f*x]))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \left(a^3(4A-5B)c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \left(7a(4A-\right.$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{7(4A-5}{4f(a+}$$

$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-}{f(a+}$$

$$= -\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af}$$

$$= -\frac{35(4A-5B)c^4x}{8a} - \frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(}{8a}$$

Mathematica [A] time = 2.29211, size = 274, normalized size = 1.44

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3072(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-420(4A-5B)(e+fx)\left(\sin\left(\frac{1}{2}(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(3072*(A - B)
*Sin[(e + f*x)/2] - 420*(4*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2]) - 24*(47*A - 75*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]) + 8*(A - 5*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) +
24*(5*A - 12*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)] + 3*
B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[4*(e + f*x)]))/(96*a*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.134, size = 678, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

```
[Out] -5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*A+47/4/f*c^4/a/(
1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*B-22/f*c^4/a/(1+tan(1/2*f*x+
1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^6*A+30/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*ta
n(1/2*f*x+1/2*e)^6*B-5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e
)^5*A+55/4/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^5*B-70/f*c
^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*A+110/f*c^4/a/(1+tan(1
/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*B+5/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2
)^4*tan(1/2*f*x+1/2*e)^3*A-55/4/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*
f*x+1/2*e)^3*B-214/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^
2*A+350/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^2*B+5/f*c^4
/a/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)*A-47/4/f*c^4/a/(1+tan(1/2*
f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)*B-70/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^
4*A+110/3/f*c^4/a/(1+tan(1/2*f*x+1/2*e)^2)^4*B-35/f*c^4/a*arctan(tan(1/2*f*
x+1/2*e))*A+175/4/f*c^4/a*arctan(tan(1/2*f*x+1/2*e))*B-32/f*c^4/a/(tan(1/2*
f*x+1/2*e)+1)*A+32/f*c^4/a/(tan(1/2*f*x+1/2*e)+1)*B
```

Maxima [B] time = 1.57989, size = 2425, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/12*(B*c^4*((19*sin(f*x + e)/(cos(f*x + e) + 1) + 211*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 91*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 219*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 165*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 165*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 64)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 4*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 6*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 4*a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a*sin(f*x + e)^8/(cos(f*x + e) + 1)^
```

```
8 + a*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 45*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a) - 4*A*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 9*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) + 16*B*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)
 + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4
8*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 72*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f
*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/a) - 144*A*c^4*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 96*B*c^4*((sin(f*x + e)/(cos(f
*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)
/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 96
*A*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1))) + 24*B*c^4*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 24*A*c^4/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.50523, size = 651, normalized size = 3.43

$$-\frac{6\,Bc^4\cos\left(fx+e\right)^5-8\,(A-5\,B)c^4\cos\left(fx+e\right)^4+(52\,A-113\,B)c^4\cos\left(fx+e\right)^3+105\,(4\,A-5\,B)c^4fx+96\,(3\,A-5\,B\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/24*(6*B*c^4*cos(f*x + e)^5 - 8*(A - 5*B)*c^4*cos(f*x + e)^4 + (52*A - 11
3*B)*c^4*cos(f*x + e)^3 + 105*(4*A - 5*B)*c^4*f*x + 96*(3*A - 5*B)*c^4*cos(
f*x + e)^2 + 384*(A - B)*c^4 + 3*(35*(4*A - 5*B)*c^4*f*x + (204*A - 239*B)*
c^4)*cos(f*x + e) - (6*B*c^4*cos(f*x + e)^4 + 2*(4*A - 17*B)*c^4*cos(f*x +
e)^3 - 105*(4*A - 5*B)*c^4*f*x + 3*(20*A - 49*B)*c^4*cos(f*x + e)^2 - 3*(76
*A - 111*B)*c^4*cos(f*x + e) + 384*(A - B)*c^4)*sin(f*x + e))/(a*f*cos(f*x
+ e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23664, size = 463, normalized size = 2.44

$$-\frac{\frac{105\left(4\,Ac^4-5\,Bc^4\right)\left(fx+e\right)}{a}+\frac{768\left(Ac^4-Bc^4\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(60\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-141\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+264\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-360\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/24*(105*(4*A*c^4 - 5*B*c^4)*(f*x + e)/a + 768*(A*c^4 - B*c^4)/(a*(tan(1/
2*f*x + 1/2*e) + 1)) + 2*(60*A*c^4*tan(1/2*f*x + 1/2*e)^7 - 141*B*c^4*tan(1
/2*f*x + 1/2*e)^7 + 264*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 360*B*c^4*tan(1/2*f*
x + 1/2*e)^6 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 165*B*c^4*tan(1/2*f*x + 1/
2*e)^5 + 840*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1320*B*c^4*tan(1/2*f*x + 1/2*e)
^4 - 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*B*c^4*tan(1/2*f*x + 1/2*e)^3 + 8
56*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1400*B*c^4*tan(1/2*f*x + 1/2*e)^2 - 60*A*
c^4*tan(1/2*f*x + 1/2*e) + 141*B*c^4*tan(1/2*f*x + 1/2*e) + 280*A*c^4 - 440
*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a))/f
```

3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=157

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{2a^3c^3(3A-4B)\cos^5(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{5c^3(3A-4B)\cos^3(e+fx)}{3af}-\frac{5c^3(3A-4B)\sin(e+fx)\cos(}{2af}$$

```
[Out] (-5*(3*A - 4*B)*c^3*x)/(2*a) - (5*(3*A - 4*B)*c^3*Cos[e + f*x]^3)/(3*a*f) -
 (5*(3*A - 4*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - (a^3*(A - B)*c^3*C
os[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) - (2*a^3*(3*A - 4*B)*c^3*Cos[e +
f*x]^5)/(f*(a^2 + a^2*Sin[e + f*x])^2)
```

Rubi [A] time = 0.317891, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$-\frac{a^3c^3(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}-\frac{2a^3c^3(3A-4B)\cos^5(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{5c^3(3A-4B)\cos^3(e+fx)}{3af}-\frac{5c^3(3A-4B)\sin(e+fx)\cos(}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-5*(3*A - 4*B)*c^3*x)/(2*a) - (5*(3*A - 4*B)*c^3*Cos[e + f*x]^3)/(3*a*f) -
 (5*(3*A - 4*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - (a^3*(A - B)*c^3*C
os[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) - (2*a^3*(3*A - 4*B)*c^3*Cos[e +
f*x]^5)/(f*(a^2 + a^2*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \left(a^2(3A-4B)c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c^3\cos^5(e+fx)}{f(a+a\sin(e+fx))^2} - \left(5(3A-4B)\right.$$
$$= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} - \frac{2a(3A-4B)c}{f(a+a\sin}$$
$$= -\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af} - \frac{a}{}$$
$$= -\frac{5(3A-4B)c^3x}{2a} - \frac{5(3A-4B)c^3\cos^3(e+fx)}{3af} - \frac{5(3A-4B)c^3\cos(e+f}{2af}$$

Mathematica [A] time = 1.36298, size = 220, normalized size = 1.4

$$\frac{c^3(\sin(e+fx)-1)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A-4B)(e+fx)-3(A-4B)\sin(2(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*(e
 + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 + 5*
e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.121, size = 449, normalized size = 2.9

$$-\frac{Ac^3}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-3}+4\,\frac{c^3\left(\tan\left(1/2\,fx+e/2\right)\right)^5B}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3}-8\,\frac{c^3\left(\tan\left(1/2\,fx+e/2\right)\right)^4A}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -1/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A+4/f*c^3/a/(1+t
an(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B-8/f*c^3/a/(1+tan(1/2*f*x+1/2*
e)^2)^3*tan(1/2*f*x+1/2*e)^4*A+14/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/
2*f*x+1/2*e)^4*B-16/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2
*A+32/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B+1/f*c^3/a/(
1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A-4/f*c^3/a/(1+tan(1/2*f*x+1/2
*e)^2)^3*tan(1/2*f*x+1/2*e)*B-8/f*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*A+46/3/f
*c^3/a/(1+tan(1/2*f*x+1/2*e)^2)^3*B-15/f*c^3/a*arctan(tan(1/2*f*x+1/2*e))*A
+20/f*c^3/a*arctan(tan(1/2*f*x+1/2*e))*B-16/f*c^3/a/(tan(1/2*f*x+1/2*e)+1)*
A+16/f*c^3/a/(tan(1/2*f*x+1/2*e)+1)*B
```

Maxima [B] time = 1.53447, size = 1512, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/3*(B*c^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*c^3
*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/a) + 9*B*c^3*((sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/
(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin
```

```
(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/a) - 18*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a) + 18*B*c^3*((sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x
 + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 18*A*c^3*(a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x +
 e) + 1))) + 6*B*c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*c^3/(a + a*sin(f*x + e)/(cos(f*x +
e) + 1)))/f
```

Fricas [A] time = 1.5018, size = 533, normalized size = 3.39

$$-\frac{2\,Bc^3\cos\left(fx+e\right)^4+(3\,A-10\,B)c^3\cos\left(fx+e\right)^3+15\,(3\,A-4\,B)c^3fx+24\,(A-2\,B)c^3\cos\left(fx+e\right)^2+48\,(A-B)c^3+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/6*(2*B*c^3*cos(f*x + e)^4 + (3*A - 10*B)*c^3*cos(f*x + e)^3 + 15*(3*A -
4*B)*c^3*f*x + 24*(A - 2*B)*c^3*cos(f*x + e)^2 + 48*(A - B)*c^3 + 3*(5*(3*A
 - 4*B)*c^3*f*x + (23*A - 28*B)*c^3)*cos(f*x + e) + (2*B*c^3*cos(f*x + e)^3
 + 15*(3*A - 4*B)*c^3*f*x - 3*(A - 4*B)*c^3*cos(f*x + e)^2 + 3*(7*A - 12*B)
*c^3*cos(f*x + e) - 48*(A - B)*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*s
in(f*x + e) + a*f)
```

Sympy [A] time = 31.7337, size = 4255, normalized size = 27.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 +
6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f
*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*
tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)
**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x
*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6
 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2
 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f)
 - 135*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*ta
n(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x
/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*ta
n(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**
5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x/(6*a*f*tan(
e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 102*A*c**3*tan(e/2 + f*x/2)**
6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 +
 f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a
*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 54*A*c**3*tan(e/
2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a
*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/
2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 336*
A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x
/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*t
an(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
6*a*f) - 96*A*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) - 378*A*c**3*tan(e/2 + f*x/2)**2/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 42*A*c**3*tan(e/2 + f*x/2)/(6*a*f*tan(e/2
 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*
a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x
/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 144*A*c**3/(6*a*f*tan(e/2 + f*x/
```

```
2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan
(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*
f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)
**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*tan(e/2 +
 f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*
tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)*
*3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c
**3*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*
x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*
tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) +
 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6
*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*
x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/
2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18
*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)
**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*
tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6
0*B*c**3*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*
f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 120*B
*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) + 108*B*c**3*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
 + f*x/2) + 6*a*f) + 372*B*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 192*B*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan
(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
 f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 456*B*c**3*tan(e/2 + f*x/2)*
*2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 68*B*c**3*tan(e
/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
```

```
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 188*B*
c**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f), Ne(f, 0)), (x*
(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a), True))
```

Giac [A] time = 1.20191, size = 317, normalized size = 2.02

$$-\frac{\frac{15\left(3Ac^3-4Bc^3\right)(fx+e)}{a}+\frac{96\left(Ac^3-Bc^3\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(3Ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12Bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+24Ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-42Bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+\cdots\right.}{\left(\tan\left(\frac{1}{2}\cdots\right.\right.}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(15*(3*A*c^3 - 4*B*c^3)*(f*x + e)/a + 96*(A*c^3 - B*c^3)/(a*(tan(1/2*f
*x + 1/2*e) + 1)) + 2*(3*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 12*B*c^3*tan(1/2*f*
x + 1/2*e)^5 + 24*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*c^3*tan(1/2*f*x + 1/2
*e)^4 + 48*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*c^3*tan(1/2*f*x + 1/2*e)^2 -
 3*A*c^3*tan(1/2*f*x + 1/2*e) + 12*B*c^3*tan(1/2*f*x + 1/2*e) + 24*A*c^3 -
46*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{3c^2(2A-3B)\cos(e+fx)}{2af}-\frac{c^2(2A-3B)\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{3c^2x(2A-3B)}{2a}$$

```
[Out] (-3*(2*A - 3*B)*c^2*x)/(2*a) - (3*(2*A - 3*B)*c^2*Cos[e + f*x])/(2*a*f) - (
a^2*(A - B)*c^2*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3) - ((2*A - 3*B)*c
^2*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.27847, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2679, 2682, 8}

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}-\frac{3c^2(2A-3B)\cos(e+fx)}{2af}-\frac{c^2(2A-3B)\cos^3(e+fx)}{2f(a\sin(e+fx)+a)}-\frac{3c^2x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-3*(2*A - 3*B)*c^2*x)/(2*a) - (3*(2*A - 3*B)*c^2*Cos[e + f*x])/(2*a*f) - (
a^2*(A - B)*c^2*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3) - ((2*A - 3*B)*c
^2*Cos[e + f*x]^3)/(2*f*(a + a*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \left(a(2A-3B)c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))} - \frac{1}{2}\left(3(2A-3B\right.$$
$$= -\frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3} - \frac{(2A-3B)c^2}{2f(a+a\sin}$$
$$= -\frac{3(2A-3B)c^2x}{2a} - \frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos^5(e+f}{f(a+a\sin(e+fx))^3}$$

Mathematica [A] time = 1.27359, size = 188, normalized size = 1.59

$$-\frac{c^2(\sin(e+fx)-1)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A-3B)(e+fx)+4(A-3B)\cos(e+fx)\right.}{4af(\sin(e+fx)+1)\left(\cos\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[(e +
 f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e +
 f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e + 9*f*x
) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(4*a*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.106, size = 299, normalized size = 2.5

$$\frac{Bc^2}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}-2\,\frac{c^2\left(\tan\left(1/2\,fx+e/2\right)\right)^2A}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}+6\,\frac{c^2\left(\tan\left(1/2\,fx+e/2\right)\right)^2B}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2/f*c^2/a/(1+ta
n(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+6/f*c^2/a/(1+tan(1/2*f*x+1/2*e
)^2)^2*tan(1/2*f*x+1/2*e)^2*B-1/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/
2*f*x+1/2*e)-2/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*A+6/f*c^2/a/(1+tan(1/2*f*
x+1/2*e)^2)^2*B+9/f*c^2/a*arctan(tan(1/2*f*x+1/2*e))*B-6/f*c^2/a*arctan(tan
(1/2*f*x+1/2*e))*A-8/f*c^2/a/(tan(1/2*f*x+1/2*e)+1)*A+8/f*c^2/a/(tan(1/2*f*
x+1/2*e)+1)*B
```

Maxima [B] time = 1.49755, size = 821, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 4*B*
c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) - 4*A*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*c^2*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.41923, size = 423, normalized size = 3.58

$$\frac{Bc^2\cos\left(fx+e\right)^3 - 3\,(2\,A-3\,B)c^2fx - 2\,(A-3\,B)c^2\cos\left(fx+e\right)^2 - 8\,(A-B)c^2 - \left(3\,(2\,A-3\,B)c^2fx + (10\,A-13\,B)c^2\right.}{2\left(af\cos\left(fx+e\right) + af\,\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*x
 + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos(f
*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^2*c
os(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x
+ e) + a*f)
```

Sympy [A] time = 14.9269, size = 2365, normalized size = 20.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2
*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/
2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan(e/2 + f*x/2)**4/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*
A*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) - 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) - 6*A*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 16*A*c**2*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 28*A*c**2*tan(e/2 + f*x/2)*
*3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
 f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) -
4*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f
*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*ta
n(e/2 + f*x/2) + 2*a*f) + 12*A*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2
)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/
2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2/(2*a*f*tan(e/2 +
 f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*
tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/
2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*f*x*tan(e/2 + f*x/2)**3/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*
B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
 f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 +
```

```
f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*t
an(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 18*B*c**2*
tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) - 22*B*c**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 +
 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*c**2*tan(e/2 + f*x/2)**2/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c
**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
f*x/2) + 2*a*f) + 10*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**2/(a*
sin(e) + a), True))
```

Giac [A] time = 1.18918, size = 221, normalized size = 1.87

$$-\frac{\frac{3\left(2Ac^2-3Bc^2\right)(fx+e)}{a}+\frac{16\left(Ac^2-Bc^2\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}-\frac{2\left(Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2Ac^2+6B\cdots\right.}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2a}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f*
x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x +
1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e)^2 - B*c^2*tan(1/2*f*x + 1/2*e) - 2*
A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=57

$$-\frac{2c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx(A-2B)}{a}+\frac{Bc\cos(e+fx)}{af}$$

```
[Out] -(((A - 2*B)*c*x)/a) + (B*c*Cos[e + f*x])/(a*f) - (2*(A - B)*c*Cos[e + f*x]
)/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.155184, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {2967, 2857, 2638}

$$-\frac{2c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}-\frac{cx(A-2B)}{a}+\frac{Bc\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 2*B)*c*x)/a) + (B*c*Cos[e + f*x])/(a*f) - (2*(A - B)*c*Cos[e + f*x]
)/(f*(a + a*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx\\
&= -\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}-\frac{c\int (aA-2aB+aB\sin(e+fx))\,dx}{a^2}\\
&= -\frac{(A-2B)cx}{a}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}-\frac{(Bc)\int \sin(e+fx)\,dx}{a}\\
&= -\frac{(A-2B)cx}{a}+\frac{Bc\cos(e+fx)}{af}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}
\end{aligned}$$

Mathematica [B] time = 0.561802, size = 127, normalized size = 2.23

$$\frac{(c-c\sin(e+fx))\left(\frac{4(A-B)\sin\left(\frac{fx}{2}\right)}{f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}+x(-(A-2B))-\frac{B\sin(e)\sin(fx)}{f}+\frac{B\cos(e)\cos(fx)}{f}\right)}{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] ((-((A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A -
B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)
```

Maple [A] time = 0.096, size = 113, normalized size = 2.

$$2\,\frac{Bc}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}-2\,\frac{c\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)A}{af}+4\,\frac{c\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)B}{af}-4\,\frac{}{af\left(\tan\left(1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/f*c/a*B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*c/a*arctan(tan(1/2*f*x+1/2*e))*A+4/f
*c/a*arctan(tan(1/2*f*x+1/2*e))*B-4/f*c/a/(tan(1/2*f*x+1/2*e)+1)*A+4/f*c/a/
(tan(1/2*f*x+1/2*e)+1)*B
```

Maxima [B] time = 1.45172, size = 346, normalized size = 6.07

$$\frac{2\left(Bc\left(\frac{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+2}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{a\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}\right)-Ac\left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}+\frac{1}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)+Bc\left(\frac{\arctan\left(\frac{\sin\left(f\right.}{\cos\left(fx\right.}\right.}{a}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(a + a*
sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] time = 1.42437, size = 289, normalized size = 5.07

$$-\frac{(A-2B)cfx-Bc\cos\left(fx+e\right)^2+2(A-B)c+\left((A-2B)cfx+(2A-3B)c\right)\cos\left(fx+e\right)+\left((A-2B)cfx-Bc\cos\left(f\right.\right.}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x + (
2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A - B
```

```
)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 7.1462, size = 830, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(
e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2)**3
+ a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e/2 + f
*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 +
 f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*
f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2
+ a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f
*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*
x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
 + a*f) - 2*B*c*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2
+ f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)**2/(a*f*
tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f)
 + 4*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)
, True))
```

Giac [B] time = 1.15299, size = 165, normalized size = 2.89

$$-\frac{\frac{(Ac-2\,Bc)(fx+e)}{a}+\frac{2\left(2\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac-3\,Bc\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(1
/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f*x
 + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=35

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

```
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Rubi [A] time = 0.135558, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2669, 3767, 8}

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

```
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\,dx}{ac}\\ &= \frac{B\sec(e+fx)}{acf}+\frac{A\int \sec^2(e+fx)\,dx}{ac}\\ &= \frac{B\sec(e+fx)}{acf}-\frac{A\,\mathrm{Subst}(\int 1\,dx,x,-\tan(e+fx))}{acf}\\ &= \frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}\end{aligned}$$

Mathematica [A] time = 0.0283891, size = 35, normalized size = 1.

$$\frac{A\tan(e+fx)}{acf}+\frac{B\sec(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),
x]
```

```
[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Maple [A] time = 0.056, size = 57, normalized size = 1.6

$$2\,\frac{1}{acf}\left(-\frac{A/2-B/2}{\tan\left(1/2\,fx+e/2\right)+1}-\frac{A/2+B/2}{\tan\left(1/2\,fx+e/2\right)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/c/a*(-(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)-(1/2*A+1/2*B)/(tan(1/2*f*x+1
/2*e)-1))
```

Maxima [A] time = 0.9652, size = 47, normalized size = 1.34

$$\frac{\frac{A\tan\left(fx+e\right)}{ac}+\frac{B}{ac\cos\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] (A*tan(f*x + e)/(a*c) + B/(a*c*cos(f*x + e)))/f
```

Fricas [A] time = 1.35681, size = 58, normalized size = 1.66

$$\frac{A\sin\left(fx+e\right)+B}{acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] (A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))
```

Sympy [A] time = 4.16627, size = 83, normalized size = 2.37

$$\begin{cases} -\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-acf}-\frac{2B}{acf\tan^{2}\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f\neq 0\\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B/
(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e
) + a)*(-c*sin(e) + c)), True))
```

Giac [A] time = 1.20056, size = 55, normalized size = 1.57

$$-\frac{2\left(A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+B\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)
```

3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=63

$$\frac{(2A-B)\tan(e+fx)}{3ac^2f}+\frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

```
[Out] ((A + B)*Sec[e + f*x])/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + ((2*A - B)*Tan[e
+ f*x])/(3*a*c^2*f)
```

Rubi [A] time = 0.202196, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 3767, 8}

$$\frac{(2A-B)\tan(e+fx)}{3ac^2f}+\frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```



```
[Out] ((A + B)*Sec[e + f*x])/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + ((2*A - B)*Tan[e
+ f*x])/(3*a*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{(2A-B)\int \sec^2(e+fx)\,dx}{3ac^2}\\ &= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}-\frac{(2A-B)\,\mathrm{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3ac^2f}\\ &= \frac{(A+B)\sec(e+fx)}{3af\left(c^2-c^2\sin(e+fx)\right)}+\frac{(2A-B)\tan(e+fx)}{3ac^2f}\end{aligned}$$

Mathematica [A] time = 0.569136, size = 108, normalized size = 1.71

$$\frac{\cos(e+fx)(-2(A+B)\cos(e+fx)+(4A-2B)\cos(2(e+fx))+8A\sin(e+fx)+A\sin(2(e+fx))-4B\sin(e+fx)+B}{12ac^2f(\sin(e+fx)-1)^2(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2
),x]
```

```
[Out] (Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)]
+ 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e + f
*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.076, size = 93, normalized size = 1.5

$$2\,\frac{1}{afc^2}\left(-1/3\,\frac{A+B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^3}-1/2\,\frac{A+B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-\frac{3/4\,A+B/4}{\tan\left(1/2\,fx+e/2\right)-1}-\frac{A/4-B/4}{\tan\left(1/2\,fx+e/2\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a/c^2*(-1/3*(A+B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(A+B)/(tan(1/2*f*x+1/2*e
)-1)^2-(3/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*
e)+1))
```

Maxima [B] time = 0.99171, size = 359, normalized size = 5.7

$$-\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+1\right)}{ac^2-\frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin(f*x
+ e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c
^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A] time = 1.40067, size = 171, normalized size = 2.71

$$-\frac{(2A-B)\cos\left(fx+e\right)^2+(2A-B)\sin\left(fx+e\right)-A+2B}{3\left(ac^2f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*f
*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [A] time = 16.2408, size = 578, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c
**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*A
*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 +
 f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 + f*x/
2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c
**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/2)**4
- 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*ta
n(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*B*tan(e/2
 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3
+ 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*tan(e/2 + f*x
/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**2),
 True))
```

Giac [A] time = 1.19162, size = 138, normalized size = 2.19

$$-\frac{\frac{3\,(A-B)}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A+B}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)/(
a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}+\frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)}+\frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

```
[Out] ((A + B)*Sec[e + f*x])/(5*a*c*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*Sec[
e + f*x])/(15*a*f*(c^3 - c^3*Sin[e + f*x])) + (2*(3*A - 2*B)*Tan[e + f*x])/
(15*a*c^3*f)
```

Rubi [A] time = 0.257064, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}+\frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)}+\frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(5*a*c*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*Sec[
e + f*x])/(15*a*f*(c^3 - c^3*Sin[e + f*x])) + (2*(3*A - 2*B)*Tan[e + f*x])/
(15*a*c^3*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}\,dx}{5ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + \frac{(2(3A-2B))\int \ldots}{15a\ldots}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} - \frac{(2(3A-2B))\,\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2} + \frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)} + \frac{2(3A-2B)\tan\ldots}{15ac^3f\ldots}$$

Mathematica [A] time = 0.842252, size = 157, normalized size = 1.54

$$-\frac{\cos(e+fx)(5(B-9A)\cos(e+fx)+32(3A-2B)\cos(2(e+fx))+120A\sin(e+fx)+36A\sin(2(e+fx))-24A\sin\ldots}{240ac^3f(\sin(e+fx\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3
),x]
```

```
[Out] -(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Cos[2*(e
+ f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e + f*x] -
80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] - 24*A*Sin
[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a*c^3*f*(-1 + Sin[e + f*x])^3*
(1 + Sin[e + f*x]))
```

Maple [A] time = 0.085, size = 145, normalized size = 1.4

$$2\,\frac{1}{afc^3}\left(-1/5\,\frac{2\,A+2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-1/4\,\frac{4\,A+4\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^4}-1/2\,\frac{5/2\,A+3/2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-\frac{1}{\tan\left(1/2\,fx+e/2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a/c^3*(-1/5*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/4*(4*A+4*B)/(tan(1/2*f
*x+1/2*e)-1)^4-1/2*(5/2*A+3/2*B)/(tan(1/2*f*x+1/2*e)-1)^2-(7/8*A+1/8*B)/(ta
n(1/2*f*x+1/2*e)-1)-1/3*(9/2*A+7/2*B)/(tan(1/2*f*x+1/2*e)-1)^3-(1/8*A-1/8*B
)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.02646, size = 571, normalized size = 5.6

$$-\frac{2\left(\frac{B\left(\frac{4\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{20\,\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{20\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}-\frac{15\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-1\right)}{ac^3-\frac{4\,ac^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5\,ac^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5\,ac^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{4\,ac^3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}-\frac{ac^3\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6}}+\frac{3\,A\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{10\,\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}-\frac{5\,\sin}{\left(\cos\left(f\right.\right.}\right.}{ac^3-\frac{4\,ac^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5\,ac^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}-\frac{5\,ac^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{4\,ac^3\sin\left(fx-\right.}{\left(\cos\left(fx+e\right)+\right.}}\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6) + 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*sin(f*x + e
)/(cos(f*x + e) + 1) + 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [A] time = 1.31978, size = 265, normalized size = 2.6

$$-\frac{4\,(3\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(2\,(3\,A-2\,B)\cos\left(fx+e\right)^{2}-9\,A+6\,B\right)\sin\left(fx+e\right)-6\,A+9\,B}{15\left(ac^{3}f\cos\left(fx+e\right)^{3}+2\,ac^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,ac^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A +
 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
 + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [A] time = 31.9582, size = 1732, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-2*A*tan(e/2 + f*x/2)**6/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a
*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f
*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 22*A*t
an(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 +
```

```
 f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)
**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 50*A*tan(e/2 + f*x/2)**
4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a
*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f
*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f*x/2)**3/(15*a*c**3*f*ta
n(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 +
 f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2)
 - 15*a*c**3*f) + 10*A*tan(e/2 + f*x/2)**2/(15*a*c**3*f*tan(e/2 + f*x/2)**6
 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a
*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) +
 10*A*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e
/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*
x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 10*A/(15*a*c**3*f*t
an(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2
+ f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2
) - 15*a*c**3*f) - 7*B*tan(e/2 + f*x/2)**6/(15*a*c**3*f*tan(e/2 + f*x/2)**6
 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a
*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) +
 28*B*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*ta
n(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 +
 f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 65*B*tan(e/2 + f
*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5
 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a
*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)**3/(15*a*c*
*3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*ta
n(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 +
 f*x/2) - 15*a*c**3*f) - 5*B*tan(e/2 + f*x/2)**2/(15*a*c**3*f*tan(e/2 + f*x
/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4
- 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**
3*f) - 20*B*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f
*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/
2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 5*B/(15*a*c**
3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan
(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 +
f*x/2) - 15*a*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin
(e) + c)**3), True))
```

Giac [A] time = 1.22914, size = 239, normalized size = 2.34

$$-\frac{\frac{15\,(A-B)}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-40\,B\tan\left(\frac{1}{2}\,f\cdots\right.}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A - B)/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 + 15*B*tan(1/2*f*x + 1/2*e)^4 - 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 - 40*B*tan(1/2*f*x
 + 1/2*e)^2 - 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 - 7*B)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=142

$$\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

```
[Out] ((A + B)*Sec[e + f*x])/(7*a*c*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*Sec[
e + f*x])/(35*a*f*(c^2 - c^2*Sin[e + f*x])^2) + ((4*A - 3*B)*Sec[e + f*x])/
(35*a*f*(c^4 - c^4*Sin[e + f*x])) + (2*(4*A - 3*B)*Tan[e + f*x])/(35*a*c^4*
f)
```

Rubi [A] time = 0.306909, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x])/(7*a*c*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*Sec[
e + f*x])/(35*a*f*(c^2 - c^2*Sin[e + f*x])^2) + ((4*A - 3*B)*Sec[e + f*x])/
(35*a*f*(c^4 - c^4*Sin[e + f*x])) + (2*(4*A - 3*B)*Tan[e + f*x])/(35*a*c^4*
f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{7ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(3(4A-3B))\int}{35ac}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(}{35af\left(c^4-c^4\sin\right.}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(}{35af\left(c^4-c^4\sin\right.}$$

$$= \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3} + \frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(4A-3B)\sec(}{35af\left(c^4-c^4\sin\right.}$$

Mathematica [A] time = 1.09769, size = 240, normalized size = 1.69

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((182B-406A)\cos(e+fx)+224(4A-3B)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4
),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x)]
 + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] +
 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*A*Si
n[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288*B*Si
n[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(2240*a*c^
4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.087, size = 189, normalized size = 1.3

$$2\,{\frac {1}{af{c}^{4}} \left(-1/7\,{\frac {4\,A+4\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{7}}}-1/6\,{\frac {12\,A+12\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{6}}}-1/4\,{\frac {18\,A+14\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{4}}}-1/5\,{\frac {19\,A}{ \left(\tan \left(1/2\,f \right. \right. }}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a/c^4*(-1/7*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(12*A+12*B)/(tan(1/2
*f*x+1/2*e)-1)^6-1/4*(18*A+14*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(19*A+17*B)/(
tan(1/2*f*x+1/2*e)-1)^5-(15/16*A+1/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(17/4*A
+7/4*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(45/4*A+27/4*B)/(tan(1/2*f*x+1/2*e)-1)
^3-(1/16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.07238, size = 836, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="maxima")
```

```
[Out] -2/35*(A*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 13)/(
a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 56*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 70*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(a*c^4 - 6*a*c^4*sin(f*
x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a*c^4*s
in(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1
```

```
)^8))/f
```

Fricas [A] time = 1.30648, size = 350, normalized size = 2.46

$$\frac{2\,(4\,A-3\,B)\cos\left(fx+e\right)^{4}-9\,(4\,A-3\,B)\cos\left(fx+e\right)^{2}+\left(6\,(4\,A-3\,B)\cos\left(fx+e\right)^{2}-20\,A+15\,B\right)\sin\left(fx+e\right)+15\,A}{35\left(3\,ac^{4}f\cos\left(fx+e\right)^{3}-4\,ac^{4}f\cos\left(fx+e\right)-\left(ac^{4}f\cos\left(fx+e\right)^{3}-4\,ac^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="fricas")
```

```
[Out] 1/35*(2*(4*A - 3*B)*cos(f*x + e)^4 - 9*(4*A - 3*B)*cos(f*x + e)^2 + (6*(4*A
 - 3*B)*cos(f*x + e)^2 - 20*A + 15*B)*sin(f*x + e) + 15*A - 20*B)/(3*a*c^4*
f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f*cos(f*x + e)^3 - 4*a*c
^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19567, size = 320, normalized size = 2.25

$$-\frac{\frac{35\,(A-B)}{ac^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{525\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+35\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-1960\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+4025\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-665\,B\tan}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm
="giac")
```

```
[Out] -1/280*(35*(A - B)/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*A*tan(1/2*f*x
+ 1/2*e)^6 + 35*B*tan(1/2*f*x + 1/2*e)^6 - 1960*A*tan(1/2*f*x + 1/2*e)^5 +
280*B*tan(1/2*f*x + 1/2*e)^5 + 4025*A*tan(1/2*f*x + 1/2*e)^4 - 665*B*tan(1/
2*f*x + 1/2*e)^4 - 4480*A*tan(1/2*f*x + 1/2*e)^3 + 1120*B*tan(1/2*f*x + 1/2
*e)^3 + 3143*A*tan(1/2*f*x + 1/2*e)^2 - 791*B*tan(1/2*f*x + 1/2*e)^2 - 1176
*A*tan(1/2*f*x + 1/2*e) + 392*B*tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/(a*c^4
*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=240

$$\frac{35c^5(4A-7B)\cos^3(e+fx)}{4a^2f}-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}+\frac{2a^3c^5(4A-7B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{6a^4c^5(4A-7B)\cos^7(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^3}$$

```
[Out] (105*(4*A - 7*B)*c^5*x)/(8*a^2) + (35*(4*A - 7*B)*c^5*Cos[e + f*x]^3)/(4*a^
2*f) + (105*(4*A - 7*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f) - (a^5*(A
- B)*c^5*Cos[e + f*x]^11)/(3*f*(a + a*Sin[e + f*x])^7) + (2*a^3*(4*A - 7*B)
*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^5) + (6*a^4*(4*A - 7*B)*c^5*
Cos[e + f*x]^7)/(f*(a^2 + a^2*Sin[e + f*x])^3) + (21*(4*A - 7*B)*c^5*Cos[e
+ f*x]^5)/(4*f*(a^2 + a^2*Sin[e + f*x]))
```

Rubi [A] time = 0.410041, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2967, 2859, 2680, 2679, 2682, 2635, 8}

$$\frac{35c^5(4A-7B)\cos^3(e+fx)}{4a^2f}-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}+\frac{2a^3c^5(4A-7B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{6a^4c^5(4A-7B)\cos^7(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] (105*(4*A - 7*B)*c^5*x)/(8*a^2) + (35*(4*A - 7*B)*c^5*Cos[e + f*x]^3)/(4*a^
2*f) + (105*(4*A - 7*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f) - (a^5*(A
- B)*c^5*Cos[e + f*x]^11)/(3*f*(a + a*Sin[e + f*x])^7) + (2*a^3*(4*A - 7*B)
*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^5) + (6*a^4*(4*A - 7*B)*c^5*
Cos[e + f*x]^7)/(f*(a^2 + a^2*Sin[e + f*x])^3) + (21*(4*A - 7*B)*c^5*Cos[e
+ f*x]^5)/(4*f*(a^2 + a^2*Sin[e + f*x]))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} - \frac{1}{3}\left(a^4(4A-7B)c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))^6}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \left(3a^2(4A-\right.$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7}{f(a+a}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{6a(4A-7}{f(a+a}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B}{3f(a+a\sin}$$

$$= \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f} -$$

$$= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} + \frac{105(4A-7B)c^5\cos(e}{8a^2f}$$

Mathematica [A] time = 1.98599, size = 354, normalized size = 1.48

$$\frac{(c-c\sin(e+fx))^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2048(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+1260(4A-7B)(e+fx)\left(\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B)
*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 10
24*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(9
```

```
5*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*(A -
7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7*A - 2
4*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)]))/(96*a^2*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.161, size = 778, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 7/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*A-95/4/f*c^5/a^
2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^7*B+46/f*c^5/a^2/(1+tan(1/2
*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^6*A-98/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^
2)^4*tan(1/2*f*x+1/2*e)^6*B+7/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*
f*x+1/2*e)^5*A-103/4/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e
)^5*B+142/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*A-322/f
*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^4*B-7/f*c^5/a^2/(1+t
an(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^3*A+103/4/f*c^5/a^2/(1+tan(1/2*f*
x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)^3*B+430/3/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^
2)^4*tan(1/2*f*x+1/2*e)^2*A-994/3/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(
1/2*f*x+1/2*e)^2*B-7/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e
)*A+95/4/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*tan(1/2*f*x+1/2*e)*B+142/3/f*
c^5/a^2/(1+tan(1/2*f*x+1/2*e)^2)^4*A-322/3/f*c^5/a^2/(1+tan(1/2*f*x+1/2*e)^
2)^4*B+105/f*c^5/a^2*arctan(tan(1/2*f*x+1/2*e))*A-735/4/f*c^5/a^2*arctan(ta
n(1/2*f*x+1/2*e))*B+64/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A-64/f*c^5/a^2/(t
an(1/2*f*x+1/2*e)+1)^2*B+96/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)*A-160/f*c^5/a^
2/(tan(1/2*f*x+1/2*e)+1)*B-128/3/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A+128/3
/f*c^5/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B
```

Maxima [B] time = 1.76675, size = 4026, normalized size = 16.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/12*(B*c^5*((603*sin(f*x + e)/(cos(f*x + e) + 1) + 1297*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2228*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2628*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3014*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2618*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1980*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 1100*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 495*sin(f*x + e)^9/(c
os(f*x + e) + 1)^9 + 165*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 256)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^2*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 18*a^2*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 22*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a
^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 18*a^2*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 13*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7*a^2*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 3*a^2*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + a^2*
sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 165*arctan(sin(f*x + e)/(cos(f*x +
 e) + 1))/a^2) - 20*A*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^2) + 40*B*c^5*((75*sin(f*x + e)/(cos(f*x + e)
 + 1) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)
 + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*A*c^5*((57*sin(f*x +
 e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
 + a^2*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) + 40*B*c^5*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin
```

```
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2
*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 160*A*c^5*
((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*
a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 160*B*c^5*((12*s
in(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 80*A*c^5*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^2) + 40*B*c^5*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 8
*A*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 40*A
*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos
(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3) + 8*B*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [A] time = 1.52659, size = 922, normalized size = 3.84

$$-\frac{6\,Bc^5\cos\left(fx+e\right)^6+4\,(2\,A-11\,B)c^5\cos\left(fx+e\right)^5+(76\,A-241\,B)c^5\cos\left(fx+e\right)^4-2\,(212\,A-431\,B)c^5\cos\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A -
 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(4*
A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A - 3
485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 2141*B)
*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(f*x +
 e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x + 3*(16
4*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)*c^5*f*
x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22966, size = 556, normalized size = 2.32

$$\frac{315\,(4\,Ac^5-7\,Bc^5)(fx+e)}{a^2}+\frac{256\left(9\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+11\,Ac^5-17\,Bc^5\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}+\frac{2\,(84\,Ac^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1/
2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e) -
36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x + 1/
2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f*x +
 1/2*e)^7 + 552*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*tan(1/2*f*x + 1/2
*e)^6 + 84*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*tan(1/2*f*x + 1/2*e)^5
+ 1704*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*tan(1/2*f*x + 1/2*e)^4 - 8
4*A*c^5*tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 1720*A*
c^5*tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 84*A*c^5*t
an(1/2*f*x + 1/2*e) + 285*B*c^5*tan(1/2*f*x + 1/2*e) + 568*A*c^5 - 1288*B*c
^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f
```

3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=180

$$\frac{35c^4(A-2B)\cos^3(e+fx)}{3a^2f}-\frac{a^4c^4(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{2a^2c^4(A-2B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{35c^4(A-2B)\sin(e+fx)\cos(e+fx)}{2a^2f}$$

```
[Out] (35*(A - 2*B)*c^4*x)/(2*a^2) + (35*(A - 2*B)*c^4*Cos[e + f*x]^3)/(3*a^2*f)
+ (35*(A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(2*a^2*f) - (a^4*(A - B)*c^4
*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^6) + (2*a^2*(A - 2*B)*c^4*Cos[e
+ f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (14*(A - 2*B)*c^4*Cos[e + f*x]^5)/(f
*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.36157, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{35c^4(A-2B)\cos^3(e+fx)}{3a^2f}-\frac{a^4c^4(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}+\frac{2a^2c^4(A-2B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}+\frac{35c^4(A-2B)\sin(e+fx)\cos(e+fx)}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x]
```



```
[Out] (35*(A - 2*B)*c^4*x)/(2*a^2) + (35*(A - 2*B)*c^4*Cos[e + f*x]^3)/(3*a^2*f)
+ (35*(A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(2*a^2*f) - (a^4*(A - B)*c^4
*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^6) + (2*a^2*(A - 2*B)*c^4*Cos[e
+ f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (14*(A - 2*B)*c^4*Cos[e + f*x]^5)/(f
*(a + a*Sin[e + f*x])^2)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} - \left(a^3(A-2B)c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \left(7a(A-2B)\right.$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4} + \frac{14(A-2B)c}{f(a+a\,\mathrm{si}}$$

$$= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6} + \frac{2a^2(A-2B)c^4}{f(a+a\sin(}$$

$$= \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f} - \frac{a^4(}{3}$$

$$= \frac{35(A-2B)c^4x}{2a^2} + \frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f} + \frac{35(A-2B)c^4\cos(e+fx)}{2a^2f}$$

Mathematica [A] time = 1.25873, size = 311, normalized size = 1.73

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(128(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+210(A-2B)(e+fx)\left(\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)*
Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(
5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(
A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A - 71*B
)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e + f*x)]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.148, size = 549, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 1/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A-6/f*c^4/a^2/(
1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B+12/f*c^4/a^2/(1+tan(1/2*f*
x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*A-34/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^
3*tan(1/2*f*x+1/2*e)^4*B+24/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*
x+1/2*e)^2*A-72/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2*B
-1/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A+6/f*c^4/a^2/(1
+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B+12/f*c^4/a^2/(1+tan(1/2*f*x+1
/2*e)^2)^3*A-106/3/f*c^4/a^2/(1+tan(1/2*f*x+1/2*e)^2)^3*B+35/f*c^4/a^2*arct
an(tan(1/2*f*x+1/2*e))*A-70/f*c^4/a^2*arctan(tan(1/2*f*x+1/2*e))*B+32/f*c^4
/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A-32/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B+32/
f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)*A-64/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)*B-64
/3/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A+64/3/f*c^4/a^2/(tan(1/2*f*x+1/2*e)+
1)^3*B
```

Maxima [B] time = 1.65923, size = 2827, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(A*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 4*B*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) +
```

1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*B*c^4*((57*sin(f*x + e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 15*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 12*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 6*a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^2*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 16*A*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 24*B*c^4*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 12*A*c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 8*B*c^4*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 8*A*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*B*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f

Fricas [A] time = 1.82285, size = 786, normalized size = 4.37

$$\frac{2\,Bc^4\cos\left(fx+e\right)^5-(3\,A-16\,B)c^4\cos\left(fx+e\right)^4+2\,(15\,A-38\,B)c^4\cos\left(fx+e\right)^3-210\,(A-2\,B)c^4fx+32\,(A-B)c^4 \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(2*B*c^4*cos(f*x + e)^5 - (3*A - 16*B)*c^4*cos(f*x + e)^4 + 2*(15*A - 3
8*B)*c^4*cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(A
- 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*cos(f*x + e)^2 - (105*(A - 2*B)*c^4*f
*x + 2*(97*A - 202*B)*c^4)*cos(f*x + e) - (2*B*c^4*cos(f*x + e)^4 + (3*A -
14*B)*c^4*cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4*cos(
f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218*B)*c^
4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) -
 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22226, size = 498, normalized size = 2.77

$$\frac{\frac{105\left(Ac^4-2\,Bc^4\right)\left(fx+e\right)}{a^2}+\frac{2\left(99\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-210\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+333\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-636\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+533\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*tan(1/2*f*x + 1/2*e)
^8 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*tan(1/2*f*x + 1/2*e)^7 -
636*B*c^4*tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 1160*
B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 1980*B*c
^4*tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1980*B*c^4*t
an(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^4*tan(
1/2*f*x + 1/2*e)^3 + 651*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*tan(1/2*
f*x + 1/2*e)^2 + 393*A*c^4*tan(1/2*f*x + 1/2*e) - 780*B*c^4*tan(1/2*f*x + 1
/2*e) + 164*A*c^4 - 330*B*c^4)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2
*e)^2 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{5c^3(2A-5B)\cos(e+fx)}{2a^2f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{5c^3(2A-5B)\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5c^3x(2A-5B)}{2a^2}+\frac{2ac^3(2A-5B)}{3f(a\sin(e+}$$

```
[Out] (5*(2*A - 5*B)*c^3*x)/(2*a^2) + (5*(2*A - 5*B)*c^3*Cos[e + f*x])/(2*a^2*f)
- (a^3*(A - B)*c^3*Cos[e + f*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (2*a*(2*A
 - 5*B)*c^3*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^3) + (5*(2*A - 5*B)*c
^3*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f*x]))
```

Rubi [A] time = 0.332406, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$\frac{5c^3(2A-5B)\cos(e+fx)}{2a^2f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}+\frac{5c^3(2A-5B)\cos^3(e+fx)}{6f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5c^3x(2A-5B)}{2a^2}+\frac{2ac^3(2A-5B)}{3f(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (5*(2*A - 5*B)*c^3*x)/(2*a^2) + (5*(2*A - 5*B)*c^3*Cos[e + f*x])/(2*a^2*f)
- (a^3*(A - B)*c^3*Cos[e + f*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (2*a*(2*A
 - 5*B)*c^3*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^3) + (5*(2*A - 5*B)*c
^3*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} - \frac{1}{3}\left(a^2(2A-5B)c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^4}$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{1}{3}\left(5(2A-\right.$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{5(2A-5B}{6f\left(a^2+\right.}$$

$$= \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5} + \frac{2a(2A-5B)c^3}{3f(a+a\sin}$$

$$= \frac{5(2A-5B)c^3x}{2a^2} + \frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}$$

Mathematica [A] time = 0.845507, size = 274, normalized size = 1.69

$$\frac{(c-c\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+30(2A-5B)(e+fx)\left(\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*S
in[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(7*
A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*(2*
A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*B)*C
os[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.129, size = 399, normalized size = 2.5

$$-\frac{Bc^3}{a^2f}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2} + 2\,\frac{c^3\left(\tan\left(1/2\,fx+e/2\right)\right)^2A}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2} - 10\,\frac{c^3\left(\tan\left(1/2\,fx+e/2\right)\right)^2}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -1/f*c^3/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B+2/f*c^3/a^2/
(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A-10/f*c^3/a^2/(1+tan(1/2*f
*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B+1/f*c^3/a^2/(1+tan(1/2*f*x+1/2*e)^2)^
2*B*tan(1/2*f*x+1/2*e)+2/f*c^3/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*A-10/f*c^3/a^
2/(1+tan(1/2*f*x+1/2*e)^2)^2*B-25/f*c^3/a^2*arctan(tan(1/2*f*x+1/2*e))*B+10
/f*c^3/a^2*arctan(tan(1/2*f*x+1/2*e))*A+16/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+1)
^2*A-16/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B+8/f*c^3/a^2/(tan(1/2*f*x+1/2*e
)+1)*A-24/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+1)*B-32/3/f*c^3/a^2/(tan(1/2*f*x+1/
2*e)+1)^3*A+32/3/f*c^3/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B
```

Maxima [B] time = 1.58791, size = 1860, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -1/3*(B*c^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f
*x + e) + 1))/a^2) - 4*A*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*
x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a^2) + 12*B*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x
```

```
 + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a
^2) - 6*A*c^3*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*B*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3
*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a^2) + 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3) - 6*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2
+ 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*B*c^3*(3*sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3))/f
```

Fricas [A] time = 1.81425, size = 687, normalized size = 4.24

$$\frac{3 Bc^3 \cos\left(fx+e\right)^4 + 6\,(A-4\,B)c^3 \cos\left(fx+e\right)^3 - 30\,(2\,A-5\,B)c^3 fx + 16\,(A-B)c^3 + \left(15\,(2\,A-5\,B)c^3 fx - (62\,A-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/6*(3*B*c^3*cos(f*x + e)^4 + 6*(A - 4*B)*c^3*cos(f*x + e)^3 - 30*(2*A - 5*
B)*c^3*f*x + 16*(A - B)*c^3 + (15*(2*A - 5*B)*c^3*f*x - (62*A - 131*B)*c^3)
*cos(f*x + e)^2 - (15*(2*A - 5*B)*c^3*f*x + 2*(26*A - 71*B)*c^3)*cos(f*x +
e) + (3*B*c^3*cos(f*x + e)^3 - 30*(2*A - 5*B)*c^3*f*x - 3*(2*A - 9*B)*c^3*c
os(f*x + e)^2 - 16*(A - B)*c^3 - (15*(2*A - 5*B)*c^3*f*x + 2*(34*A - 79*B)*
c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
 - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19919, size = 315, normalized size = 1.94

$$\frac{\frac{15\left(2\,Ac^3-5\,Bc^3\right)(fx+e)}{a^2}-\frac{6\left(Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^3+10\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}+\frac{16\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx\right.\right.}{}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(15*(2*A*c^3 - 5*B*c^3)*(f*x + e)/a^2 - 6*(B*c^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 10*B*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c^3
*tan(1/2*f*x + 1/2*e) - 2*A*c^3 + 10*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2
*a^2) + 16*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 12*A*c^3*tan(1/2*f*x + 1/2*e) - 24*B*c^3*tan(1/2*f*x + 1/2*e) + 5*A*c^3
- 11*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{c^2(A-4B)\cos(e+fx)}{a^2f}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{c^2x(A-4B)}{a^2}+\frac{2c^2(A-4B)\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] ((A - 4*B)*c^2*x)/a^2 + ((A - 4*B)*c^2*Cos[e + f*x])/(a^2*f) - (a^2*(A - B)
*c^2*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^4) + (2*(A - 4*B)*c^2*Cos[e
+ f*x]^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.276867, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{c^2(A-4B)\cos(e+fx)}{a^2f}-\frac{a^2c^2(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}+\frac{c^2x(A-4B)}{a^2}+\frac{2c^2(A-4B)\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] ((A - 4*B)*c^2*x)/a^2 + ((A - 4*B)*c^2*Cos[e + f*x])/(a^2*f) - (a^2*(A - B)
*c^2*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^4) + (2*(A - 4*B)*c^2*Cos[e
+ f*x]^3)/(3*f*(a + a*Sin[e + f*x])^2)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx \\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}-\frac{1}{3}\left(a(A-4B)c^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}+\frac{\left((A-4B)c^2\right)\int
\\
&= \frac{(A-4B)c^2\cos(e+fx)}{a^2f}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(
\\
&= \frac{(A-4B)c^2x}{a^2}+\frac{(A-4B)c^2\cos(e+fx)}{a^2f}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(
\end{aligned}$$

Mathematica [B] time = 0.571783, size = 234, normalized size = 2.17

$$\frac{(c-c\sin(e+fx))^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+3(A-4B)(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A -
 B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3)*(c - c*Sin[e + f*x])^2)/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^4*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.117, size = 198, normalized size = 1.8

$$-2\,\frac{Bc^2}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}+2\,\frac{c^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)A}{a^2f}-8\,\frac{c^2\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)B}{a^2f}+8\,\frac{}{a^2f\,(\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/f*c^2/a^2*B/(1+tan(1/2*f*x+1/2*e)^2)+2/f*c^2/a^2*arctan(tan(1/2*f*x+1/2*
e))*A-8/f*c^2/a^2*arctan(tan(1/2*f*x+1/2*e))*B+8/f*c^2/a^2/(tan(1/2*f*x+1/2
*e)+1)^2*A-8/f*c^2/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B-16/3/f*c^2/a^2/(tan(1/2*f
*x+1/2*e)+1)^3*A+16/3/f*c^2/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B-8/f*c^2/a^2*B/(t
an(1/2*f*x+1/2*e)+1)
```

Maxima [B] time = 1.52802, size = 1125, normalized size = 10.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*
c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 2*B*c^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.70713, size = 568, normalized size = 5.26

$$-\frac{3\,Bc^2\cos\left(fx+e\right)^3+6\,(A-4\,B)c^2fx-4\,(A-B)c^2-\left(3\,(A-4\,B)c^2fx-(8\,A-23\,B)c^2\right)\cos\left(fx+e\right)^2+\left(3\,(A-4\,B)c^2\right.}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A
- 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x +
2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*x +
 e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(f*x
+ e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f -
```

```
(a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 30.2976, size = 2474, normalized size = 22.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x*t
an(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2
)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a*
*2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a*
*2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5
 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*x
*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)
**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**
2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*A*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)*
*5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*A*c**2*
tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f
*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) + 24*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 - 12*B*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(e/2
 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*t
```

```
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x
/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(
e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 12*B*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c**2*tan(
e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*
f*tan(e/2 + f*x/2) + 3*a**2*f) - 78*B*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) - 74*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 90*B*c**2*tan(e/2 + f*x
/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*
f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) - 38*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.18346, size = 184, normalized size = 1.7

$$\frac{\frac{3\left(Ac^2-4Bc^2\right)(fx+e)}{a^2}-\frac{6Bc^2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^2}-\frac{8\left(3Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+9Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Ac^2+4Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2 +
 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*e)
 + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1/2*
e) + 1)^3))/f
```

3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=72

$$\frac{c(A-7B)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{Bcx}{a^2}-\frac{2c(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -((B*c*x)/a^2) + ((A - 7*B)*c*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) -
(2*(A - B)*c*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.207294, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2735, 2648}

$$\frac{c(A-7B)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}-\frac{Bcx}{a^2}-\frac{2c(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((B*c*x)/a^2) + ((A - 7*B)*c*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) -
(2*(A - B)*c*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx &= (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx\\
&= -\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{c\int \frac{aA-4aB+3aB\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}\\
&= -\frac{Bcx}{a^2} - \frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{((A-7B)c)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}\\
&= -\frac{Bcx}{a^2} - \frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(A-7B)c\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}
\end{aligned}$$

Mathematica [B] time = 0.559679, size = 156, normalized size = 2.17

$$\frac{c\left(-6(A-3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)-9Bfx\sin\left(e+\frac{fx}{2}\right)-3Bfx\sin\left(e+\frac{3fx}{2}\right)-14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\right.}{6a^2f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*f
*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin[
(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^2*f
*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] time = 0.104, size = 160, normalized size = 2.2

$$-2\,{\frac {Bc\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) }{{a}^{2}f}}+4\,{\frac {Ac}{{a}^{2}f \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{2}}}-4\,{\frac {Bc}{{a}^{2}f \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{2}}}-2\,{\frac {}{{a}^{2}f \left(\tan \left(1/2 \right. \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/f*c/a^2*B*arctan(tan(1/2*f*x+1/2*e))+4/f*c/a^2/(tan(1/2*f*x+1/2*e)+1)^2*
A-4/f*c/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B-2/f*c/a^2/(tan(1/2*f*x+1/2*e)+1)*A-2
/f*c/a^2/(tan(1/2*f*x+1/2*e)+1)*B-8/3/f*c/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A+8/
3/f*c/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B
```

Maxima [B] time = 1.48853, size = 610, normalized size = 8.47

$$-\frac{2\left(Bc\left(\frac{\frac{9\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+4}{a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}+\frac{3\,\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^{2}}\right)+\frac{Ac\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2\right)}{a^{2}+\frac{3\,a^{2}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{2}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{2}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}-\frac{}{a^{2}+\frac{3\,a^{2}\sin}{\cos(f}}\right.}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(
f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.6239, size = 401, normalized size = 5.57

$$\frac{6\,Bcfx - \left(3\,Bcfx + (A - 7\,B)c\right)\cos\left(fx + e\right)^2 + 2\,(A - B)c + \left(3\,Bcfx + (A + 5\,B)c\right)\cos\left(fx + e\right) + \left(6\,Bcfx - 2\,(A - B)c\right.}{3\left(a^2f\cos\left(fx + e\right)^2 - a^2f\cos\left(fx + e\right) - 2\,a^2f - \left(a^2f\cos\left(fx + e\right) + 2\,a^2f\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c + (
3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c*f
*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f
*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 16.2594, size = 711, normalized size = 9.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*A*c*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*A*c*tan(
e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a
**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f
*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**
2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*
tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*
a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/
2 + f*x/2) + 3*a**2*f) + 2*B*c*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/
2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) - 18*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 8*B*c/(3*a**2*f*tan
(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
 + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)**
```

```
2, True))
```

Giac [A] time = 1.19561, size = 124, normalized size = 1.72

$$-\frac{\frac{3\,(fx+e)Bc}{a^2}+\frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ac+5\,Bc\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/2
*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/2*
f*x + 1/2*e) + 1)^3))/f
```

3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=62

$$\frac{(2A+B)\tan(e+fx)}{3a^2cf}-\frac{(A-B)\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

```
[Out] -((A - B)*Sec[e + f*x])/(3*c*f*(a^2 + a^2*Sin[e + f*x])) + ((2*A + B)*Tan[e
 + f*x])/(3*a^2*c*f)
```

Rubi [A] time = 0.196517, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 3767, 8}

$$\frac{(2A+B)\tan(e+fx)}{3a^2cf}-\frac{(A-B)\sec(e+fx)}{3cf\left(a^2\sin(e+fx)+a^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

```
[Out] -((A - B)*Sec[e + f*x])/(3*c*f*(a^2 + a^2*Sin[e + f*x])) + ((2*A + B)*Tan[e
 + f*x])/(3*a^2*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx &= \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{ac}\\ &= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}+\frac{(2A+B)\int \sec^2(e+fx)\,dx}{3a^2c}\\ &= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}-\frac{(2A+B)\,\mathrm{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3a^2cf}\\ &= -\frac{(A-B)\sec(e+fx)}{3cf\left(a^2+a^2\sin(e+fx)\right)}+\frac{(2A+B)\tan(e+fx)}{3a^2cf}\end{aligned}$$

Mathematica [A] time = 0.480835, size = 110, normalized size = 1.77

$$\frac{\cos(e+fx)(-2(A-B)\cos(e+fx)+2(2A+B)\cos(2(e+fx))-8A\sin(e+fx)-A\sin(2(e+fx))-4B\sin(e+fx)+}{12a^2cf(\sin(e+fx)-1)(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)]
 - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e +
f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.072, size = 97, normalized size = 1.6

$$2\,\frac{1}{a^2cf}\left(-\frac{A/4+B/4}{\tan\left(1/2\,fx+e/2\right)-1}-1/2\,\frac{-A+B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-1/3\,\frac{A-B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^3}-\frac{3/4\,A-B/4}{\tan\left(1/2\,fx+e/2\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2/c*(-(1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-A+B)/(tan(1/2*f*x+1/
2*e)+1)^2-1/3*(A-B)/(tan(1/2*f*x+1/2*e)+1)^3-(3/4*A-1/4*B)/(tan(1/2*f*x+1/2
*e)+1))
```

Maxima [B] time = 1.00891, size = 358, normalized size = 5.77

$$\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(f*x +
 e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*
c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A] time = 1.56936, size = 171, normalized size = 2.76

$$-\frac{(2\,A+B)\cos\left(fx+e\right)^2-(2\,A+B)\sin\left(fx+e\right)-A-2\,B}{3\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*f
*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [A] time = 16.2528, size = 578, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*A
*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 +
 f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 + f*x/
2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**
2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/2)**4
+ 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*ta
n(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*B*tan(e/2
 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3
- 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*tan(e/2 + f*x
/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)),
 True))
```

Giac [A] time = 1.17437, size = 138, normalized size = 2.23

$$-\frac{\frac{3\,(A+B)}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A-B}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/2
*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)/(
a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=62

$$\frac{A\tan^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*Tan[e + f*x])/(a^2*c^2*f) + (A*Tan[e
+ f*x]^3)/(3*a^2*c^2*f)
```

Rubi [A] time = 0.139538, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2669, 3767}

$$\frac{A\tan^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x]
```


```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*Tan[e + f*x])/(a^2*c^2*f) + (A*Tan[e
+ f*x]^3)/(3*a^2*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\,dx}{a^2c^2}\\ &= \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\int \sec^4(e+fx)\,dx}{a^2c^2}\\ &= \frac{B\sec^3(e+fx)}{3a^2c^2f}-\frac{A\,\mathrm{Subst}\left(\int\left(1+x^2\right)dx,x,-\tan(e+fx)\right)}{a^2c^2f}\\ &= \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{A\tan^3(e+fx)}{3a^2c^2f}\end{aligned}$$

Mathematica [A] time = 0.118258, size = 53, normalized size = 0.85

$$\frac{A\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(a
^2*c^2*f)
```

Maple [B] time = 0.066, size = 145, normalized size = 2.3

$$2\,\frac{1}{fc^2a^2}\left(-1/3\,\frac{A/2+B/2}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^3}-1/2\,\frac{A/2+B/2}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-\frac{A/2+B/4}{\tan\left(1/2\,fx+e/2\right)-1}-1/2\,\frac{-A/2+B/2}{\left(\tan\left(1/2\,fx+e/2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/c^2/a^2*(-1/3*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/2*A+1/2*B)/
(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-1/2*A+1
/2*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^3-(
1/2*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [A] time = 0.987443, size = 63, normalized size = 1.02

$$\frac{\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)A}{a^2c^2}+\frac{B}{a^2c^2\cos\left(fx+e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x + e
)^3))/f
```

Fricas [A] time = 1.61172, size = 103, normalized size = 1.66

$$\frac{\left(2\,A\cos\left(fx+e\right)^2+A\right)\sin\left(fx+e\right)+B}{3\,a^2c^2f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] 1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [A] time = 17.455, size = 651, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*
a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**
2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**
2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c
**2*f) + B*tan(e/2 + f*x/2)**6/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*
c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**
2*f) - 9*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*
c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**
2*f) + 3*B*tan(e/2 + f*x/2)**2/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*
c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**
2*f) - 3*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f*x
/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne(f, 0)), (x*
(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

Giac [A] time = 1.19815, size = 117, normalized size = 1.89

$$-\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-2\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+B\right)}{3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}a^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/2
*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 -
1)^3*a^2*c^2*f)
```

3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

```
[Out] ((A + B)*Sec[e + f*x]^3)/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + ((4*A - B)*Ta
n[e + f*x])/(5*a^2*c^3*f) + ((4*A - B)*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```

Rubi [A] time = 0.220292, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 3767}

$$\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]
```


```
[Out] ((A + B)*Sec[e + f*x]^3)/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + ((4*A - B)*Ta
n[e + f*x])/(5*a^2*c^3*f) + ((4*A - B)*Tan[e + f*x]^3)/(15*a^2*c^3*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^2c^2}$$
$$= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{(4A-B)\int \sec^4(e+fx)\,dx}{5a^2c^3}$$
$$= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} - \frac{(4A-B)\,\mathrm{Subst}\left(\int\left(1+x^2\right)dx, x, -\tan(e+\ldots\right.}{5a^2c^3f}$$
$$= \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)} + \frac{(4A-B)\tan(e+fx)}{5a^2c^3f} + \frac{(4A-B)\tan^3(e+\ldots}{15a^2c^3f}$$

Mathematica [B] time = 0.960834, size = 237, normalized size = 2.55

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(54(A+B)\cos(e+fx)-32(4A-B)\cos(2(e+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*A*
Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*Cos
[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e + f*
x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)
] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1 + Sin[
e + f*x])^3*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.095, size = 183, normalized size = 2.

$$2\,\frac{1}{a^{2}fc^{3}}\left(-1/5\,\frac{A+B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{5}}-1/4\,\frac{2\,A+2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{4}}-1/2\,\frac{3/2\,A+B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{2}}-1/3\,\frac{5/2\,}{\left(\tan\left(1/2\,f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^2/c^3*(-1/5*(A+B)/(tan(1/2*f*x+1/2*e)-1)^5-1/4*(2*A+2*B)/(tan(1/2*f*x
+1/2*e)-1)^4-1/2*(3/2*A+B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(5/2*A+2*B)/(tan(1/
2*f*x+1/2*e)-1)^3-(11/16*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-1/4*A+1/4*B
)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)^3-(5/16
*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.07258, size = 879, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/(a^2*c^
3 - 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*
c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1) - 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)/(a^2*c^3 - 2*a^2*c^3*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
```

```
 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

Fricas [A] time = 1.62225, size = 262, normalized size = 2.82

$$-\frac{2\,(4\,A-B)\cos\left(fx+e\right)^{4}-(4\,A-B)\cos\left(fx+e\right)^{2}+\left(2\,(4\,A-B)\cos\left(fx+e\right)^{2}+4\,A-B\right)\sin\left(fx+e\right)-A+4\,B}{15\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-a^{2}c^{3}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B)
*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e)^
3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23233, size = 317, normalized size = 3.41

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A-7\,B\right)}{a^{2}c^{3}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+45\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-480\,A}{}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 - 9*B*tan(1/2*f*x + 1/2*e)^2 + 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A - 7*B)/(a^2*c^3*(ta
n(1/2*f*x + 1/2*e) + 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 + 45*B*tan(1/2*f
*x + 1/2*e)^4 - 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 + 70*B*tan(1/2*f*x + 1/2*e)^2 - 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A + 13*B)/(a^2*c^3*(tan(1/
2*f*x + 1/2*e) - 1)^5))/f
```

3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=135

$$\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

```
[Out] ((A + B)*Sec[e + f*x]^3)/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + ((5*A - 2*B
)*Sec[e + f*x]^3)/(35*a^2*f*(c^4 - c^4*Sin[e + f*x])) + (4*(5*A - 2*B)*Tan[
e + f*x])/(35*a^2*c^4*f) + (4*(5*A - 2*B)*Tan[e + f*x]^3)/(105*a^2*c^4*f)
```

Rubi [A] time = 0.270265, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)}+\frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^3)/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + ((5*A - 2*B
)*Sec[e + f*x]^3)/(35*a^2*f*(c^4 - c^4*Sin[e + f*x])) + (4*(5*A - 2*B)*Tan[
e + f*x])/(35*a^2*c^4*f) + (4*(5*A - 2*B)*Tan[e + f*x]^3)/(105*a^2*c^4*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
```

```
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}\,dx}{7a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(4(5A-2B}{}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} - \frac{(4(5A-2B}{}$$

$$= \frac{(A+B)\sec^3(e+fx)}{7a^2f\left(c^2-c^2\sin(e+fx)\right)^2} + \frac{(5A-2B)\sec^3(e+fx)}{35a^2f\left(c^4-c^4\sin(e+fx)\right)} + \frac{4(5A-2B)}{35a}$$

Mathematica [B] time = 0.924016, size = 285, normalized size = 2.11

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(42(25A+4B)\cos(e+fx)-512(5A-2B)\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[2*(e + f*x
)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos[4*(e + f*x
)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f*x)]
 - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e + f*x)] - 96*
B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e + f*x)] - 300*
A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e + f*x)] - 128*B
*Sin[5*(e + f*x)]))/(13440*a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x
])^2)
```

Maple [A] time = 0.116, size = 233, normalized size = 1.7

$$2\,\frac{1}{a^{2}fc^{4}}\left(-1/7\,\frac{2\,A+2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{7}}-1/6\,\frac{6\,A+6\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{6}}-1/4\,\frac{10\,A+8\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{4}}-1/5\,\frac{10\,A}{\left(\tan\left(1/2\,fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a^2/c^4*(-1/7*(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(6*A+6*B)/(tan(1/2
*f*x+1/2*e)-1)^6-1/4*(10*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(10*A+9*B)/(ta
n(1/2*f*x+1/2*e)-1)^5-(13/16*A+1/8*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(23/8*A+11
/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(55/8*A+35/8*B)/(tan(1/2*f*x+1/2*e)-1)^3
-1/2*(-1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/8*A-1/8*B)/(tan(1/2*f*x
+1/2*e)+1)^3-(3/16*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.09332, size = 1127, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(36*sin(f*x + e)/(cos(f*x + e) + 1) - 132*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 68*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 84*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 140*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 140*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 105
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)
/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2
*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x
 + e)^10/(cos(f*x + e) + 1)^10) + 5*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
24*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 76*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 28*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 42*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 56*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 28*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 42*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 21*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9 - 6)/(a^2*c^4 - 4*a^2*c^4*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^2*c^4*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8*a^2*c^4*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c^4*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10))/f
```

Fricas [A] time = 1.65329, size = 371, normalized size = 2.75

$$-\frac{16\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-8\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(8\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-12\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}}{105\left(a^{2}c^{4}f\cos\left(fx+e\right)^{5}+2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\,a^{2}c^{4}f\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*(
5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*si
n(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*x +
 e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23712, size = 398, normalized size = 2.95

$$-\frac{\frac{35\left(9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-6\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,A-5\,B\right)}{a^{2}c^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{1365\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-5775\,A}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/840*(35*(9*A*tan(1/2*f*x + 1/2*e)^2 - 6*B*tan(1/2*f*x + 1/2*e)^2 + 15*A*
tan(1/2*f*x + 1/2*e) - 9*B*tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(tan(
1/2*f*x + 1/2*e) + 1)^3) + (1365*A*tan(1/2*f*x + 1/2*e)^6 + 210*B*tan(1/2*f
*x + 1/2*e)^6 - 5775*A*tan(1/2*f*x + 1/2*e)^5 - 105*B*tan(1/2*f*x + 1/2*e)^
5 + 12250*A*tan(1/2*f*x + 1/2*e)^4 - 175*B*tan(1/2*f*x + 1/2*e)^4 - 14350*A
*tan(1/2*f*x + 1/2*e)^3 + 910*B*tan(1/2*f*x + 1/2*e)^3 + 10185*A*tan(1/2*f*
x + 1/2*e)^2 - 756*B*tan(1/2*f*x + 1/2*e)^2 - 3955*A*tan(1/2*f*x + 1/2*e) +
 427*B*tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(tan(1/2*f*x + 1/2*e)
- 1)^7))/f
```

3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=175

$$\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(A}{9a^2c^2}$$

```
[Out] ((A + B)*Sec[e + f*x]^3)/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*
Sec[e + f*x]^3)/(21*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + ((2*A - B)*Sec[e +
f*x]^3)/(21*a^2*f*(c^5 - c^5*Sin[e + f*x])) + (4*(2*A - B)*Tan[e + f*x])/(2
1*a^2*c^5*f) + (4*(2*A - B)*Tan[e + f*x]^3)/(63*a^2*c^5*f)
```

Rubi [A] time = 0.324576, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}+\frac{4(2A-B)\tan(e+fx)}{21a^2c^5f}+\frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2}+\frac{(A}{9a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
```


```
[Out] ((A + B)*Sec[e + f*x]^3)/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*
Sec[e + f*x]^3)/(21*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + ((2*A - B)*Sec[e +
f*x]^3)/(21*a^2*f*(c^5 - c^5*Sin[e + f*x])) + (4*(2*A - B)*Tan[e + f*x])/(2
1*a^2*c^5*f) + (4*(2*A - B)*Tan[e + f*x]^3)/(63*a^2*c^5*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^2c^2}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{3a^2c^3}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(5(2A-B))\int}{21}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\,\mathrm{s}}{21a^2f\left(c^5-c\right.}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\,\mathrm{s}}{21a^2f\left(c^5-c\right.}$$

$$= \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\,\mathrm{s}}{21a^2f\left(c^5-c\right.}$$

Mathematica [A] time = 1.10471, size = 329, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(180(31A-5B)\cos(e+fx)-6912(2A-B)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f*x
)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e + f*x
)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*
x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e + f*x
] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e + f*x)]
- 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e + f*x)
] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5*(e + f*
x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*c^5*f*(-1
 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.118, size = 277, normalized size = 1.6

$$2\,\frac{1}{a^2fc^5}\left(-1/9\,\frac{4\,A+4\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^9}-1/8\,\frac{16\,A+16\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^8}-1/7\,\frac{34\,A+32\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^7}-1/6\,\frac{46\,A}{\left(\tan\left(1/2\,f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f/a^2/c^5*(-1/9*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(16*A+16*B)/(tan(1
/2*f*x+1/2*e)-1)^8-1/7*(34*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(46*A+40*B)
/(tan(1/2*f*x+1/2*e)-1)^6-1/2*(9/2*A+13/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(57/6
4*A+5/64*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(59/2*A+39/2*B)/(tan(1/2*f*x+1/2*e)-
1)^4-1/3*(57/4*A+59/8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/5*(175/4*A+135/4*B)/(ta
n(1/2*f*x+1/2*e)-1)^5-1/2*(-1/16*A+1/16*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/
16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)^3-(7/64*A-5/64*B)/(tan(1/2*f*x+1/2*e)+1
))
```

Maxima [B] time = 1.14971, size = 1347, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1
)^9 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 - 19)/(a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
+ 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 36
*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 2*a^2
*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^2*c^
5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) + B*(6*sin(f*x + e)/(cos(f*x + e)
+ 1) - 75*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 128*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 - 162*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 36*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5 + 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 189*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8 + 126*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 63
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1)/(a^2*c^5 - 6*a^2*c^5*sin(f*x +
e)/(cos(f*x + e) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*
a^2*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 36*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2
*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 2*a^2*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5
*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x
 + e) + 1)^11 - a^2*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12))/f
```

Fricas [A] time = 1.77727, size = 439, normalized size = 2.51

$$\frac{8\,(2\,A-B)\cos\left(fx+e\right)^{6}-36\,(2\,A-B)\cos\left(fx+e\right)^{4}+15\,(2\,A-B)\cos\left(fx+e\right)^{2}+\left(24\,(2\,A-B)\cos\left(fx+e\right)^{4}-20\,(2\,A\right.}{63\left(3\,a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}-\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] 1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A -
B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x + e
)^2 - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a^2*c^5*f*cos(f*x + e)^5 -
4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^5*f*cos(f*
x + e)^3)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27133, size = 479, normalized size = 2.74

$$-\frac{\frac{21\left(21\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+36\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-24\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,A-13\,B\right)}{a^{2}c^{5}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}+\frac{3591\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+315\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 36
*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c^5
*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*tan
(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*x +
1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e)^6 -
 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 107730*A
*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*tan(1/2*
f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f*x + 1/2
```

```
*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*e) + 2196
*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x + 1/2*e) -
1)^9))/f
```

3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=243

$$-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f}-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}+\frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6}-\frac{6a^5c^5(3A-8B)\cos^7(e+fx)}{5f\left(a^2\sin(e+fx)+a^2\right)^4}$$

```
[Out] (-21*(3*A - 8*B)*c^5*x)/(2*a^3) - (7*(3*A - 8*B)*c^5*Cos[e + f*x]^3)/(a^3*f
) - (21*(3*A - 8*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(2*a^3*f) - (a^5*(A - B)
*c^5*Cos[e + f*x]^11)/(5*f*(a + a*Sin[e + f*x])^8) + (2*a^3*(3*A - 8*B)*c^5
*Cos[e + f*x]^9)/(15*f*(a + a*Sin[e + f*x])^6) - (6*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^7)/(5*f*(a^2 + a^2*Sin[e + f*x])^4) - (42*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^5)/(5*f*(a^4 + a^4*Sin[e + f*x])^2)
```

Rubi [A] time = 0.411668, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f}-\frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}+\frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6}-\frac{6a^5c^5(3A-8B)\cos^7(e+fx)}{5f\left(a^2\sin(e+fx)+a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (-21*(3*A - 8*B)*c^5*x)/(2*a^3) - (7*(3*A - 8*B)*c^5*Cos[e + f*x]^3)/(a^3*f
) - (21*(3*A - 8*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(2*a^3*f) - (a^5*(A - B)
*c^5*Cos[e + f*x]^11)/(5*f*(a + a*Sin[e + f*x])^8) + (2*a^3*(3*A - 8*B)*c^5
*Cos[e + f*x]^9)/(15*f*(a + a*Sin[e + f*x])^6) - (6*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^7)/(5*f*(a^2 + a^2*Sin[e + f*x])^4) - (42*a^5*(3*A - 8*B)*c^5*Cos
[e + f*x]^5)/(5*f*(a^4 + a^4*Sin[e + f*x])^2)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \left(a^5c^5\right)\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^8}\,dx$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} - \frac{1}{5}\left(a^4(3A-8B)c^5\right)\int \frac{\cos^{10}(e+fx)}{(a+a\sin(e+fx))}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} + \frac{1}{5}\left(3a^2(3\right.$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} - \frac{6a(3A-}{5f(a-}$$

$$= -\frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} - \frac{6a(3A-}{5f(a-}$$

$$= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8}{15f(a+}$$

$$= -\frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f} -$$

$$= -\frac{21(3A-8B)c^5x}{2a^3} - \frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} - \frac{21(3A-8B)c^5\cos(e}{2a^3f}$$

Mathematica [A] time = 2.62741, size = 388, normalized size = 1.6

$$\frac{(c-c\sin(e+fx))^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(768(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-630(3A-8B)(e+fx)\left(\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)*
Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64
*(21*A - 31*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 128*(54*A - 119*B)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 630*(3*A - 8*B)
*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*(32*A - 127*B)*Cos[
e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 5*B*Cos[3*(e + f*x)]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(A - 8*B)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e +
```

```
f*x)/2])^10*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.184, size = 649, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -1/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A+8/f*c^5/a^3/
(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B-16/f*c^5/a^3/(1+tan(1/2*f
*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^4*A+62/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)
^3*tan(1/2*f*x+1/2*e)^4*B-32/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f
*x+1/2*e)^2*A+128/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^2
*B+1/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A-8/f*c^5/a^3/
(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B-16/f*c^5/a^3/(1+tan(1/2*f*x
+1/2*e)^2)^3*A+190/3/f*c^5/a^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B-63/f*c^5/a^3*ar
ctan(tan(1/2*f*x+1/2*e))*A+168/f*c^5/a^3*arctan(tan(1/2*f*x+1/2*e))*B+128/f
*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^4*A-128/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^4*
B-32/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^2*A+96/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+
1)^2*B-64/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)*A+160/f*c^5/a^3/(tan(1/2*f*x+1/2
*e)+1)*B-64/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^3*A+64/3/f*c^5/a^3/(tan(1/2*f*
x+1/2*e)+1)^3*B-256/5/f*c^5/a^3/(tan(1/2*f*x+1/2*e)+1)^5*A+256/5/f*c^5/a^3/
(tan(1/2*f*x+1/2*e)+1)^5*B
```

Maxima [B] time = 1.80959, size = 4431, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^5*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(f*x
```

+ e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 544)/(
a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*a^3*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
+ a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a^3) - A*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(
a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 20*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3
) + 5*B*c^5*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5
*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 30*A*c^5*((10
5*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*si
n(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
 1))/a^3) + 60*B*c^5*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(
cos(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)

```
^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) - 20*A*c^5*((95*sin(f*x + e)/(cos(f*x +
e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) + 20*B*c^5*((95*sin(f*x + e)/(cos(f*x +
e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^5*(20*sin(f*x + e)/(cos(f*x + e)
 + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 40*A*c^5*(5*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a
^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 20*
B*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5
*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5) + 30*A*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a
^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^5*(5*
sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A] time = 1.89307, size = 1080, normalized size = 4.44

$$-\frac{10\,Bc^{5}\cos\left(fx+e\right)^{6}+15\,(A-6\,B)c^{5}\cos\left(fx+e\right)^{5}+10\,(21\,A-74\,B)c^{5}\cos\left(fx+e\right)^{4}-1260\,(3\,A-8\,B)c^{5}fx-192\,(A\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/30*(10*B*c^5*cos(f*x + e)^6 + 15*(A - 6*B)*c^5*cos(f*x + e)^5 + 10*(21*A
 - 74*B)*c^5*cos(f*x + e)^4 - 1260*(3*A - 8*B)*c^5*f*x - 192*(A - B)*c^5 +
(315*(3*A - 8*B)*c^5*f*x + (2373*A - 6128*B)*c^5)*cos(f*x + e)^3 + (945*(3*
A - 8*B)*c^5*f*x - 2*(753*A - 2248*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8
*B)*c^5*f*x + 2*(323*A - 848*B)*c^5)*cos(f*x + e) + (10*B*c^5*cos(f*x + e)^
5 - 5*(3*A - 20*B)*c^5*cos(f*x + e)^4 + 5*(39*A - 128*B)*c^5*cos(f*x + e)^3
 - 1260*(3*A - 8*B)*c^5*f*x + 192*(A - B)*c^5 + (315*(3*A - 8*B)*c^5*f*x -
2*(1089*A - 2744*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8*B)*c^5*f*x + 2*(3
07*A - 832*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^
3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2
 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28761, size = 508, normalized size = 2.09

$$-\frac{\frac{315\left(3\,Ac^{5}-8\,Bc^{5}\right)\left(fx+e\right)}{a^{3}}+\frac{10\left(3\,Ac^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-24\,Bc^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+48\,Ac^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-186\,Bc^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+96\,Ac^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{\cdots}\cdots}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)^{3}a^{3}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(315*(3*A*c^5 - 8*B*c^5)*(f*x + e)/a^3 + 10*(3*A*c^5*tan(1/2*f*x + 1/
2*e)^5 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^5 + 48*A*c^5*tan(1/2*f*x + 1/2*e)^4
- 186*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 96*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 384*
B*c^5*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^5*tan(1/2*f*x + 1/2*e) + 24*B*c^5*tan(
1/2*f*x + 1/2*e) + 48*A*c^5 - 190*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a^
3) + 64*(30*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 75*B*c^5*tan(1/2*f*x + 1/2*e)^4
+ 135*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 345*B*c^5*tan(1/2*f*x + 1/2*e)^3 + 255
*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 595*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 165*A*c^
5*tan(1/2*f*x + 1/2*e) - 395*B*c^5*tan(1/2*f*x + 1/2*e) + 39*A*c^5 - 94*B*c
^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=201

$$-\frac{7c^4(2A-7B)\cos(e+fx)}{2a^3f}-\frac{a^4c^4(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}+\frac{2a^2c^4(2A-7B)\cos^7(e+fx)}{15f(a\sin(e+fx)+a)^5}-\frac{7c^4(2A-7B)\cos^3(e+fx)}{6f\left(a^3\sin(e+fx)+a^3\right)}-$$

```
[Out] (-7*(2*A - 7*B)*c^4*x)/(2*a^3) - (7*(2*A - 7*B)*c^4*Cos[e + f*x])/(2*a^3*f)
 - (a^4*(A - B)*c^4*Cos[e + f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (2*a^2*(
2*A - 7*B)*c^4*Cos[e + f*x]^7)/(15*f*(a + a*Sin[e + f*x])^5) - (14*(2*A - 7
*B)*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^3) - (7*(2*A - 7*B)*c^4*
Cos[e + f*x]^3)/(6*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.391648, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$-\frac{7c^4(2A-7B)\cos(e+fx)}{2a^3f}-\frac{a^4c^4(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}+\frac{2a^2c^4(2A-7B)\cos^7(e+fx)}{15f(a\sin(e+fx)+a)^5}-\frac{7c^4(2A-7B)\cos^3(e+fx)}{6f\left(a^3\sin(e+fx)+a^3\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (-7*(2*A - 7*B)*c^4*x)/(2*a^3) - (7*(2*A - 7*B)*c^4*Cos[e + f*x])/(2*a^3*f)
 - (a^4*(A - B)*c^4*Cos[e + f*x]^9)/(5*f*(a + a*Sin[e + f*x])^7) + (2*a^2*(
2*A - 7*B)*c^4*Cos[e + f*x]^7)/(15*f*(a + a*Sin[e + f*x])^5) - (14*(2*A - 7
*B)*c^4*Cos[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^3) - (7*(2*A - 7*B)*c^4*
Cos[e + f*x]^3)/(6*f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = \left(a^4c^4\right)\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^7}\,dx$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} - \frac{1}{5}\left(a^3(2A-7B)c^4\right)\int \frac{\cos^8(e+fx)}{(a+a\sin(e+fx))^6}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} + \frac{1}{15}\left(7a(2\right.$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14(2A-}{15f(a+}$$

$$= -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} - \frac{14(2A-}{15f(a+}$$

$$= -\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7} + \frac{2a^2(2A-7B}{15f(a+a}$$

$$= -\frac{7(2A-7B)c^4x}{2a^3} - \frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} - \frac{a^4(A-B)c^4\cos^9(e+f}{5f(a+a\sin(e+fx))^7}$$

Mathematica [A] time = 1.53674, size = 348, normalized size = 1.73

$$\frac{(c-c\sin(e+fx))^4\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(384(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-210(2A-7B)(e+fx)\left(\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)*
Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*
(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64*
(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)*Sin
[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 210*(2*A - 7*B)*(e
+ f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 60*(A - 7*B)*Cos[e + f*x]*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^8*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.155, size = 474, normalized size = 2.4

$$\frac{Bc^4}{fa^3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2}-2\,\frac{c^4\left(\tan\left(1/2\,fx+e/2\right)\right)^2A}{fa^3\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}+14\,\frac{c^4\left(\tan\left(1/2\,fx+e/2\right)\right)^2B}{fa^3\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 1/f*c^4/a^3/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2/f*c^4/a^3/(
1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+14/f*c^4/a^3/(1+tan(1/2*f*
x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B-1/f*c^4/a^3/(1+tan(1/2*f*x+1/2*e)^2)^2
*B*tan(1/2*f*x+1/2*e)-2/f*c^4/a^3/(1+tan(1/2*f*x+1/2*e)^2)^2*A+14/f*c^4/a^3
/(1+tan(1/2*f*x+1/2*e)^2)^2*B+49/f*c^4/a^3*arctan(tan(1/2*f*x+1/2*e))*B-14/
f*c^4/a^3*arctan(tan(1/2*f*x+1/2*e))*A+64/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)^
4*A-64/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B-16/f*c^4/a^3/(tan(1/2*f*x+1/2*e
)+1)*A+48/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)*B-128/5/f*c^4/a^3/(tan(1/2*f*x+1
/2*e)+1)^5*A+128/5/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B-128/3/f*c^4/a^3/(ta
n(1/2*f*x+1/2*e)+1)^3*A+64/3/f*c^4/a^3/(tan(1/2*f*x+1/2*e)+1)^3*B+32/f*c^4/
a^3*B/(tan(1/2*f*x+1/2*e)+1)^2
```

Maxima [B] time = 1.71925, size = 3232, normalized size = 16.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*(B*c^4*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 5
```

```
*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*A*c^4*((105
*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 11*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^3*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x + e)/(cos(f*x + e) +
1))/a^3) + 24*B*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(c
os(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(
f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) - 8*A*c^4*((95*sin(f*x + e)/(cos(f*x + e)
 + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) + 12*B*c^4*((95*sin(f*x + e)/(cos(f*x + e)
 + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/a^3) - 2*A*c^4*(20*sin(f*x + e)/(cos(f*x + e) +
 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 24*A*c^4*(5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 16*B*
c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) + 24*A*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(co
```

```
s(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^4*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.83411, size = 961, normalized size = 4.78

$$\frac{15\,Bc^4\cos\left(fx+e\right)^5-30\,(A-6\,B)c^4\cos\left(fx+e\right)^4+420\,(2\,A-7\,B)c^4fx+96\,(A-B)c^4-\left(105\,(2\,A-7\,B)c^4fx+(554\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/30*(15*B*c^4*cos(f*x + e)^5 - 30*(A - 6*B)*c^4*cos(f*x + e)^4 + 420*(2*A
- 7*B)*c^4*f*x + 96*(A - B)*c^4 - (105*(2*A - 7*B)*c^4*f*x + (554*A - 1819*
B)*c^4)*cos(f*x + e)^3 - (315*(2*A - 7*B)*c^4*f*x - 2*(134*A - 619*B)*c^4)*
cos(f*x + e)^2 + 6*(35*(2*A - 7*B)*c^4*f*x + 2*(74*A - 249*B)*c^4)*cos(f*x
+ e) - (15*B*c^4*cos(f*x + e)^4 + 15*(2*A - 11*B)*c^4*cos(f*x + e)^3 - 420*
(2*A - 7*B)*c^4*f*x + 96*(A - B)*c^4 + (105*(2*A - 7*B)*c^4*f*x - 2*(262*A
- 827*B)*c^4)*cos(f*x + e)^2 - 6*(35*(2*A - 7*B)*c^4*f*x + 2*(66*A - 241*B)
*c^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
 e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*co
s(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23538, size = 412, normalized size = 2.05

$$-\frac{\frac{105\left(2\,Ac^4-7\,Bc^4\right)\left(fx+e\right)}{a^3}-\frac{30\left(Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^4+14\,Bc^4\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}+\frac{32\left(15\,Ac^4\,\mathrm{ta}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/30*(105*(2*A*c^4 - 7*B*c^4)*(f*x + e)/a^3 - 30*(B*c^4*tan(1/2*f*x + 1/2*
e)^3 - 2*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 14*B*c^4*tan(1/2*f*x + 1/2*e)^2 - B
*c^4*tan(1/2*f*x + 1/2*e) - 2*A*c^4 + 14*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 +
1)^2*a^3) + 32*(15*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^4*tan(1/2*f*x + 1/
2*e)^4 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^3
 + 130*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 380*B*c^4*tan(1/2*f*x + 1/2*e)^2 + 80
*A*c^4*tan(1/2*f*x + 1/2*e) - 250*B*c^4*tan(1/2*f*x + 1/2*e) + 19*A*c^4 - 5
9*B*c^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{c^3(A-6B)\cos(e+fx)}{a^3f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{2a^3c^3(A-6B)\cos^3(e+fx)}{3f\left(a^3\sin(e+fx)+a^3\right)^2}-\frac{c^3x(A-6B)}{a^3}+\frac{2ac^3(A-6B)\cos^5(}{15f(a\sin(e+fx)}$$

```
[Out] -(((A - 6*B)*c^3*x)/a^3) - ((A - 6*B)*c^3*Cos[e + f*x])/(a^3*f) - (a^3*(A -
 B)*c^3*Cos[e + f*x]^7)/(5*f*(a + a*Sin[e + f*x])^6) + (2*a*(A - 6*B)*c^3*C
os[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^4) - (2*a^3*(A - 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(a^3 + a^3*Sin[e + f*x])^2)
```

Rubi [A] time = 0.331005, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2680, 2682, 8}

$$-\frac{c^3(A-6B)\cos(e+fx)}{a^3f}-\frac{a^3c^3(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}-\frac{2a^3c^3(A-6B)\cos^3(e+fx)}{3f\left(a^3\sin(e+fx)+a^3\right)^2}-\frac{c^3x(A-6B)}{a^3}+\frac{2ac^3(A-6B)\cos^5(}{15f(a\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(((A - 6*B)*c^3*x)/a^3) - ((A - 6*B)*c^3*Cos[e + f*x])/(a^3*f) - (a^3*(A -
 B)*c^3*Cos[e + f*x]^7)/(5*f*(a + a*Sin[e + f*x])^6) + (2*a*(A - 6*B)*c^3*C
os[e + f*x]^5)/(15*f*(a + a*Sin[e + f*x])^4) - (2*a^3*(A - 6*B)*c^3*Cos[e +
 f*x]^3)/(3*f*(a^3 + a^3*Sin[e + f*x])^2)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^6}\,dx$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} - \frac{1}{5}\left(a^2(A-6B)c^3\right)\int \frac{\cos^6(e+fx)}{(a+a\sin(e+fx))^5}\,d$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} + \frac{1}{3}\left((A-6B)\right.$$

$$= -\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4} - \frac{2(A-6B)c^3}{3af(a+a\,\mathrm{s}}$$

$$= -\frac{(A-6B)c^3\cos(e+fx)}{a^3f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} + \frac{2a(A-6B)c^3\,\mathrm{c}}{15f(a+a\sin}$$

$$= -\frac{(A-6B)c^3x}{a^3} - \frac{(A-6B)c^3\cos(e+fx)}{a^3f} - \frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6} +$$

Mathematica [B] time = 1.04783, size = 308, normalized size = 2.01

$$\frac{(c-c\sin(e+fx))^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-15(A-6B)(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*x)
/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5)*(c - c*Sin[e + f*x])^3)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
6*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.141, size = 323, normalized size = 2.1

$$2\,\frac{Bc^3}{fa^3\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}-2\,\frac{c^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)A}{fa^3}+12\,\frac{c^3\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)B}{fa^3}+32\,\frac{}{fa^3\left(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c^3/a^3*B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*c^3/a^3*arctan(tan(1/2*f*x+1/2*e
))*A+12/f*c^3/a^3*arctan(tan(1/2*f*x+1/2*e))*B+32/f*c^3/a^3/(tan(1/2*f*x+1/
2*e)+1)^4*A-32/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B+8/f*c^3/a^3/(tan(1/2*f*
x+1/2*e)+1)^2*A+8/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)^2*B-4/f*c^3/a^3/(tan(1/2
*f*x+1/2*e)+1)*A+12/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)*B-64/5/f*c^3/a^3/(tan(
1/2*f*x+1/2*e)+1)^5*A+64/5/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B-80/3/f*c^3/
a^3/(tan(1/2*f*x+1/2*e)+1)^3*A+16/f*c^3/a^3/(tan(1/2*f*x+1/2*e)+1)^3*B
```

Maxima [B] time = 1.6299, size = 2267, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(3*B*c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
 + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11
*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^3) - A*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) +
145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
 e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a^3) + 3*B*c^3*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 1
45*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x +
e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/a^3) - A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*A*c^3*(5*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^3*(5*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^
3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*
c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1
```

```
)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^3*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.73465, size = 821, normalized size = 5.37

$$\frac{15\,Bc^3\cos\left(fx+e\right)^4+60\,(A-6\,B)c^3fx+24\,(A-B)c^3-\left(15\,(A-6\,B)c^3fx+(46\,A-231\,B)c^3\right)\cos\left(fx+e\right)^3-\left(45\,(A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*(15*B*c^3*cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (15
*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*cos(f*x + e)^3 - (45*(A - 6*B)*c^3
*f*x - 2*(A - 66*B)*c^3)*cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*A -
 31*B)*c^3)*cos(f*x + e) + (15*B*c^3*cos(f*x + e)^3 + 60*(A - 6*B)*c^3*f*x
- 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*cos(f*x +
e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*cos(f*x + e))*sin(f*x +
 e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e)
- 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x
 + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28588, size = 305, normalized size = 1.99

$$\frac{\frac{30\,Bc^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3}-\frac{15\left(Ac^3-6\,Bc^3\right)\left(fx+e\right)}{a^3}-\frac{4\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+}{a^3\left(\mathrm{ta}\right.}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(30*B*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(f
*x + e)/a^3 - 4*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*tan(1/2*f*x + 1
/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*tan(1/2*f*x + 1/2*e)^
3 + 100*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 5
0*A*c^3*tan(1/2*f*x + 1/2*e) - 270*B*c^3*tan(1/2*f*x + 1/2*e) + 13*A*c^3 -
63*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=110

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}+\frac{2Bc^2\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bc^2x}{a^3}-\frac{2Bc^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

```
[Out] (B*c^2*x)/a^3 - (a^2*(A - B)*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^
5) - (2*B*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*B*c^2*Cos[e
 + f*x])/(f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.264867, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2680, 8}

$$-\frac{a^2c^2(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}+\frac{2Bc^2\cos(e+fx)}{f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bc^2x}{a^3}-\frac{2Bc^2\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*c^2*x)/a^3 - (a^2*(A - B)*c^2*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^
5) - (2*B*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^3) + (2*B*c^2*Cos[e
 + f*x])/(f*(a^3 + a^3*Sin[e + f*x]))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
```

```
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^5}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} + \left(aBc^2\right)\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^4}\,dx$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} - \frac{\left(Bc^2\right)\int \frac{\cos^2(e+}{(a+a\sin(e}}{a}$$

$$= -\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2Bc^2\cos(e+}{f\left(a^3+a^3\sin(e\right.}$$

$$= \frac{Bc^2x}{a^3} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3} + \frac{2Bc^2 c}{f\left(a^3+a^3\right.}$$

Mathematica [B] time = 0.684357, size = 272, normalized size = 2.47

$$\frac{(c-c\sin(e+fx))^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+2(3A-43B)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(A
 - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^5)*(c - c*Sin[e + f*x])^2)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.123, size = 249, normalized size = 2.3

$$2\,{\frac{B{c}^{2}\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) }{f{a}^{3}}}+16\,{\frac{A{c}^{2}}{f{a}^{3} \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{4}}}-16\,{\frac{B{c}^{2}}{f{a}^{3} \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{4}}}-2\,{\frac{}{f{a}^{3} \left(\tan \left(1/2 \right. \right. }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c^2/a^3*B*arctan(tan(1/2*f*x+1/2*e))+16/f*c^2/a^3/(tan(1/2*f*x+1/2*e)+1
)^4*A-16/f*c^2/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B-2/f*c^2/a^3/(tan(1/2*f*x+1/2*
e)+1)*A+2/f*c^2/a^3/(tan(1/2*f*x+1/2*e)+1)*B-32/5/f*c^2/a^3/(tan(1/2*f*x+1/
2*e)+1)^5*A+32/5/f*c^2/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B-16/f*c^2/a^3/(tan(1/2
*f*x+1/2*e)+1)^3*A+32/3/f*c^2/a^3/(tan(1/2*f*x+1/2*e)+1)^3*B+8/f*c^2/a^3*A/
(tan(1/2*f*x+1/2*e)+1)^2
```

Maxima [B] time = 1.57486, size = 1531, normalized size = 13.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*c^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
```

```
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 2*A*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

Fricas [B] time = 1.74236, size = 656, normalized size = 5.96

$$-\frac{60\,Bc^2fx - \left(15\,Bc^2fx - (3\,A - 43\,B)c^2\right)\cos\left(fx+e\right)^3 - 12\,(A-B)c^2 - \left(45\,Bc^2fx - (9\,A+11\,B)c^2\right)\cos\left(fx+e\right)^2 + 6\,(}{15\left(a^3f\cos\left(fx+e\right)^3 + 3\,a^3f\cos\left(fx+e\right)^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 12
*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^
```

```
2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 - (15
*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9*B)*
c^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x +
e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos
(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21275, size = 215, normalized size = 1.95

$$\frac{\frac{15\left(fx+e\right)Bc^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 60\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 170\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 100\,Bc^2\tan\left(\frac{1}{2} \ldots\right.}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*c^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*c^
2*tan(1/2*f*x + 1/2*e)^4 - 60*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c^2*tan(1
/2*f*x + 1/2*e)^2 - 170*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*c^2*tan(1/2*f*
x + 1/2*e) + 3*A*c^2 - 23*B*c^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{c(A+4B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{ac(A-11B)\cos(e+fx)}{15f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{2c(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] (-2*(A - B)*c*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) + (a*(A - 11*B)*c*
Cos[e + f*x])/(15*f*(a^2 + a^2*Sin[e + f*x])^2) + ((A + 4*B)*c*Cos[e + f*x]
)/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.225012, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2967, 2857, 2750, 2648}

$$\frac{c(A+4B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{ac(A-11B)\cos(e+fx)}{15f\left(a^2\sin(e+fx)+a^2\right)^2}-\frac{2c(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (-2*(A - B)*c*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) + (a*(A - 11*B)*c*
Cos[e + f*x])/(15*f*(a^2 + a^2*Sin[e + f*x])^2) + ((A + 4*B)*c*Cos[e + f*x]
)/(15*f*(a^3 + a^3*Sin[e + f*x]))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
```

```
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^4}\,dx$$
$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{c\int \frac{aA-6aB+5aB\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$
$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{((A+4B)c)\int \frac{}{a+a\,\mathrm{si}}}{15a^2}$$
$$= -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3} + \frac{(A-11B)c\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \frac{(A+4B)c\cos(e+}{15f\left(a^3+a^3\sin(e\right.}$$

Mathematica [A] time = 0.772458, size = 139, normalized size = 1.35

$$\frac{c\left(-15(A+B)\cos\left(e+\frac{fx}{2}\right)+5(A+B)\cos\left(e+\frac{3fx}{2}\right)+A\sin\left(2e+\frac{5fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)-15B\sin\left(2e+\frac{3fx}{2}\right)+4B\sin\left(2e\right.\right.}{30a^3f\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(
f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*
x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.11, size = 115, normalized size = 1.1

$$2\,{\frac {c}{f{a}^{3}}\left(-1/4\,{\frac {-16\,A+16\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{4}}}-1/5\,{\frac {8\,A-8\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) +1 \right) ^{5}}}-{\frac {A}{\tan \left(1/2\,fx+e/2 \right) +1}}-1/3\,{\frac {14\,A-10\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) \right.}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f*c/a^3*(-1/4*(-16*A+16*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(8*A-8*B)/(tan(1/
2*f*x+1/2*e)+1)^5-A/(tan(1/2*f*x+1/2*e)+1)-1/3*(14*A-10*B)/(tan(1/2*f*x+1/2
*e)+1)^3-1/2*(-6*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] time = 1.04638, size = 990, normalized size = 9.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) - 2*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
```

```
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A] time = 1.61019, size = 464, normalized size = 4.5

$$\frac{(A+4B)c\cos\left(fx+e\right)^3-(2A-7B)c\cos\left(fx+e\right)^2+3(A-B)c\cos\left(fx+e\right)+6(A-B)c-\left((A+4B)c\cos\left(fx+e\right)^2\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3a^3f\cos\left(fx+e\right)^2-2a^3f\cos\left(fx+e\right)-4a^3f+\left(a^3f\cos\left(fx+e\right)^2-2a^3f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] 1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B)
*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c*c
os(f*x + e) + 6*(A - B)*c)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*co
s(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a
^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 34.1097, size = 1035, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
```

```
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 10*B*c*tan(e
/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)*
*4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f*x/2)/(15*a**3*f*ta
n(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*
x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*
a**3*f) - 2*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(
e) + c)/(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.20507, size = 186, normalized size = 1.81

$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+25\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-5\,Bc\tan\left(\right.\right.}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 15*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 25*A*c*tan(1/2*f*x + 1/2*e)^2 - 5*B*c*tan(1/2*f
*x + 1/2*e)^2 + 5*A*c*tan(1/2*f*x + 1/2*e) + 5*B*c*tan(1/2*f*x + 1/2*e) + 4
*A*c + B*c)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal. Leaf size=102

$$\frac{2(3A+2B)\tan(e+fx)}{15a^3cf}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{(A-B)\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

```
[Out] -((A - B)*Sec[e + f*x])/(5*a*c*f*(a + a*Sin[e + f*x])^2) - ((3*A + 2*B)*Sec
[e + f*x])/(15*c*f*(a^3 + a^3*Sin[e + f*x])) + (2*(3*A + 2*B)*Tan[e + f*x])
/(15*a^3*c*f)
```

Rubi [A] time = 0.248533, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(3A+2B)\tan(e+fx)}{15a^3cf}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3\sin(e+fx)+a^3\right)}-\frac{(A-B)\sec(e+fx)}{5acf(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

```
[Out] -((A - B)*Sec[e + f*x])/(5*a*c*f*(a + a*Sin[e + f*x])^2) - ((3*A + 2*B)*Sec
[e + f*x])/(15*c*f*(a^3 + a^3*Sin[e + f*x])) + (2*(3*A + 2*B)*Tan[e + f*x])
/(15*a^3*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
```

```
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx=\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx}{ac}$$

$$=-\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}+\frac{(3A+2B)\int \frac{\sec^2(e+fx)}{a+a\sin(e+fx)}\,dx}{5a^2c}$$

$$=-\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3+a^3\sin(e+fx)\right)}+\frac{(2(3A+2B))}{1}$$

$$=-\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3+a^3\sin(e+fx)\right)}-\frac{(2(3A+2B))}{}$$

$$=-\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2}-\frac{(3A+2B)\sec(e+fx)}{15cf\left(a^3+a^3\sin(e+fx)\right)}+\frac{2(3A+2B)\,\mathrm{ta}}{15a^3c}$$

Mathematica [A] time = 0.791835, size = 156, normalized size = 1.53

$$\frac{\cos(e+fx)(-5(9A+B)\cos(e+fx)+32(3A+2B)\cos(2(e+fx))-120A\sin(e+fx)-36A\sin(2(e+fx))+24A\sin}{240a^3cf(\sin(e+fx)-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
),x]
```

```
[Out] (Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e +
 f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] - 8
0*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*Sin[
3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a^3*c*f*(-1 + Sin[e + f*x])*(1
+ Sin[e + f*x])^3)
```

Maple [A] time = 0.087, size = 145, normalized size = 1.4

$$2\,\frac{1}{a^3cf}\left(-\frac{A/8+B/8}{\tan\left(1/2\,fx+e/2\right)-1}-1/4\,\frac{-4\,A+4\,B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^4}-1/5\,\frac{2\,A-2\,B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}-1/2\,\frac{-5/2\,A+3/2\,B}{\left(\tan\left(1/2\,fx+e/2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^3/c*(-(1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(-4*A+4*B)/(tan(1/2*f*
x+1/2*e)+1)^4-1/5*(2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-5/2*A+3/2*B)/(ta
n(1/2*f*x+1/2*e)+1)^2-(7/8*A-1/8*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(9/2*A-7/2*B
)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.01068, size = 571, normalized size = 5.6

$$\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1\right)}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{10\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{5\sin(f}{(\cos(fx+}\right.}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)}{(\cos(fx+e)+1)}}\right.}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a
^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6) - 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*sin(f*x + e)
/(cos(f*x + e) + 1) + 5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - a^3*c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [A] time = 1.61257, size = 263, normalized size = 2.58

$$\frac{4\,(3\,A+2\,B)\cos\left(fx+e\right)^2+\left(2\,(3\,A+2\,B)\cos\left(fx+e\right)^2-9\,A-6\,B\right)\sin\left(fx+e\right)-6\,A-9\,B}{15\left(a^3cf\cos\left(fx+e\right)^3-2\,a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3cf\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A -
6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x
+ e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

Sympy [A] time = 54.0912, size = 1732, normalized size = 16.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((2*A*tan(e/2 + f*x/2)**6/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a*
*3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*
tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 22*A*ta
n(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 +
```

```
f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)*
*2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 50*A*tan(e/2 + f*x/2)**4
/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*
tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f*x/2)**3/(15*a**3*c*f*tan
(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 +
f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2)
- 15*a**3*c*f) - 10*A*tan(e/2 + f*x/2)**2/(15*a**3*c*f*tan(e/2 + f*x/2)**6
+ 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a*
*3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) +
10*A*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/
2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x
/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 10*A/(15*a**3*c*f*ta
n(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 +
 f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2)
 - 15*a**3*c*f) - 7*B*tan(e/2 + f*x/2)**6/(15*a**3*c*f*tan(e/2 + f*x/2)**6
+ 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a*
*3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) -
28*B*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan
(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 +
f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 65*B*tan(e/2 + f*
x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5
+ 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a*
*3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**3/(15*a**3*
c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan
(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 +
f*x/2) - 15*a**3*c*f) - 5*B*tan(e/2 + f*x/2)**2/(15*a**3*c*f*tan(e/2 + f*x/
2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 -
 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c
*f) + 20*B*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*
tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2
 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 5*B/(15*a**3*c
*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(
e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f
*x/2) - 15*a**3*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*s
in(e) + c)), True))
```

Giac [A] time = 1.21241, size = 236, normalized size = 2.31

$$-\frac{\frac{15\,(A+B)}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+40\,B\tan\left(\frac{1}{2}\right.}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/60*(15*(A + B)/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (105*A*tan(1/2*f*x +
 1/2*e)^4 - 15*B*tan(1/2*f*x + 1/2*e)^4 + 270*A*tan(1/2*f*x + 1/2*e)^3 + 30
*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 + 40*B*tan(1/2*f*x
 + 1/2*e)^2 + 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) + 63*A
 + 7*B)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}-\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

```
[Out] -((A - B)*Sec[e + f*x]^3)/(5*c^2*f*(a^3 + a^3*Sin[e + f*x])) + ((4*A + B)*T
an[e + f*x])/(5*a^3*c^2*f) + ((4*A + B)*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```

Rubi [A] time = 0.204458, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 3767}

$$\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}-\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3\sin(e+fx)+a^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x]
```


```
[Out] -((A - B)*Sec[e + f*x]^3)/(5*c^2*f*(a^3 + a^3*Sin[e + f*x])) + ((4*A + B)*T
an[e + f*x])/(5*a^3*c^2*f) + ((4*A + B)*Tan[e + f*x]^3)/(15*a^3*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{(4A+B)\int \sec^4(e+fx)\,dx}{5a^3c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} - \frac{(4A+B)\,\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(\right.}{5a^3c^2f}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{5c^2f\left(a^3+a^3\sin(e+fx)\right)} + \frac{(4A+B)\tan(e+fx)}{5a^3c^2f} + \frac{(4A+B)\tan^3(e}{15a^3c^2f}$$

Mathematica [B] time = 0.985886, size = 237, normalized size = 2.63

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(54(A-B)\cos(e+fx)-32(4A+B)\cos(2(e+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A*C
os[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*Cos[
4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e + f*x
)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)]
 + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 + Sin[e
 + f*x])^2*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.08, size = 185, normalized size = 2.1

$$2\,\frac{1}{fa^3c^2}\left(-1/3\,\frac{A/4+B/4}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^3}-1/2\,\frac{A/4+B/4}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}-\frac{1}{\tan\left(1/2\,fx+e/2\right)-1}\left(\frac{5\,A}{16}+3/16\,B\right)-1/4\,\frac{}{(\mathrm{ta}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^3/c^2*(-1/3*(1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/4*A+1/4*B)/
(tan(1/2*f*x+1/2*e)-1)^2-(5/16*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(-2*A+2
*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(A-B)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-3/2*A
+B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(5/2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^3-(11/1
6*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.06267, size = 878, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/(a^3*c^
2 + 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*
c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8) + B*(6*sin(f*x + e)/(cos(f*x + e) + 1) + 9
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3)/(a^3*c^2 + 2*a^3*c^2*
sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*c^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
```

```
2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8))/f
```

Fricas [A] time = 1.75245, size = 262, normalized size = 2.91

$$-\frac{2\,(4\,A+B)\cos\left(fx+e\right)^{4}-(4\,A+B)\cos\left(fx+e\right)^{2}-\left(2\,(4\,A+B)\cos\left(fx+e\right)^{2}+4\,A+B\right)\sin\left(fx+e\right)-A-4\,B}{15\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+a^{3}c^{2}f\cos\left(fx+e\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B)
*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e)^
3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20683, size = 317, normalized size = 3.52

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A+7\,B\right)}{a^{3}c^{2}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^{3}}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-45\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+480}{}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 + 9*B*tan(1/2*f*x + 1/2*e)^2 - 24*A*
tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(ta
n(1/2*f*x + 1/2*e) - 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 - 45*B*tan(1/2*f
*x + 1/2*e)^4 + 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e)^3
+ 650*A*tan(1/2*f*x + 1/2*e)^2 - 70*B*tan(1/2*f*x + 1/2*e)^2 + 400*A*tan(1/
2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(tan(1/
2*f*x + 1/2*e) + 1)^5))/f
```

3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=84

$$\frac{A\tan^5(e+fx)}{5a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*Tan[e + f*x])/(a^3*c^3*f) + (2*A*Tan[
e + f*x]^3)/(3*a^3*c^3*f) + (A*Tan[e + f*x]^5)/(5*a^3*c^3*f)
```

Rubi [A] time = 0.151853, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2669, 3767}

$$\frac{A\tan^5(e+fx)}{5a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*Tan[e + f*x])/(a^3*c^3*f) + (2*A*Tan[
e + f*x]^3)/(3*a^3*c^3*f) + (A*Tan[e + f*x]^5)/(5*a^3*c^3*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx &= \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\,dx}{a^3c^3}\\ &= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\int \sec^6(e+fx)\,dx}{a^3c^3}\\ &= \frac{B\sec^5(e+fx)}{5a^3c^3f}-\frac{A\,\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(e+fx)\right)}{a^3c^3f}\\ &= \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan^5(e+fx)}{5a^3c^3f}\end{aligned}$$

Mathematica [A] time = 0.198097, size = 65, normalized size = 0.77

$$\frac{A\left(\frac{1}{5}\tan^5(e+fx)+\frac{2}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{a^3c^3f}+\frac{B\sec^5(e+fx)}{5a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^3),x]
```

```
[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3
+ Tan[e + f*x]^5/5))/(a^3*c^3*f)
```

Maple [B] time = 0.08, size = 227, normalized size = 2.7

$$2\,\frac{1}{fa^3c^3}\left(-1/4\,\frac{A+B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^4}-1/5\,\frac{A/2+B/2}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^5}-1/2\,\frac{1}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^2}\left(\frac{7\,A}{8}+5/8\,B\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3/c^3*(-1/4*(A+B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(1/2*A+1/2*B)/(tan(1/2
*f*x+1/2*e)-1)^5-1/2*(7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+3/16*B)/
(tan(1/2*f*x+1/2*e)-1)-1/3*(11/8*A+9/8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/4*(-A+
B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/5
*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^5-(1/2*A-3/16*B)/(tan(1/2*f*x+1/2*e)+
1)-1/3*(11/8*A-9/8*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [A] time = 0.976002, size = 81, normalized size = 0.96

$$\frac{\frac{\left(3\,\tan\left(fx+e\right)^{5}+10\,\tan\left(fx+e\right)^{3}+15\,\tan\left(fx+e\right)\right)A}{a^{3}c^{3}}+\frac{3\,B}{a^{3}c^{3}\cos\left(fx+e\right)^{5}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*A/(a^3*c^3)
+ 3*B/(a^3*c^3*cos(f*x + e)^5))/f
```

Fricas [A] time = 1.86647, size = 138, normalized size = 1.64

$$\frac{\left(8\,A\cos\left(fx+e\right)^{4}+4\,A\cos\left(fx+e\right)^{2}+3\,A\right)\sin\left(fx+e\right)+3\,B}{15\,a^{3}c^{3}f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/(
a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [A] time = 104.988, size = 1506, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-60*A*tan(e/2 + f*x/2)**9/(30*a**3*c**3*f*tan(e/2 + f*x/2)**10 -
 150*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2)**6
- 300*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**2
 - 30*a**3*c**3*f) + 80*A*tan(e/2 + f*x/2)**7/(30*a**3*c**3*f*tan(e/2 + f*x
/2)**10 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f
*x/2)**6 - 300*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 +
f*x/2)**2 - 30*a**3*c**3*f) - 232*A*tan(e/2 + f*x/2)**5/(30*a**3*c**3*f*tan
(e/2 + f*x/2)**10 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*t
an(e/2 + f*x/2)**6 - 300*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*
tan(e/2 + f*x/2)**2 - 30*a**3*c**3*f) + 80*A*tan(e/2 + f*x/2)**3/(30*a**3*c
**3*f*tan(e/2 + f*x/2)**10 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3
*c**3*f*tan(e/2 + f*x/2)**6 - 300*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*c**3*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**3*f) - 60*A*tan(e/2 + f*x/2)/(30*
a**3*c**3*f*tan(e/2 + f*x/2)**10 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 30
0*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 300*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**3*f) + 3*B*tan(e/2 + f*x/2)
**10/(30*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 150*a**3*c**3*f*tan(e/2 + f*x/2
)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 300*a**3*c**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**3*f) - 75*B*tan(e/
2 + f*x/2)**8/(30*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 150*a**3*c**3*f*tan(e/
2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 300*a**3*c**3*f*tan(e
/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**3*f) + 30
*B*tan(e/2 + f*x/2)**6/(30*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 150*a**3*c**3
*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 300*a**3*c**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**
3*f) - 150*B*tan(e/2 + f*x/2)**4/(30*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 150
*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 30
0*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 3
0*a**3*c**3*f) + 15*B*tan(e/2 + f*x/2)**2/(30*a**3*c**3*f*tan(e/2 + f*x/2)*
*10 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2
)**6 - 300*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/
2)**2 - 30*a**3*c**3*f) - 15*B/(30*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 150*a
**3*c**3*f*tan(e/2 + f*x/2)**8 + 300*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 300*
a**3*c**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 30*
a**3*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e) +
c)**3), True))
```

Giac [A] time = 1.24934, size = 181, normalized size = 2.15

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-20\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+58\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+30\,B\tan\left(\frac{1}{2}\,f\right.\right.}{15\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{5}a^{3}c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*tan
(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/2*
e)^4 - 20*A*tan(1/2*f*x + 1/2*e)^3 + 15*A*tan(1/2*f*x + 1/2*e) + 3*B)/((tan
(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)
```

3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

Optimal. Leaf size=121

$$\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

```
[Out] ((A + B)*Sec[e + f*x]^5)/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + ((6*A - B)*Ta
n[e + f*x])/(7*a^3*c^4*f) + (2*(6*A - B)*Tan[e + f*x]^3)/(21*a^3*c^4*f) + (
(6*A - B)*Tan[e + f*x]^5)/(35*a^3*c^4*f)
```

Rubi [A] time = 0.223278, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2859, 3767}

$$\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}+\frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + ((6*A - B)*Ta
n[e + f*x])/(7*a^3*c^4*f) + (2*(6*A - B)*Tan[e + f*x]^3)/(21*a^3*c^4*f) + (
(6*A - B)*Tan[e + f*x]^5)/(35*a^3*c^4*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
```

```
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx}{a^3c^3}$$
$$= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(6A-B)\int \sec^6(e+fx)\,dx}{7a^3c^4}$$
$$= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} - \frac{(6A-B)\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -$$
$$= \frac{(A+B)\sec^5(e+fx)}{7a^3f\left(c^4-c^4\sin(e+fx)\right)} + \frac{(6A-B)\tan(e+fx)}{7a^3c^4f} + \frac{2(6A-B)\tan^3(e$$

Mathematica [B] time = 1.09089, size = 325, normalized size = 2.69

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(1500(A+B)\cos(e+fx)-640(6A-B)\cos($$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^4),x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e + f*x)] +
750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(e + f*x)] +
 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*x)] -
 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin[e + f*x] + 2
560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e + f*x)] - 7680
```

```
*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*(e + f*x)] - 30
0*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[5*(e + f*x)] - 7
5*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)]))/(53760*a^3*c^4*f*(-1 + Sin[e
 + f*x])^4*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.1, size = 271, normalized size = 2.2

$$2\,\frac{1}{f a^3 c^4}\left(-1/7\,\frac{A+B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{7}}-1/6\,\frac{3\,A+3\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{6}}-1/4\,\frac{11/2\,A+9/2\,B}{\left(\tan\left(1/2\,fx+e/2\right)-1\right)^{4}}-1/2\,\frac{}{\left(\tan\left(1/2\,fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

```
[Out] 2/f/a^3/c^4*(-1/7*(A+B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(3*A+3*B)/(tan(1/2*f*x
+1/2*e)-1)^6-1/4*(11/2*A+9/2*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(15/8*A+B)/(ta
n(1/2*f*x+1/2*e)-1)^2-(21/32*A+5/32*B)/(tan(1/2*f*x+1/2*e)-1)-1/5*(21/4*A+1
9/4*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(33/8*A+11/4*B)/(tan(1/2*f*x+1/2*e)-1)^
3-1/2*(-1/2*A+3/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/2*A+1/2*B)/(tan(1/2*f
*x+1/2*e)+1)^4-1/5*(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(3/4*A-5/8*B)
/(tan(1/2*f*x+1/2*e)+1)^3-(11/32*A-5/32*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.14742, size = 1376, normalized size = 11.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] -2/105*(B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 45*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 80*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 110*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 188*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 266*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 112*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 35
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 70*sin(f*x + e)^9/(cos(f*x + e) + 1)
^9 - 105*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3*c^4*s
in(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*sin(f*x +
```

```
e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
+ 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 4*
a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f*x + e)^11/(
cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) - 3*A
*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 105*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 35*s
in(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 - 5*a^3*c^4*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 + 2*a^3*c^4*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12))/f
```

Fricas [A] time = 2.09174, size = 350, normalized size = 2.89

$$-\frac{8\,(6\,A-B)\cos\left(fx+e\right)^{6}-4\,(6\,A-B)\cos\left(fx+e\right)^{4}-(6\,A-B)\cos\left(fx+e\right)^{2}+\left(8\,(6\,A-B)\cos\left(fx+e\right)^{4}+4\,(6\,A-\right.}{105\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}c^{4}f\cos\left(fx+e\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] -1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B)
*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^2
+ 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f*x
+ e) - a^3*c^4*f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21273, size = 479, normalized size = 3.96

$$-\frac{\frac{7\left(165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-75\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+540\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+750\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+480\,A\tan\left(\frac{1}{2}\cdots\right)\cdots\right)}{a^{3}c^{4}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] -1/1680*(7*(165*A*tan(1/2*f*x + 1/2*e)^4 - 75*B*tan(1/2*f*x + 1/2*e)^4 + 54
0*A*tan(1/2*f*x + 1/2*e)^3 - 210*B*tan(1/2*f*x + 1/2*e)^3 + 750*A*tan(1/2*f
*x + 1/2*e)^2 - 280*B*tan(1/2*f*x + 1/2*e)^2 + 480*A*tan(1/2*f*x + 1/2*e) -
 170*B*tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e)
+ 1)^5) + (2205*A*tan(1/2*f*x + 1/2*e)^6 + 525*B*tan(1/2*f*x + 1/2*e)^6 - 1
0080*A*tan(1/2*f*x + 1/2*e)^5 - 1470*B*tan(1/2*f*x + 1/2*e)^5 + 21945*A*tan
(1/2*f*x + 1/2*e)^4 + 2555*B*tan(1/2*f*x + 1/2*e)^4 - 26460*A*tan(1/2*f*x +
 1/2*e)^3 - 2240*B*tan(1/2*f*x + 1/2*e)^3 + 18963*A*tan(1/2*f*x + 1/2*e)^2
+ 1407*B*tan(1/2*f*x + 1/2*e)^2 - 7476*A*tan(1/2*f*x + 1/2*e) - 434*B*tan(1
/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal. Leaf size=162

$$\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(A}{9a^3c^3}$$

```
[Out] ((A + B)*Sec[e + f*x]^5)/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B
)*Sec[e + f*x]^5)/(63*a^3*f*(c^5 - c^5*Sin[e + f*x])) + (2*(7*A - 2*B)*Tan[
e + f*x])/(21*a^3*c^5*f) + (4*(7*A - 2*B)*Tan[e + f*x]^3)/(63*a^3*c^5*f) +
(2*(7*A - 2*B)*Tan[e + f*x]^5)/(105*a^3*c^5*f)
```

Rubi [A] time = 0.288791, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)}+\frac{(A}{9a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B
)*Sec[e + f*x]^5)/(63*a^3*f*(c^5 - c^5*Sin[e + f*x])) + (2*(7*A - 2*B)*Tan[
e + f*x])/(21*a^3*c^5*f) + (4*(7*A - 2*B)*Tan[e + f*x]^3)/(63*a^3*c^5*f) +
(2*(7*A - 2*B)*Tan[e + f*x]^5)/(105*a^3*c^5*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
```

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}\,dx}{9a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{(2(7A-2B))}{2\ldots}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} - \frac{(2(7A-2B))}{\ldots}$$

$$= \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2} + \frac{(7A-2B)\sec^5(e+fx)}{63a^3f\left(c^5-c^5\sin(e+fx)\right)} + \frac{2(7A-2B)\,\mathrm{ta}\ldots}{21a^3c\ldots}$$

Mathematica [B] time = 1.3207, size = 373, normalized size = 2.3

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(1125(49A+13B)\cos(e+fx)-20480(7A-2B\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^5),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*(e
 + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A*Co
s[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 325*B
*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)] - 12
25*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x] + 92
160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*x)] - 1
36192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4*(e + f*
x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*Sin[5*(e +
 f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168*A*Sin[7*(
e + f*x)] - 2048*B*Sin[7*(e + f*x)]))/(1290240*a^3*c^5*f*(-1 + Sin[e + f*x]
)^5*(1 + Sin[e + f*x])^3)
```

Maple [F] time = 180., size = 0, normalized size = 0.

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

```
[Out] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

Maxima [B] time = 1.18959, size = 1621, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="maxima")
```

```
[Out] -2/315*(B*(100*sin(f*x + e)/(cos(f*x + e) + 1) - 340*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 55*sin(f*x + e)^4/
```

```
(cos(f*x + e) + 1)^4 - 88*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1608*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1032*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
483*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 588*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 - 420*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 420*sin(f*x + e)^11/(c
os(f*x + e) + 1)^11 - 315*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 25)/(a^3*
c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^
3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c
^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^
5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x
+ e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*si
n(f*x + e)^14/(cos(f*x + e) + 1)^14) - 7*A*(5*sin(f*x + e)/(cos(f*x + e) +
1) - 80*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 190*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 269*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 96*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 516*sin(f*x +
 e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 69*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 240*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 + 30*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 90*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 + 45*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 10)/(a^3*c^5 -
4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 19*a^3*c^5*
sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 45*a^3*c^5*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 16*a^3*c^5*sin(
f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e)^12/(cos(f*x + e) +
 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - a^3*c^5*sin(f*x
+ e)^14/(cos(f*x + e) + 1)^14))/f
```

Fricas [A] time = 2.03789, size = 459, normalized size = 2.83

$$-\frac{32\,(7\,A-2\,B)\cos\left(fx+e\right)^{6}-16\,(7\,A-2\,B)\cos\left(fx+e\right)^{4}-4\,(7\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(16\,(7\,A-2\,B)\cos\left(fx+e\right)^{6}\right.}{315\left(a^{3}c^{5}f\cos\left(fx+e\right)^{7}+2\,a^{3}c^{5}f\cos\left(fx+e\right)^{5}\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="fricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*(
7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^6 - 24*(7*A - 2*B)
*cos(f*x + e)^4 - 10*(7*A - 2*B)*cos(f*x + e)^2 - 49*A + 14*B)*sin(f*x + e)
 - 14*A + 49*B)/(a^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*sin(
f*x + e) - 2*a^3*c^5*f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25432, size = 560, normalized size = 3.46

$$-\frac{\frac{21\left(435\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-225\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+1470\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-690\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2060\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-940\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1330\ldots}{a^{3}c^{5}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorit
hm="giac")
```

```
[Out] -1/20160*(21*(435*A*tan(1/2*f*x + 1/2*e)^4 - 225*B*tan(1/2*f*x + 1/2*e)^4 +
 1470*A*tan(1/2*f*x + 1/2*e)^3 - 690*B*tan(1/2*f*x + 1/2*e)^3 + 2060*A*tan(
1/2*f*x + 1/2*e)^2 - 940*B*tan(1/2*f*x + 1/2*e)^2 + 1330*A*tan(1/2*f*x + 1/
2*e) - 590*B*tan(1/2*f*x + 1/2*e) + 353*A - 163*B)/(a^3*c^5*(tan(1/2*f*x +
1/2*e) + 1)^5) + (31185*A*tan(1/2*f*x + 1/2*e)^8 + 4725*B*tan(1/2*f*x + 1/2
*e)^8 - 185220*A*tan(1/2*f*x + 1/2*e)^7 - 11340*B*tan(1/2*f*x + 1/2*e)^7 +
546840*A*tan(1/2*f*x + 1/2*e)^6 + 15120*B*tan(1/2*f*x + 1/2*e)^6 - 961380*A
*tan(1/2*f*x + 1/2*e)^5 + 3780*B*tan(1/2*f*x + 1/2*e)^5 + 1101618*A*tan(1/2
*f*x + 1/2*e)^4 - 24318*B*tan(1/2*f*x + 1/2*e)^4 - 828492*A*tan(1/2*f*x + 1
/2*e)^3 + 33852*B*tan(1/2*f*x + 1/2*e)^3 + 404208*A*tan(1/2*f*x + 1/2*e)^2
- 19368*B*tan(1/2*f*x + 1/2*e)^2 - 116172*A*tan(1/2*f*x + 1/2*e) + 6732*B*t
```

```
an(1/2*f*x + 1/2*e) + 16373*A - 223*B)/(a^3*c^5*(tan(1/2*f*x + 1/2*e) - 1)^
9))/f
```

3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal. Leaf size=205

$$\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{(8A}{99a^3f}$$

```
[Out] ((A + B)*Sec[e + f*x]^5)/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + ((8*A - 3*
B)*Sec[e + f*x]^5)/(99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + ((8*A - 3*B)*Sec
[e + f*x]^5)/(99*a^3*f*(c^6 - c^6*Sin[e + f*x])) + (2*(8*A - 3*B)*Tan[e + f
*x])/(33*a^3*c^6*f) + (4*(8*A - 3*B)*Tan[e + f*x]^3)/(99*a^3*c^6*f) + (2*(8
*A - 3*B)*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rubi [A] time = 0.34542, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2859, 2672, 3767}

$$\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{(8A}{99a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]
```

```
[Out] ((A + B)*Sec[e + f*x]^5)/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + ((8*A - 3*
B)*Sec[e + f*x]^5)/(99*a^3*f*(c^3 - c^3*Sin[e + f*x])^2) + ((8*A - 3*B)*Sec
[e + f*x]^5)/(99*a^3*f*(c^6 - c^6*Sin[e + f*x])) + (2*(8*A - 3*B)*Tan[e + f
*x])/(33*a^3*c^6*f) + (4*(8*A - 3*B)*Tan[e + f*x]^3)/(99*a^3*c^6*f) + (2*(8
*A - 3*B)*Tan[e + f*x]^5)/(165*a^3*c^6*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx = \frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx}{a^3c^3}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}\,dx}{11a^3c^4}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{(7(8A-}{}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{(8A-}{99a^3f\left(c^6\right.}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{(8A-}{99a^3f\left(c^6\right.}$$

$$= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3} + \frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2} + \frac{(8A-}{99a^3f\left(c^6\right.}$$

Mathematica [A] time = 3.25535, size = 401, normalized size = 1.96

$$\frac{-3850(107A-3B)\cos(e+fx)+135168(8A-3B)\cos(2(e+fx))+1802240A\sin(e+fx)+247170A\sin(2(e+fx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^6),x]
```

```
[Out] (1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e
+ f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*Co
s[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] - 105
0*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f*x)]
 + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(e + f
*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Sin[e +
f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*A*Sin[3
*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)] - 5250*
B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e + f*x)] +
 37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin[7*(e + f*
x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B*Sin[8*(e +
```

```
f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.128, size = 365, normalized size = 1.8

$$2\,{\frac {1}{f{a}^{3}{c}^{6}} \left(-1/11\,{\frac {4\,A+4\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{11}}}-1/10\,{\frac {20\,A+20\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{10}}}-1/9\,{\frac {53\,A+51\,B}{ \left(\tan \left(1/2\,fx+e/2 \right) -1 \right) ^{9}}}-1/8\,{\frac {92}{ \left(\tan \left(1/ \right. \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

```
[Out] 2/f/a^3/c^6*(-1/11*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/10*(20*A+20*B)/(ta
n(1/2*f*x+1/2*e)-1)^10-1/9*(53*A+51*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(92*A+8
4*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/4*(169/4*A+99/4*B)/(tan(1/2*f*x+1/2*e)-1)^4
-1/6*(217/2*A+84*B)/(tan(1/2*f*x+1/2*e)-1)^6-(219/256*A+21/256*B)/(tan(1/2*
f*x+1/2*e)-1)-1/7*(231/2*A+98*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/2*(303/64*A+99/
64*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(623/8*A+427/8*B)/(tan(1/2*f*x+1/2*e)-1)
^5-1/3*(1095/64*A+507/64*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(-5/32*A+1/8*B)/(t
an(1/2*f*x+1/2*e)+1)^2-1/4*(-1/8*A+1/8*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/1
6*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(7/32*A-3/16*B)/(tan(1/2*f*x+1/2*e
)+1)^3-(37/256*A-21/256*B)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] time = 1.22911, size = 1872, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="maxima")
```

```
[Out] -2/495*(A*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 8217*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(cos(f
*x + e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(f*x +
e)^11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 -
```

```
1815*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(cos(f*x
+ e) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/(a^3*c^6 -
6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*a^3*c
^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^3*c^6
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^3*c^6
*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(cos(f*
x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10*a^3*
c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/(cos(
f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16) + 3*B*(30
*sin(f*x + e)/(cos(f*x + e) + 1) - 215*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 245*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 434*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 231*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 880*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1815*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 + 330*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 99*
sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 264*sin(f*x + e)^11/(cos(f*x + e) +
 1)^11 - 495*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 330*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 - 165*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 - 5)/(a^3*c
^6 - 6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 50*
a^3*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 110*a^
3*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 34*a^
3*c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x + e)^12/(c
os(f*x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 - 10
*a^3*c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin(f*x + e)^15/
(cos(f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e) + 1)^16))/f
```

Fricas [A] time = 2.02722, size = 543, normalized size = 2.65

$$\frac{16\,(8\,A-3\,B)\cos\left(fx+e\right)^{8}-72\,(8\,A-3\,B)\cos\left(fx+e\right)^{6}+30\,(8\,A-3\,B)\cos\left(fx+e\right)^{4}+7\,(8\,A-3\,B)\cos\left(fx+e\right)^{2}+}{495\left(3\,a^{3}c^{6}f\cos\left(fx+e\right)^{7}-4\,a^{3}c^{6}f\cos\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="fricas")
```

```
[Out] 1/495*(16*(8*A - 3*B)*cos(f*x + e)^8 - 72*(8*A - 3*B)*cos(f*x + e)^6 + 30*(
8*A - 3*B)*cos(f*x + e)^4 + 7*(8*A - 3*B)*cos(f*x + e)^2 + (48*(8*A - 3*B)*
cos(f*x + e)^6 - 40*(8*A - 3*B)*cos(f*x + e)^4 - 14*(8*A - 3*B)*cos(f*x + e
)^2 - 72*A + 27*B)*sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*cos(f*x + e)^7
- 4*a^3*c^6*f*cos(f*x + e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f*cos(
f*x + e)^5)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27891, size = 641, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorit
hm="giac")
```

```
[Out] -1/63360*(33*(555*A*tan(1/2*f*x + 1/2*e)^4 - 315*B*tan(1/2*f*x + 1/2*e)^4 +
 1920*A*tan(1/2*f*x + 1/2*e)^3 - 1020*B*tan(1/2*f*x + 1/2*e)^3 + 2710*A*tan
(1/2*f*x + 1/2*e)^2 - 1410*B*tan(1/2*f*x + 1/2*e)^2 + 1760*A*tan(1/2*f*x +
1/2*e) - 900*B*tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(tan(1/2*f*x
+ 1/2*e) + 1)^5) + (108405*A*tan(1/2*f*x + 1/2*e)^10 + 10395*B*tan(1/2*f*x
+ 1/2*e)^10 - 784080*A*tan(1/2*f*x + 1/2*e)^9 - 5940*B*tan(1/2*f*x + 1/2*e)
^9 + 2901195*A*tan(1/2*f*x + 1/2*e)^8 - 79695*B*tan(1/2*f*x + 1/2*e)^8 - 66
52800*A*tan(1/2*f*x + 1/2*e)^7 + 388080*B*tan(1/2*f*x + 1/2*e)^7 + 10407474
*A*tan(1/2*f*x + 1/2*e)^6 - 816354*B*tan(1/2*f*x + 1/2*e)^6 - 11435424*A*ta
n(1/2*f*x + 1/2*e)^5 + 1114344*B*tan(1/2*f*x + 1/2*e)^5 + 8949270*A*tan(1/2
*f*x + 1/2*e)^4 - 990990*B*tan(1/2*f*x + 1/2*e)^4 - 4899840*A*tan(1/2*f*x +
 1/2*e)^3 + 609840*B*tan(1/2*f*x + 1/2*e)^3 + 1816265*A*tan(1/2*f*x + 1/2*e
)^2 - 235785*B*tan(1/2*f*x + 1/2*e)^2 - 411664*A*tan(1/2*f*x + 1/2*e) + 563
```

```
64*B*tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(tan(1/2*f*x + 1/2*e
) - 1)^11))/f
```

3.81 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=198

$$\frac{256ac^5(11A-5B)\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4(11A-5B)\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(11A-5B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f}+\frac{2ac}{}$$

```
[Out] (256*a*(11*A - 5*B)*c^5*Cos[e + f*x]^3)/(3465*f*(c - c*Sin[e + f*x])^(3/2))
 + (64*a*(11*A - 5*B)*c^4*Cos[e + f*x]^3)/(1155*f*Sqrt[c - c*Sin[e + f*x]])
 + (8*a*(11*A - 5*B)*c^3*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(231*f) +
 (2*a*(11*A - 5*B)*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(99*f) -
(2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f)
```

Rubi [A] time = 0.487202, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2856, 2674, 2673}

$$\frac{256ac^5(11A-5B)\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}}+\frac{64ac^4(11A-5B)\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(11A-5B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f}+\frac{2ac}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (256*a*(11*A - 5*B)*c^5*Cos[e + f*x]^3)/(3465*f*(c - c*Sin[e + f*x])^(3/2))
 + (64*a*(11*A - 5*B)*c^4*Cos[e + f*x]^3)/(1155*f*Sqrt[c - c*Sin[e + f*x]])
 + (8*a*(11*A - 5*B)*c^3*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(231*f) +
 (2*a*(11*A - 5*B)*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(99*f) -
(2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+$$

$$= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f} + \frac{1}{11}(a(11A$$

$$= \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} -$$

$$= \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2a}{}$$

$$= \frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8a(11A-5B)c^3\cos}{}$$

$$= \frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}} + \frac{64a(11A-5B)c^4}{1155f\sqrt{c-c\sin}}$$

Mathematica [A] time = 2.88061, size = 149, normalized size = 0.75

$$-\frac{ac^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(60(121A-202B)\cos(2(e+fx))+30558A\sin(e+fx)-7}{13860f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-
35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*Cos[4*(e +
f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[3*(e + f*x)
] + 2870*B*Sin[3*(e + f*x)]))/(13860*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
))
```

Maple [A] time = 1.046, size = 119, normalized size = 0.6

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^4\left(1+\sin\left(fx+e\right)\right)^2a\left(\left(-385A+1435B\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+\left(3916A-4300B\right)\sin\left(f\right.\right.}{3465f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/3465*(-1+sin(f*x+e))*c^4*(1+sin(f*x+e))^2*a*((-385*A+1435*B)*sin(f*x+e)*c
os(f*x+e)^2+(3916*A-4300*B)*sin(f*x+e)+315*B*cos(f*x+e)^4+(1815*A-3345*B)*c
os(f*x+e)^2-5324*A+4940*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [A] time = 1.78942, size = 749, normalized size = 3.78

$$\frac{2\left(315\,Bac^3\cos\left(fx+e\right)^6-35\,(11\,A-32\,B)ac^3\cos\left(fx+e\right)^5+5\,(209\,A-221\,B)ac^3\cos\left(fx+e\right)^4+2\,(1243\,A-1195\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5
+ 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(f*
x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3*co
s(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B*a*c^3*cos(f*x + e)^5 + 35*(11*
A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)^3 -
96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x + e)
- 256*(11*A - 5*B)*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
7/2), x)
```

3.82 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=157

$$\frac{64ac^4(3A-B)\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3(3A-B)\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2(3A-B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc\cos^3}{}$$

```
[Out] (64*a*(3*A - B)*c^4*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (1
6*a*(3*A - B)*c^3*Cos[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*(
3*A - B)*c^2*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(21*f) - (2*a*B*c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f)
```

Rubi [A] time = 0.412332, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2856, 2674, 2673}

$$\frac{64ac^4(3A-B)\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16ac^3(3A-B)\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2ac^2(3A-B)\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc\cos^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (64*a*(3*A - B)*c^4*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (1
6*a*(3*A - B)*c^3*Cos[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*(
3*A - B)*c^2*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(21*f) - (2*a*B*c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx &= (ac)\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+f\\
&= -\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}+\frac{1}{3}(a(3A-\\
&= \frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc\\
&= \frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2a(3A-B)c^2\cos^3(e+}{21\\
&= \frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16a(3A-B)c^3\cos^3(e+}{105f\sqrt{c-c\sin(e+f}}
\end{aligned}$$

Mathematica [A] time = 1.44892, size = 123, normalized size = 0.78

$$-\frac{ac^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3((648A-741B)\sin(e+fx)+30(3A-8B)\cos(2(e+fx))-}{630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-
942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*Sin[e + f
*x] + 35*B*Sin[3*(e + f*x)]))/(630*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 1.018, size = 103, normalized size = 0.7

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{2}a\left(-35B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\left(-162A+194B\right)\sin\left(fx+e\right)+(-\right.}{315f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/315*(-1+sin(f*x+e))*c^3*(1+sin(f*x+e))^2*a*(-35*B*cos(f*x+e)^2*sin(f*x+e
)+(-162*A+194*B)*sin(f*x+e)+(-45*A+120*B)*cos(f*x+e)^2+258*A-226*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [A] time = 1.75947, size = 591, normalized size = 3.76

$$2\left(35\,Bac^2\cos\left(fx+e\right)^5+5\,(9\,A-10\,B)ac^2\cos\left(fx+e\right)^4+(117\,A-109\,B)ac^2\cos\left(fx+e\right)^3-8\,(3\,A-B)ac^2\cos\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (1
17*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 32*
(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x + e
)^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x + e)
^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
5/2), x)
```

3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{2ac^2(7A-B)\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(7A-B)\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

```
[Out] (8*a*(7*A - B)*c^3*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (2*
a*(7*A - B)*c^2*Cos[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*B*c*
Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f)
```

Rubi [A] time = 0.320017, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2ac^2(7A-B)\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}+\frac{8ac^3(7A-B)\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (8*a*(7*A - B)*c^3*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (2*
a*(7*A - B)*c^2*Cos[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*B*c*
Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=(ac)\int \cos^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+$$

$$=-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}+\frac{1}{7}(a(7A-B)$$

$$=\frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c}}{7f}$$

$$=\frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2a(7A-B)c^2\cos^3(e+f}{35f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.980497, size = 104, normalized size = 0.9

$$\frac{ac\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3((66B-42A)\sin(e+fx)+98A+15B\cos(2(e+fx))-59B)}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e +
 f*x)] + (-42*A + 66*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.917, size = 81, normalized size = 0.7

$$\frac{\left(-2+2\,\sin\left(fx+e\right)\right)c^2\left(1+\sin\left(fx+e\right)\right)^2 a\left(\sin\left(fx+e\right)(21\,A-33\,B)-15\,B\left(\cos\left(fx+e\right)\right)^2-49\,A+37\,B\right)}{105\,f\cos\left(fx+e\right)}\frac{}{\sqrt{c-c\,\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/105*(-1+sin(f*x+e))*c^2*(1+sin(f*x+e))^2*a*(sin(f*x+e)*(21*A-33*B)-15*B*c
os(f*x+e)^2-49*A+37*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [A] time = 1.71847, size = 454, normalized size = 3.91

$$-\frac{2\left(15\,Bac\cos\left(fx+e\right)^4-3\,(7\,A-6\,B)ac\cos\left(fx+e\right)^3+(7\,A-B)ac\cos\left(fx+e\right)^2-4\,(7\,A-B)ac\cos\left(fx+e\right)-8\,(7\right.}{105\left(f\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A -
 B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c - (
15*B*a*c*cos(f*x + e)^3 + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a*c*
cos(f*x + e) + 8*(7*A - B)*a*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=73

$$\frac{2ac^2(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*a*(5*A + B)*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2*a
*B*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.239825, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2967, 2856, 2673}

$$\frac{2ac^2(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a*(5*A + B)*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2*a
*B*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=(ac)\int\frac{\cos^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}+\frac{1}{5}(a(5A+B)c)\int\frac{\cos^2(e+}{\sqrt{c-c\sin(e}}$$
$$=\frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A] time = 0.423134, size = 87, normalized size = 1.19

$$\frac{2a\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3(5A+3B\sin(e+fx)-2B)}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (2*a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(5*A - 2*B + 3*B*Sin[e + f*x])
*Sqrt[c - c*Sin[e + f*x]])/(15*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.963, size = 63, normalized size = 0.9

$$-\frac{(-2+2\sin(fx+e))c(1+\sin(fx+e))^2a(3B\sin(fx+e)+5A-2B)}{15f\cos(fx+e)}\frac{1}{\sqrt{c-c\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*(-1+sin(f*x+e))*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [A] time = 1.66284, size = 336, normalized size = 4.6

$$-\frac{2\left(3\,Ba\cos\left(fx+e\right)^{3}+(5\,A+4\,B)a\cos\left(fx+e\right)^{2}-(5\,A+B)a\cos\left(fx+e\right)-2\,(5\,A+B)a+\left(3\,Ba\cos\left(fx+e\right)^{2}-(5\right.}{15\left(f\cos\left(fx+e\right)-f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] -2/15*(3*B*a*cos(f*x + e)^3 + (5*A + 4*B)*a*cos(f*x + e)^2 - (5*A + B)*a*co
s(f*x + e) - 2*(5*A + B)*a + (3*B*a*cos(f*x + e)^2 - (5*A + B)*a*cos(f*x +
e) - 2*(5*A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
 - f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int B\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2\sqrt{2}a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}+\frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

```
[Out] (2*Sqrt[2]*a*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin
[e + f*x]])])/(Sqrt[c]*f) - (2*a*(3*A + 5*B)*Cos[e + f*x])/(3*f*Sqrt[c - c*
Sin[e + f*x]]) + (2*a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c*f)
```

Rubi [A] time = 0.335919, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2858, 2751, 2649, 206}

$$-\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2\sqrt{2}a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}+\frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[2]*a*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin
[e + f*x]])])/(Sqrt[c]*f) - (2*a*(3*A + 5*B)*Cos[e + f*x])/(3*f*Sqrt[c - c*
Sin[e + f*x]]) + (2*a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2858

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(
```

```
a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$= \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} - \frac{(2a)\int \frac{-\frac{3Ac}{2}-\frac{Bc}{2}+\left(-\frac{3Ac}{2}-\frac{5Bc}{2}\right)\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{3c}$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} + (2a(A$$

$$= -\frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf} - \frac{(4a(A}{}$$

$$= \frac{2\sqrt{2}a(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB}{}$$

Mathematica [A] time = 1.26508, size = 166, normalized size = 1.36

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6\sqrt{2}(A+B)\sqrt{-c(\sin(e+fx)+1)}\tan^{-1}\left(\frac{\sqrt{-c(\sin(e+fx)+1)}}{\sqrt{2}\sqrt{c}}\right)+\sqrt{c}(2(3A+5B)\sin\right.}{3\sqrt{c}f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*Sqrt[2]*(A + B)*ArcTan[Sqrt[-(
c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x]))] + S
qrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x])))/(3*S
qrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 1.253, size = 159, normalized size = 1.3

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)a}{3c^{2}\cos\left(fx+e\right)f}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\left(3c^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)A+3c^{3/2}\sqrt{2}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/3*(-1+sin(f*x+e))*(c*(1+sin(f*x+e)))^(1/2)*a*(3*c^(3/2)*2^(1/2)*arctanh(
1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+3*c^(3/2)*2^(1/2)*arctanh(1
/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-B*(c*(1+sin(f*x+e)))^(3/2)-3
*A*c*(c*(1+sin(f*x+e)))^(1/2)-3*B*c*(c*(1+sin(f*x+e)))^(1/2))/c^2/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] time = 1.77234, size = 690, normalized size = 5.66

$$\frac{\frac{3\sqrt{2}\left((A+B)ac\cos\left(fx+e\right)-(A+B)ac\sin\left(fx+e\right)+(A+B)ac\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{c}}+}{3\left(cf\cos\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A +
B)*a*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos
(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2))/sqrt(c) + 2*(B*a*cos(f*x + e)^2 - (3*A + 4*B)*a*cos(f*x + e) - (
3*A + 5*B)*a - (B*a*cos(f*x + e) + (3*A + 5*B)*a)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{A\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{B\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int\frac{B\sin^{2}\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt(
-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c
```

```
), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))
```

Giac [B] time = 2.41465, size = 541, normalized size = 4.43

$$\frac{12\sqrt{2}(Aa+Ba)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}-\sqrt{c}\right)}{2\sqrt{-c}}\right)}{\sqrt{-c}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}+\frac{\left(\left(\left(\frac{\left(3Aac\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)+4Bac\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{c^{6}}+\frac{3\left(Aac\,\mathrm{sgn}\left(\cdots\right.\right.}{\cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/3*(12*sqrt(2)*(A*a + B*a)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c)*sgn(
tan(1/2*f*x + 1/2*e) - 1)) + ((((3*A*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1) + 4*
B*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e)/c^6 + 3*(A*a*c*sg
n(tan(1/2*f*x + 1/2*e) - 1) + 2*B*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^6)*t
an(1/2*f*x + 1/2*e) + 3*(A*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1) + 2*B*a*c*sgn(
tan(1/2*f*x + 1/2*e) - 1))/c^6)*tan(1/2*f*x + 1/2*e) + (3*A*a*c*sgn(tan(1/2
*f*x + 1/2*e) - 1) + 4*B*a*c*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^6)/(c*tan(1/2
*f*x + 1/2*e)^2 + c)^(3/2) - (12*sqrt(2)*A*a*c^7*arctan(sqrt(c)/sqrt(-c)) +
 12*sqrt(2)*B*a*c^7*arctan(sqrt(c)/sqrt(-c)) + 3*sqrt(2)*A*a*sqrt(-c)*sqrt(
c) + 5*sqrt(2)*B*a*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(-c
)*c^7))/f
```

3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}+\frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((a*(A + 5*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[2]*c^(3/2)*f)) + (a*(A + B)*Cos[e + f*x])/(f*(c - c*Sin[e + f
*x])^(3/2)) + (2*a*B*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.318343, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2857, 2751, 2649, 206}

$$-\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}+\frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}}+\frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((a*(A + 5*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(Sqrt[2]*c^(3/2)*f)) + (a*(A + B)*Cos[e + f*x])/(f*(c - c*Sin[e + f
*x])^(3/2)) + (2*a*B*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{a\int \frac{-Ac-3Bc-2Bc\sin(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{2c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(a(A+5B))\int \frac{1}{\sqrt{c-c\sin(e+}}}{2c}$$

$$= \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+5B))\,\text{Subst}\left(\int \frac{}{2c}\right.}{c}$$

$$= -\frac{a(A+5B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\,c}{cf\sqrt{c-}}$$

Mathematica [A] time = 1.55308, size = 157, normalized size = 1.37

$$\frac{a\sec(e+fx)\left(2\sqrt{c}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2(A-2B\sin(e+fx)+3B)+\sqrt{2}(A+5B)\sqrt{-c(\sin(e+fx)+1)}\,(\right.}{2c^{3/2}f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sq
rt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin[e
 + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B - 2
*B*Sin[e + f*x])))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.958, size = 227, normalized size = 2.

$$\frac{a}{2f\cos(fx+e)}\left(A\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c(1+\sin(fx+e))}\frac{1}{\sqrt{c}}\right)\sin(fx+e)\,c+5B\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{c(1+\sin(fx+e))}}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2*a/c^(5/2)*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/
2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)-5*B*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+2*(c*(1+sin(f*x+e)))^(
1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B)*(c*(1+sin(f*x+e)))^(1/
2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [B] time = 1.76055, size = 852, normalized size = 7.41

$$\frac{\frac{\sqrt{2}\left((A+5B)ac\cos\left(fx+e\right)^{2}-(A+5B)ac\cos\left(fx+e\right)-2(A+5B)ac+\left((A+5B)ac\cos\left(fx+e\right)+2(A+5B)ac\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{\cdots}}{\cdots}}{\cos\left(fx+e\right)^{2}+\left(\cos\cdots\right.}\right.}{\sqrt{c}}\cdots}{4\left(c^{2}f\cos\left(fx+e\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) - 2
*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x + e
```

```
))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x
+ e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
- 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) + (A + B
)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x
+ e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.55831, size = 720, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -(2*(B*a*tan(1/2*f*x + 1/2*e)/(c*sgn(tan(1/2*f*x + 1/2*e) - 1)) + B*a/(c*sg
n(tan(1/2*f*x + 1/2*e) - 1)))/sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) + sqrt(2)*
(A*a + 5*B*a)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*ta
n(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c)*c*sgn(tan(1/2*f*x
+ 1/2*e) - 1)) - 2*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^3*A*a + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))^3*B*a - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^2*A*a*sqrt(c) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))^2*B*a*sqrt(c) - (sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*a*c - (sqrt(c)*tan(1/2*f*x + 1/2*e)
- sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*a*c - A*a*c^(3/2) - B*a*c^(3/2))/((
(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 - 2*(
```

```
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c)
- c)^2*c*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] -(a*(A - 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*
x]])])/(8*Sqrt[2]*c^(5/2)*f) + (a*(A + B)*Cos[e + f*x])/(2*f*(c - c*Sin[e +
 f*x])^(5/2)) - (a*(A + 9*B)*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2
))
```

Rubi [A] time = 0.334508, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2857, 2750, 2649, 206}

$$-\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f}-\frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}+\frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -(a*(A - 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*
x]])])/(8*Sqrt[2]*c^(5/2)*f) + (a*(A + B)*Cos[e + f*x])/(2*f*(c - c*Sin[e +
 f*x])^(5/2)) - (a*(A + 9*B)*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2
))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} + \frac{a\int \frac{-Ac-5Bc-4Bc\sin(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{4c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} - \frac{(a(A-7B))\int \frac{}{\sqrt{c-c\,\mathrm{s}}}}{16c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}} + \frac{(a(A-7B))\,\mathrm{Subst}}{}$$

$$= -\frac{a(A-7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+}{8cf(c-}$$

Mathematica [A] time = 2.17522, size = 199, normalized size = 1.58

$$-\frac{a(\sin(e+fx)-1)(\sin(e+fx)+1)\left(\frac{2\sqrt{c}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((A+9B)\sin(e+fx)+3A-5B)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5} + \sqrt{2}(A-7B)\sec(e+fx)\sqrt{-c(\sin}\right.}{16c^{5/2}f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[Sqrt[-
(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1 + Sin[e
 + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A - 5*B +
(A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5))/(16*c^(5
/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 1.45, size = 268, normalized size = 2.1

$$\frac{a}{\left(-16+16\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(-2\sin\left(fx+e\right)\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)c^2\left(A-7B\right)-\sqrt{2}\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/16*a*(-2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^2*(A-7*B)-2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2*(A-7*B)*cos(f*x+e)^2+2*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)
*2^(1/2)/c^(1/2))*c^2-4*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-2*A*(c+c*sin(f*x+e
))^(3/2)*c^(1/2)-14*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^
(1/2))*c^2+28*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-18*B*(c+c*sin(f*x+e))^(3/2)*
c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/c^(9/2)/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [B] time = 1.7798, size = 1035, normalized size = 8.21

$$-\frac{\sqrt{2}\Big((A-7B)a\cos\left(fx+e\right)^{3}+3(A-7B)a\cos\left(fx+e\right)^{2}-2(A-7B)a\cos\left(fx+e\right)-4(A-7B)a-\Big((A-7B)a\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/32*(sqrt(2)*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2 -
 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) - 4*(A +
B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f
)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] sage2
```

3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=163

$$-\frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] -(a*(A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*
x]])])/(32*Sqrt[2]*c^(7/2)*f) + (a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e
+ f*x])^(7/2)) - (a*(A + 13*B)*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(
5/2)) - (a*(A - 3*B)*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.372946, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2967, 2857, 2750, 2650, 2649, 206}

$$-\frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2),
x]
```


```
[Out] -(a*(A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*
x]])])/(32*Sqrt[2]*c^(7/2)*f) + (a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e
+ f*x])^(7/2)) - (a*(A + 13*B)*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(
5/2)) - (a*(A - 3*B)*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(3/2))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = (ac)\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} + \frac{a\int \frac{-Ac-7Bc-6Bc\sin(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx}{6c^2}$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{(a(A-3B))\int \frac{\ }{(c-}}{16c}$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(}{32c^2f(c-c\sin(e}$$

$$= \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}} - \frac{a(A-3B)\cos(}{32c^2f(c-c\sin(e}$$

$$= -\frac{a(A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} + \frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}} - \frac{a(A}{24cf(}$$

Mathematica [A] time = 3.31844, size = 217, normalized size = 1.33

$$-\frac{a(\sin(e+fx)-1)(\sin(e+fx)+1)\left(\frac{\sqrt{c}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(4(5A+17B)\sin(e+fx)+3(A-3B)\cos(2(e+fx))+47A-13B)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7} + 3\sqrt{2}(A\right.}{192c^{7/2}f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*Sqrt[2]*(A - 3*B)*ArcTan[Sqrt
[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1 + Sin
[e + f*x]))] + (Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*A - 13*B
+ 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7))/(192*c^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 1.416, size = 352, normalized size = 2.2

$$\frac{a}{192\left(-1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\left(-3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)c^{4}\left(A-3\,B\right)\sin\left(fx+e\right)\left(\cos\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/192*a*(-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4
*(A-3*B)*sin(f*x+e)*cos(f*x+e)^2+12*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*c^4*(A-3*B)*sin(f*x+e)+9*2^(1/2)*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-3*B)*cos(f*x+e)^2+24*A*(c+c*sin(f*x+e
))^(1/2)*c^(7/2)+32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-6*A*(c+c*sin(f*x+e))^(
5/2)*c^(3/2)-72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+32*B*(c+c*sin(f*x+e))^(3/2
)*c^(5/2)+18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-12*A*2^(1/2)*arctanh(1/2*(c+c
*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4+36*B*2^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/c^(15/2)/(-1+si
n(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [B] time = 1.71156, size = 1289, normalized size = 7.91

$$-\frac{3\sqrt{2}\left((A-3\,B)a\cos\left(fx+e\right)^{4}-3\,(A-3\,B)a\cos\left(fx+e\right)^{3}-8\,(A-3\,B)a\cos\left(fx+e\right)^{2}+4\,(A-3\,B)a\cos\left(fx+e\right)+8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] -1/384*(3*sqrt(2)*((A - 3*B)*a*cos(f*x + e)^4 - 3*(A - 3*B)*a*cos(f*x + e)^
3 - 8*(A - 3*B)*a*cos(f*x + e)^2 + 4*(A - 3*B)*a*cos(f*x + e) + 8*(A - 3*B)
*a + ((A - 3*B)*a*cos(f*x + e)^3 + 4*(A - 3*B)*a*cos(f*x + e)^2 - 4*(A - 3*
B)*a*cos(f*x + e) - 8*(A - 3*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x +
e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x
+ e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(
cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
3*(A - 3*B)*a*cos(f*x + e)^3 - (7*A + 43*B)*a*cos(f*x + e)^2 + 2*(11*A - B)
*a*cos(f*x + e) + 32*(A + B)*a + (3*(A - 3*B)*a*cos(f*x + e)^2 + 2*(5*A + 1
7*B)*a*cos(f*x + e) + 32*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
)/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 +
 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x +
 e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] sage2
```

3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=210

$$\frac{256a^2c^6(13A-3B)\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5(13A-3B)\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(13A-3B)\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3(13A-3B)\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}$$

```
[Out] (256*a^2*(13*A - 3*B)*c^6*Cos[e + f*x]^5)/(15015*f*(c - c*Sin[e + f*x])^(5/
2)) + (64*a^2*(13*A - 3*B)*c^5*Cos[e + f*x]^5)/(3003*f*(c - c*Sin[e + f*x])
^(3/2)) + (8*a^2*(13*A - 3*B)*c^4*Cos[e + f*x]^5)/(429*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*a^2*(13*A - 3*B)*c^3*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/
(143*f) - (2*a^2*B*c^2*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f)
```

Rubi [A] time = 0.554384, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{256a^2c^6(13A-3B)\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}+\frac{64a^2c^5(13A-3B)\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(13A-3B)\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^3(13A-3B)\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^2*(13*A - 3*B)*c^6*Cos[e + f*x]^5)/(15015*f*(c - c*Sin[e + f*x])^(5/
2)) + (64*a^2*(13*A - 3*B)*c^5*Cos[e + f*x]^5)/(3003*f*(c - c*Sin[e + f*x])
^(3/2)) + (8*a^2*(13*A - 3*B)*c^4*Cos[e + f*x]^5)/(429*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*a^2*(13*A - 3*B)*c^3*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/
(143*f) - (2*a^2*B*c^2*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin($$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f} + \frac{1}{13}\left(a^2\right.$$

$$= \frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f} -$$

$$= \frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}} + \frac{2a^2(13A-3B)c^3\ c}{}$$

$$= \frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2(13A-3B)c^4}{429f\sqrt{c-c\,\mathrm{si}}}$$

$$= \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}} + \frac{64a^2(13A-3B}{3003f(c-c\,\mathrm{s}}$$

Mathematica [B] time = 6.70347, size = 1355, normalized size = 6.45

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(
7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x))/2]
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((A - 4*
B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/
(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e
+ f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[
e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos[(13*(e + f*x)
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((7*
A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
)/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)
*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*(a + a*Sin[e + f*x])^2*(
c - c*Sin[e + f*x])^(7/2)*Sin[(5*(e + f*x))/2])/(160*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((A - 4*B)*(
a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(112
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4) + (A*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e +
 f*x))/2])/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4) - ((2*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(7/2)*Sin[(11*(e + f*x))/2])/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*(a + a*Sin[e + f*x])^
2*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] time = 1.12, size = 121, normalized size = 0.6

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^{4}\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(\left(-1365A+4935B\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}+\left(11180A-11820B\right)\right.}{15015f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/15015*(-1+sin(f*x+e))*c^4*(1+sin(f*x+e))^3*a^2*((-1365*A+4935*B)*sin(f*x+
e)*cos(f*x+e)^2+(11180*A-11820*B)*sin(f*x+e)+1155*B*cos(f*x+e)^4+(5915*A-10
605*B)*cos(f*x+e)^2-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [A] time = 1.54687, size = 895, normalized size = 4.26

$$\frac{2\left(1155Ba^{2}c^{3}\cos\left(fx+e\right)^{7}+105\left(13A-14B\right)a^{2}c^{3}\cos\left(fx+e\right)^{6}+35\left(91A-87B\right)a^{2}c^{3}\cos\left(fx+e\right)^{5}-20\left(13A-3B\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 2/15015*(1155*B*a^2*c^3*cos(f*x + e)^7 + 105*(13*A - 14*B)*a^2*c^3*cos(f*x
+ e)^6 + 35*(91*A - 87*B)*a^2*c^3*cos(f*x + e)^5 - 20*(13*A - 3*B)*a^2*c^3*
```

```
cos(f*x + e)^4 + 32*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^3 - 64*(13*A - 3*B)*a
^2*c^3*cos(f*x + e)^2 + 256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A -
 3*B)*a^2*c^3 + (1155*B*a^2*c^3*cos(f*x + e)^6 - 105*(13*A - 25*B)*a^2*c^3*
cos(f*x + e)^5 + 140*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^4 + 160*(13*A - 3*B)
*a^2*c^3*cos(f*x + e)^3 + 192*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^2 + 256*(13
*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A - 3*B)*a^2*c^3)*sin(f*x + e))*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=167

$$\frac{2a^2c^3(11A-B)\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}+\frac{16a^2c^4(11A-B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{64a^2c^5(11A-B)\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{}}{11f}$$

```
[Out] (64*a^2*(11*A - B)*c^5*Cos[e + f*x]^5)/(3465*f*(c - c*Sin[e + f*x])^(5/2))
+ (16*a^2*(11*A - B)*c^4*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^(3/2))
 + (2*a^2*(11*A - B)*c^3*Cos[e + f*x]^5)/(99*f*Sqrt[c - c*Sin[e + f*x]]) -
(2*a^2*B*c^2*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f)
```

Rubi [A] time = 0.452875, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^2c^3(11A-B)\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}+\frac{16a^2c^4(11A-B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{64a^2c^5(11A-B)\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (64*a^2*(11*A - B)*c^5*Cos[e + f*x]^5)/(3465*f*(c - c*Sin[e + f*x])^(5/2))
+ (16*a^2*(11*A - B)*c^4*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^(3/2))
 + (2*a^2*(11*A - B)*c^3*Cos[e + f*x]^5)/(99*f*Sqrt[c - c*Sin[e + f*x]]) -
(2*a^2*B*c^2*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \left(a^2c^2\right)\int \cos^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(\ldots}$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f} + \frac{1}{11}\left(a^2(11\ldots\right.$$

$$= \frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}} - \frac{2a^2Bc^2\cos^5(e+fx)\ldots}{11f}$$

$$= \frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(11A-B)c^3\cos^5\ldots}{99f\sqrt{c-c\sin(e\ldots}}$$

$$= \frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}} + \frac{16a^2(11A-B)c^4\cos\ldots}{693f(c-c\sin(e+\ldots}$$

Mathematica [B] time = 6.55698, size = 1173, normalized size = 7.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((6*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c
- c*Sin[e + f*x])^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*Cos[(5*(e + f*x))/2]*(a
 + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(80*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((2*A + 3*B)
*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(1
12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + ((2*A - B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^4) - (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e +
 f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((6*A - B)*Sin[(e + f*x
)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A
+ B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e + f*x))/2]
)/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4) + ((8*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5
/2)*Sin[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A + 3*B)*(a + a*Sin[e + f*x])^2
*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - B)
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(1
44*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4) + (B*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(
e + f*x))/2])/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)
```

Maple [A] time = 0.951, size = 105, normalized size = 0.6

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{3}a^{2}\left(-315\,B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\left(-1210\,A+1370\,B\right)\sin\left(fx+e\right)\right.}{3465\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/3465*(-1+sin(f*x+e))*c^3*(1+sin(f*x+e))^3*a^2*(-315*B*cos(f*x+e)^2*sin(f
*x+e)+(-1210*A+1370*B)*sin(f*x+e)+(-385*A+980*B)*cos(f*x+e)^2+1562*A-1402*B
)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] time = 1.53081, size = 740, normalized size = 4.43

$$-\frac{2\left(315\,Ba^{2}c^{2}\cos\left(fx+e\right)^{6}-35\,(11\,A-10\,B)a^{2}c^{2}\cos\left(fx+e\right)^{5}+5\,(11\,A-B)a^{2}c^{2}\cos\left(fx+e\right)^{4}-8\,(11\,A-B)a^{2}c^{2}\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*c^2*cos(f*x + e)^6 - 35*(11*A - 10*B)*a^2*c^2*cos(f*x +
e)^5 + 5*(11*A - B)*a^2*c^2*cos(f*x + e)^4 - 8*(11*A - B)*a^2*c^2*cos(f*x +
 e)^3 + 16*(11*A - B)*a^2*c^2*cos(f*x + e)^2 - 64*(11*A - B)*a^2*c^2*cos(f*
x + e) - 128*(11*A - B)*a^2*c^2 - (315*B*a^2*c^2*cos(f*x + e)^5 + 35*(11*A
- B)*a^2*c^2*cos(f*x + e)^4 + 40*(11*A - B)*a^2*c^2*cos(f*x + e)^3 + 48*(11
*A - B)*a^2*c^2*cos(f*x + e)^2 + 64*(11*A - B)*a^2*c^2*cos(f*x + e) + 128*(
11*A - B)*a^2*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
- f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(5/2), x)
```

3.91 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (8*a^2*(9*A + B)*c^4*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (
2*a^2*(9*A + B)*c^3*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) - (2*
a^2*B*c^2*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.386664, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```


```
[Out] (8*a^2*(9*A + B)*c^4*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^(5/2)) + (
2*a^2*(9*A + B)*c^3*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^(3/2)) - (2*
a^2*B*c^2*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}} + \frac{1}{9}\left(a^2(9A+B)c^2\right)\int \frac{\text{co}}{\sqrt{c-}}$$

$$= \frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{2a^2(9A+B)c^3\cos^5(}{63f(c-c\sin(e+f}$$

Mathematica [A] time = 4.64179, size = 106, normalized size = 0.88

$$\frac{a^2c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5((130B-90A)\sin(e+fx)+162A+35B\cos(2(e+fx))-8}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*(
e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*f*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 1.133, size = 83, normalized size = 0.7

$$\frac{\left(-2+2\,\sin\left(fx+e\right)\right)c^2\left(1+\sin\left(fx+e\right)\right)^3a^2\left(\sin\left(fx+e\right)\left(45\,A-65\,B\right)-35\,B\left(\cos\left(fx+e\right)\right)^2-81\,A+61\,B\right)}{315\,f\cos\left(fx+e\right)}\frac{}{\sqrt{c-c\,\mathrm{s}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/315*(-1+sin(f*x+e))*c^2*(1+sin(f*x+e))^3*a^2*(sin(f*x+e)*(45*A-65*B)-35*B
*cos(f*x+e)^2-81*A+61*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] time = 1.43657, size = 575, normalized size = 4.79

$$-\frac{2\left(35\,Ba^2c\cos\left(fx+e\right)^5+5\,(9\,A+8\,B)a^2c\cos\left(fx+e\right)^4-(9\,A+B)a^2c\cos\left(fx+e\right)^3+2\,(9\,A+B)a^2c\cos\left(fx+e\right)^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (9
*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A +
B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4 - 5
*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A
 + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(3/2), x)
```

3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{2a^2c^3(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (2*a^2*(7*A + 3*B)*c^3*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) -
(2*a^2*B*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.330345, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2856, 2673}

$$\frac{2a^2c^3(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^2*(7*A + 3*B)*c^3*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) -
(2*a^2*B*c^2*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}} + \frac{1}{7}\left(a^2(7A+3B)c^2\right)\int \frac{}{(c-}$$

$$= \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^3}$$

Mathematica [A] time = 0.592306, size = 89, normalized size = 1.1

$$\frac{2a^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5(7A+5B\sin(e+fx)-2B)}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(7*A - 2*B + 5*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.865, size = 65, normalized size = 0.8

$$-\frac{\left(-2+2\,\sin\left(fx+e\right)\right)c\left(1+\sin\left(fx+e\right)\right)^3a^2\left(5\,B\sin\left(fx+e\right)+7\,A-2\,B\right)}{35\,f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/35*(-1+sin(f*x+e))*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] time = 1.50575, size = 463, normalized size = 5.72

$$\frac{2\left(5\,Ba^{2}\cos\left(fx+e\right)^{4}-(7\,A+8\,B)a^{2}\cos\left(fx+e\right)^{3}-(21\,A+19\,B)a^{2}\cos\left(fx+e\right)^{2}+2\,(7\,A+3\,B)a^{2}\cos\left(fx+e\right)+4\,(7}{35\left(f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/35*(5*B*a^2*cos(f*x + e)^4 - (7*A + 8*B)*a^2*cos(f*x + e)^3 - (21*A + 19*
B)*a^2*cos(f*x + e)^2 + 2*(7*A + 3*B)*a^2*cos(f*x + e) + 4*(7*A + 3*B)*a^2
- (5*B*a^2*cos(f*x + e)^3 + (7*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(7*A + 3*B)
*a^2*cos(f*x + e) - 4*(7*A + 3*B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int A\sqrt{-c\sin\left(e+fx\right)+c}\,dx+\int 2A\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)dx+\int A\sqrt{-c\sin\left(e+fx\right)+c}\sin^{2}\left(e+fx\right)d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Int
egral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=161

$$-\frac{2a^2c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (4*Sqrt[2]*a^2*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^2*B*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[
e + f*x])^(5/2)) - (2*a^2*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x
])^(3/2)) - (4*a^2*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.442156, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2860, 2679, 2649, 206}

$$-\frac{2a^2c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{4\sqrt{2}a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f}-\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```


```
[Out] (4*Sqrt[2]*a^2*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^2*B*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[
e + f*x])^(5/2)) - (2*a^2*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x
])^(3/2)) - (4*a^2*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(a^2(A+B)c^2\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \left(2a^2(A+B)c\right)\int$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A+B)\cos(}{f\sqrt{c-c\sin(e}}$$

$$= -\frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A+B)\cos(}{f\sqrt{c-c\sin(e}}$$

$$= \frac{4\sqrt{2}a^2(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2}{3f}$$

Mathematica [C] time = 1.194, size = 175, normalized size = 1.09

$$-\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(2(5A+11B)\sin(e+fx)}{15f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*((120 + 12
0*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4]
)] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(70*A + 79*B - 3*B*Cos[2*(e + f*
x)] + 2*(5*A + 11*B)*Sin[e + f*x])))/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 1.164, size = 197, normalized size = 1.2

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)a^2}{15c^3\cos\left(fx+e\right)f}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\left(30c^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)A+30c^{5/2}\sqrt{2}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/15*(-1+sin(f*x+e))*(c*(1+sin(f*x+e)))^(1/2)*a^2*(30*c^(5/2)*2^(1/2)*arct
anh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+30*c^(5/2)*2^(1/2)*arct
anh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-3*B*(c*(1+sin(f*x+e)))^
(5/2)-5*A*(c*(1+sin(f*x+e)))^(3/2)*c-5*B*(c*(1+sin(f*x+e)))^(3/2)*c-30*A*c^
2*(c*(1+sin(f*x+e)))^(1/2)-30*B*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] time = 1.56018, size = 822, normalized size = 5.11

$$2\left(\frac{15\sqrt{2}\left((A+B)a^2c\cos\left(fx+e\right)-(A+B)a^2c\sin\left(fx+e\right)+(A+B)a^2c\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(f\right.}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*sqrt(2)*((A + B)*a^2*c*cos(f*x + e) - (A + B)*a^2*c*sin(f*x + e) +
 (A + B)*a^2*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*
sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c)
 + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2))/sqrt(c) + (3*B*a^2*cos(f*x + e)^3 + (5*A + 14*B)*a^2*cos
(f*x + e)^2 - (35*A + 41*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2 + (3*B*a
^2*cos(f*x + e)^2 - (5*A + 11*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2)*si
n(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e)
 + c*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.97345, size = 771, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/60*(480*sqrt(2)*(A*a^2 + B*a^2)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(-c
)*sgn(tan(1/2*f*x + 1/2*e) - 1)) + ((((((35*A*a^2*c^2*sgn(tan(1/2*f*x + 1/2
*e) - 1) + 38*B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e)
/c^9 + 15*(3*A*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) + 4*B*a^2*c^2*sgn(tan(
1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 10*(8*A*a^2*c^2*sgn(tan(
1/2*f*x + 1/2*e) - 1) + 11*B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*ta
n(1/2*f*x + 1/2*e) + 10*(8*A*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) + 11*B*a
^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 15*(3*A*a
^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) + 4*B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e)
```

```
 - 1))/c^9)*tan(1/2*f*x + 1/2*e) + (35*A*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) -
 1) + 38*B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)/(c*tan(1/2*f*x + 1/2
*e)^2 + c)^(5/2) - 4*(120*sqrt(2)*A*a^2*c^10*arctan(sqrt(c)/sqrt(-c)) + 120
*sqrt(2)*B*a^2*c^10*arctan(sqrt(c)/sqrt(-c)) + 10*sqrt(2)*A*a^2*sqrt(-c)*sq
rt(c) + 13*sqrt(2)*B*a^2*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(s
qrt(-c)*c^10))/f
```

3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{\sqrt{2}a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((Sqrt[2]*a^2*(3*A + 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
 c*Sin[e + f*x]])])/(c^(3/2)*f)) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(2*f*(c
 - c*Sin[e + f*x])^(7/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x]^3)/(6*f*(c - c*Si
n[e + f*x])^(3/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e +
 f*x]])
```

Rubi [A] time = 0.480917, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2679, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{\sqrt{2}a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((Sqrt[2]*a^2*(3*A + 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
 c*Sin[e + f*x]])])/(c^(3/2)*f)) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(2*f*(c
 - c*Sin[e + f*x])^(7/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x]^3)/(6*f*(c - c*Si
n[e + f*x])^(3/2)) + (a^2*(3*A + 7*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e +
 f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} - \frac{1}{4}\left(a^2(3A+7B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} - \frac{1}{2}\left(a^2(3A+7B\right.$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)\,\mathrm{co}}{cf\sqrt{c-c\sin(}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}} + \frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)\,\mathrm{co}}{cf\sqrt{c-c\sin(}}$$

$$= -\frac{\sqrt{2}a^2(3A+7B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}$$

Mathematica [C] time = 0.916248, size = 355, normalized size = 2.02

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(2A+7B)\cos\left(\frac{1}{2}(e+fx)\right)\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*Ar
cTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c -
c*Sin[e + f*x])^(3/2))
```

Maple [A] time = 1.065, size = 282, normalized size = 1.6

$$\frac{a^2}{3 f \cos\left(f x + e\right)} \left(\sin\left(f x + e\right)\left(9 A \sqrt{2}\text{Artanh}\left(1/2 \frac{\sqrt{c + c \sin\left(f x + e\right)}\sqrt{2}}{\sqrt{c}}\right) c^2 - 6 A \sqrt{c + c \sin\left(f x + e\right)} c^{3/2} + 21 B \sqrt{2}\text{A}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/3*a^2*(sin(f*x+e)*(9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*c^2-6*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+21*B*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*(c+c*sin(f*x+e))^(1/2)*c^(3
/2)-2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))-9*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+12*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-21*B*2
^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+24*B*(c+c*si
n(f*x+e))^(1/2)*c^(3/2)+2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))*(c*(1+sin(f*x+e
)))^(1/2)/c^(7/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \sin\left(f x + e\right) + A\right)\left(a \sin\left(f x + e\right) + a\right)^2}{\left(-c \sin\left(f x + e\right) + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] time = 1.52835, size = 988, normalized size = 5.61

$$\frac{3 \sqrt{2}\left((3 A+7 B)a^2 c \cos\left(fx+e\right)^2-(3 A+7 B)a^2 c \cos\left(fx+e\right)-2 (3 A+7 B)a^2 c+\left((3 A+7 B)a^2 c \cos\left(fx+e\right)+2 (3 A+7 B)a^2 c\right) \sin\left(fx+e\right)\right) \log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right.}{\cdots}\right.}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((3*A + 7*B)*a^2*c*cos(f*x + e)^2 - (3*A + 7*B)*a^2*c*cos(f*
x + e) - 2*(3*A + 7*B)*a^2*c + ((3*A + 7*B)*a^2*c*cos(f*x + e) + 2*(3*A + 7
*B)*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x
+ e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1
)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(B*a^2*cos(f*x + e)^3 + (3*A + 10*B
)*a^2*cos(f*x + e)^2 + 6*(A + 2*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2 + (B*a^
2*cos(f*x + e)^2 - 3*(A + 3*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2)*sin(f*x +
e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) -
 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=175

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}+\frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (3*a^2*(A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(4*Sqrt[2]*c^(5/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(4*f*(c
- c*Sin[e + f*x])^(9/2)) - (a^2*(A + 9*B)*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e
 + f*x])^(5/2)) - (3*a^2*(A + 9*B)*Cos[e + f*x])/(8*c^2*f*Sqrt[c - c*Sin[e
+ f*x]])
```

Rubi [A] time = 0.478418, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}+\frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f}-\frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (3*a^2*(A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
 f*x]])])/(4*Sqrt[2]*c^(5/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(4*f*(c
- c*Sin[e + f*x])^(9/2)) - (a^2*(A + 9*B)*Cos[e + f*x]^3)/(8*f*(c - c*Sin[e
 + f*x])^(5/2)) - (3*a^2*(A + 9*B)*Cos[e + f*x])/(8*c^2*f*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{1}{8}\left(a^2(A+9B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,d$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} + \frac{\left(3a^2(A+9B)\right)}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2(A+9B)\,\mathrm{co}}{8c^2f\sqrt{c-c\sin}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2(A+9B)\,\mathrm{co}}{8c^2f\sqrt{c-c\sin}}$$

$$= \frac{3a^2(A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2}{8}$$

Mathematica [C] time = 1.20048, size = 344, normalized size = 1.97

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-(5A+13B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (
3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*(5*A
+ 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 8*B*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] time = 1.443, size = 386, normalized size = 2.2

$$-\frac{a^2}{\left(-8+8\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(3\,A\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)\left(\sin\left(fx+e\right)\right)^2c^2+27\,B\sqrt{2}\mathrm{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/c^(9/2)*a^2*(3*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)^2*c^2+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)
*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-6*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2-16*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3
/2)*sin(f*x+e)^2-54*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/
c^(1/2))*sin(f*x+e)*c^2+10*A*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+3*A*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2+26*B*(c*(1+sin(f*x
+e)))^(3/2)*c^(1/2)+32*B*c^(3/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)+27*B*2
^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-12*A*(c*(1
+sin(f*x+e)))^(1/2)*c^(3/2)-60*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2))*(c*(1+si
n(f*x+e)))^(1/2)/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^2}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] time = 1.53259, size = 1143, normalized size = 6.53

$$\frac{3\sqrt{2}\Big((A+9B)a^2\cos\big(fx+e\big)^3+3(A+9B)a^2\cos\big(fx+e\big)^2-2(A+9B)a^2\cos\big(fx+e\big)-4(A+9B)a^2-\Big((A+9B)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x + e
)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos(f
*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e))*s
qrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)
*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2
*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) -
 cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*cos(f*x
+ e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos(f*x + e
)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*
sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*
c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=175

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}-\frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx)}$$

```
[Out] (a^2*(A - 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(16*Sqrt[2]*c^(7/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(6*f*(c
- c*Sin[e + f*x])^(11/2)) + (a^2*(A - 11*B)*Cos[e + f*x]^3)/(24*f*(c - c*Si
n[e + f*x])^(7/2)) - (a^2*(A - 11*B)*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e +
 f*x])^(3/2))
```

Rubi [A] time = 0.491015, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2680, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}-\frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}+\frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*(A - 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/(16*Sqrt[2]*c^(7/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(6*f*(c
- c*Sin[e + f*x])^(11/2)) + (a^2*(A - 11*B)*Cos[e + f*x]^3)/(24*f*(c - c*Si
n[e + f*x])^(7/2)) - (a^2*(A - 11*B)*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e +
 f*x])^(3/2))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{1}{12}\left(a^2(A-11B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{\left(a^2(A-11B)\right)}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)}{16c^2f(c-c\sin}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)}{16c^2f(c-c\sin}$$

$$= \frac{a^2(A-11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2}{2}$$

Mathematica [C] time = 1.83863, size = 342, normalized size = 1.95

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(A+21B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(7*A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
+ 3*(A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^(1/
4)*(A - 11*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(7*A + 1
9*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(A + 21*B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*
x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(
7/2))
```

Maple [B] time = 1.628, size = 354, normalized size = 2.

$$-\frac{a^{2}}{96\left(-1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\left(-3\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}c^{3}\left(A-11\,B\right)\sin\left(fx+e\right)\left(\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/96*a^2*(-3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c
^3*(A-11*B)*sin(f*x+e)*cos(f*x+e)^2+12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)/c^(1/2))*2^(1/2)*c^3*(A-11*B)*sin(f*x+e)+9*arctanh(1/2*(c+c*sin(f*x+
e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3*(A-11*B)*cos(f*x+e)^2+24*A*(c+c*sin(
f*x+e))^(1/2)*c^(5/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-6*A*(c+c*sin(f*x+
e))^(5/2)*c^(1/2)-264*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+352*B*(c+c*sin(f*x+e
))^(3/2)*c^(3/2)-126*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-12*A*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+132*B*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+e)))^(1/2)/c^(13/
2)/(-1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [B] time = 1.59873, size = 1353, normalized size = 7.73

$$-\frac{3\sqrt{2}\left(\left(A-11\,B\right)a^{2}\cos\left(fx+e\right)^{4}-3\left(A-11\,B\right)a^{2}\cos\left(fx+e\right)^{3}-8\left(A-11\,B\right)a^{2}\cos\left(fx+e\right)^{2}+4\left(A-11\,B\right)a^{2}\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*x
 + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e) +
 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*cos(f
*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x + e))
*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(
c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
 - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A + 13*B)*
a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*a^2 + (3*
(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e) - 32*(A +
B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*
c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^
4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e)
 - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=222

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos}{\sqrt{2}\sqrt{c-c\,s}}\right.}{256\sqrt{2}c^{9/2}f}$$

```
[Out] (a^2*(3*A - 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(256*Sqrt[2]*c^(9/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(8*f*
(c - c*Sin[e + f*x])^(13/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x]^3)/(48*f*(c -
 c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 13*B)*Cos[e + f*x])/(64*c^2*f*(c - c*
Sin[e + f*x])^(5/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x])/(256*c^3*f*(c - c*Si
n[e + f*x])^(3/2))
```

Rubi [A] time = 0.51109, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2650, 2649, 206}

$$\frac{a^2c^2(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}+\frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}-\frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos}{\sqrt{2}\sqrt{c-c\,s}}\right.}{256\sqrt{2}c^{9/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (a^2*(3*A - 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/(256*Sqrt[2]*c^(9/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(8*f*
(c - c*Sin[e + f*x])^(13/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x]^3)/(48*f*(c -
 c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 13*B)*Cos[e + f*x])/(64*c^2*f*(c - c*
Sin[e + f*x])^(5/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x])/(256*c^3*f*(c - c*Si
n[e + f*x])^(3/2))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^2c^2\right)\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{1}{16}\left(a^2(3A-13B)c\right)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{1}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{\left(a^2(3A-13\right.}{}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-13B}{64c^2f(c-c\,\mathrm{s}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-13B}{64c^2f(c-c\,\mathrm{s}}$$

$$= \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-13B}{64c^2f(c-c\,\mathrm{s}}$$

$$= \frac{a^2(3A-13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} +$$

<u>Mathematica</u> [C] time = 2.67365, size = 357, normalized size = 1.61

$$\frac{a^2(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-24-24i)\sqrt[4]{-1}(3A-13B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+f\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Cos
[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 791*
B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f*x)
)/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*I)*(
-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] + 1517*
B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e + f*x))/2
] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Sin[(7*(e
+ f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2] + Sin[(e
```

```
 + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] time = 1.628, size = 440, normalized size = 2.

$$\frac{a^{2}}{1536\left(-1+\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\left(-12\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)c^{5}\left(3\,A-13\,B\right)\sin\left(fx+e\right)\left(\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/1536/c^(19/2)*a^2*(-12*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*c^5*(3*A-13*B)*sin(f*x+e)*cos(f*x+e)^2+24*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5*(3*A-13*B)*sin(f*x+e)-3*2^(1/2)*ar
ctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5*(3*A-13*B)*cos(f*x+e)
^4+24*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5*(3*A-
13*B)*cos(f*x+e)^2+144*A*(c+c*sin(f*x+e))^(1/2)*c^(9/2)-264*A*(c+c*sin(f*x+
e))^(3/2)*c^(7/2)-132*A*(c+c*sin(f*x+e))^(5/2)*c^(5/2)+18*A*(c+c*sin(f*x+e)
)^(7/2)*c^(3/2)-624*B*(c+c*sin(f*x+e))^(1/2)*c^(9/2)+1144*B*(c+c*sin(f*x+e)
)^(3/2)*c^(7/2)-452*B*(c+c*sin(f*x+e))^(5/2)*c^(5/2)-78*B*(c+c*sin(f*x+e))^
(7/2)*c^(3/2)-72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^5+312*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c
^5)*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))^3/cos(f*x+e)/(c-c*sin(f*x+e))^
(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)
^(9/2), x)
```

Fricas [B] time = 1.63159, size = 1670, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/3072*(3*sqrt(2)*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*co
s(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*cos(
f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3*A
- 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*A -
13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B
)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*cos(f*x +
 e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2*cos(f*x +
 e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3*(3*A - 13*
B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 12*(19*A + 3*
B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
 c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^
3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(
f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos
(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.21462, size = 2141, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] 1/768*(3*sqrt(2)*(3*A*a^2 - 13*B*a^2)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqr
t(-c)*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 2*(1527*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^15*A*a^2 + 39*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^15*B*a^2 - 4473*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^14*A*a^2*sqrt(c
) + 2487*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^14*B*a^2*sqrt(c) + 22233*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))^13*A*a^2*c + 7593*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^13*B*a^2*c - 23811*(sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^12*A*a^2*c^(3/2) + 1293*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^12*B*a^2*c^(3/2
) - 2133*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^11*A*a^2*c^2 + 1563*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^11*B*a^2*c^2 + 68019*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*
tan(1/2*f*x + 1/2*e)^2 + c))^10*A*a^2*c^(5/2) - 10589*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^10*B*a^2*c^(5/2) - 25371*(sq
rt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*a^2*c^
3 - 9355*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^9*B*a^2*c^3 - 71487*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^8*A*a^2*c^(7/2) - 3055*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^8*B*a^2*c^(7/2) + 25173*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*a^2*c^4 - 7195*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*B*a^2*c^4 + 5
6469*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*
A*a^2*c^(9/2) + 15909*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^6*B*a^2*c^(9/2) + 10971*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*a^2*c^5 + 2123*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*a^2*c^5 - 31881*(sqrt(c)*ta
n(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*a^2*c^(11/2) -
 3673*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4
*B*a^2*c^(11/2) - 17079*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^3*A*a^2*c^6 + 5913*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*
tan(1/2*f*x + 1/2*e)^2 + c))^3*B*a^2*c^6 - 7695*(sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*a^2*c^(13/2) - 3007*(sqrt(c)*t
```

```
an(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*a^2*c^(13/2)
- 345*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A
*a^2*c^7 - 41*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
 + c))*B*a^2*c^7 - 117*A*a^2*c^(15/2) - 5*B*a^2*c^(15/2))/(((sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 - 2*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^8*c^4*sgn(
tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.98 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=210

$$\frac{2a^3c^4(15A-B)\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}+\frac{8a^3c^5(15A-B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{64a^3c^6(15A-B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{256a^3c^7(15A-B)\cos^7(}{45045f(c-c\sin(e+f}$$

```
[Out] (256*a^3*(15*A - B)*c^7*Cos[e + f*x]^7)/(45045*f*(c - c*Sin[e + f*x])^(7/2)
) + (64*a^3*(15*A - B)*c^6*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^(5/
2)) + (8*a^3*(15*A - B)*c^5*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^(3/
2)) + (2*a^3*(15*A - B)*c^4*Cos[e + f*x]^7)/(195*f*Sqrt[c - c*Sin[e + f*x]]
) - (2*a^3*B*c^3*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f)
```

Rubi [A] time = 0.534393, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(15A-B)\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}+\frac{8a^3c^5(15A-B)\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{64a^3c^6(15A-B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{256a^3c^7(15A-B)\cos^7(}{45045f(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (256*a^3*(15*A - B)*c^7*Cos[e + f*x]^7)/(45045*f*(c - c*Sin[e + f*x])^(7/2)
) + (64*a^3*(15*A - B)*c^6*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^(5/
2)) + (8*a^3*(15*A - B)*c^5*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^(3/
2)) + (2*a^3*(15*A - B)*c^4*Cos[e + f*x]^7)/(195*f*Sqrt[c - c*Sin[e + f*x]]
) - (2*a^3*B*c^3*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}+\frac{1}{15}\left(a^3(1\right.$$

$$= \frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}-\frac{2a^3Bc^3\cos^7(e+fx}{15}$$

$$= \frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}+\frac{2a^3(15A-B)c^4\cos}{195f\sqrt{c-c\sin(e}}$$

$$= \frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3(15A-B)c^5\cos}{715f(c-c\sin(e}$$

$$= \frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}+\frac{64a^3(15A-B)c^6}{6435f(c-c\sin}$$

Mathematica [B] time = 6.88615, size = 1569, normalized size = 7.47

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(
7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e + f*
x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (3
*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x
])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)*Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(704*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(6*A + B)*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(19
2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6) + (3*(10*A - 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/
2)*Sin[(5*(e + f*x))/2])/(320*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(4*A + 3*B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(7*(e + f*x))/2])/(448*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((12*A -
 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(9*(e + f*x))/2
])/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^6) + ((2*A + 5*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^
(7/2)*Sin[(11*(e + f*x))/2])/(704*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])
^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(13*(e + f*x))/2])/(832*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*(a +
```

```
 a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(15*(e + f*x))/2])/(960*f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^6)
```

Maple [A] time = 1.05, size = 121, normalized size = 0.6

$$\frac{\left(-2+2\,\sin\left(fx+e\right)\right)c^{4}\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(\left(-3465\,A+12243\,B\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}+\left(24780\,A-25676\,B\right.\right.}{45045\,f\cos\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/45045*(-1+sin(f*x+e))*c^4*(1+sin(f*x+e))^4*a^3*((-3465*A+12243*B)*sin(f*x
+e)*cos(f*x+e)^2+(24780*A-25676*B)*sin(f*x+e)+3003*B*cos(f*x+e)^4+(14175*A-
24969*B)*cos(f*x+e)^2-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [B] time = 1.52881, size = 972, normalized size = 4.63

$$-\frac{2\left(3003\,Ba^{3}c^{3}\cos\left(fx+e\right)^{8}-231\left(15\,A-14\,B\right)a^{3}c^{3}\cos\left(fx+e\right)^{7}+21\left(15\,A-B\right)a^{3}c^{3}\cos\left(fx+e\right)^{6}-28\left(15\,A-B\right)a^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -2/45045*(3003*B*a^3*c^3*cos(f*x + e)^8 - 231*(15*A - 14*B)*a^3*c^3*cos(f*x
 + e)^7 + 21*(15*A - B)*a^3*c^3*cos(f*x + e)^6 - 28*(15*A - B)*a^3*c^3*cos(
f*x + e)^5 + 40*(15*A - B)*a^3*c^3*cos(f*x + e)^4 - 64*(15*A - B)*a^3*c^3*c
os(f*x + e)^3 + 128*(15*A - B)*a^3*c^3*cos(f*x + e)^2 - 512*(15*A - B)*a^3*
c^3*cos(f*x + e) - 1024*(15*A - B)*a^3*c^3 - (3003*B*a^3*c^3*cos(f*x + e)^7
 + 231*(15*A - B)*a^3*c^3*cos(f*x + e)^6 + 252*(15*A - B)*a^3*c^3*cos(f*x +
 e)^5 + 280*(15*A - B)*a^3*c^3*cos(f*x + e)^4 + 320*(15*A - B)*a^3*c^3*cos(
f*x + e)^3 + 384*(15*A - B)*a^3*c^3*cos(f*x + e)^2 + 512*(15*A - B)*a^3*c^3
*cos(f*x + e) + 1024*(15*A - B)*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(7/2), x)
```

3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=161

$$\frac{2a^3c^4(13A+B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{16a^3c^5(13A+B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{64a^3c^6(13A+B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (64*a^3*(13*A + B)*c^6*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^(7/2))
+ (16*a^3*(13*A + B)*c^5*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^(5/2)
) + (2*a^3*(13*A + B)*c^4*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)
) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.471644, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(13A+B)\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}+\frac{16a^3c^5(13A+B)\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{64a^3c^6(13A+B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```


```
[Out] (64*a^3*(13*A + B)*c^6*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^(7/2))
+ (16*a^3*(13*A + B)*c^5*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^(5/2)
) + (2*a^3*(13*A + B)*c^4*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)
) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx &= \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx\\
&= -\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}+\frac{1}{13}\left(a^3(13A+B)c^3\right)\int \frac{}{\sqrt{c}}\\
&= \frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx}}\\
&= \frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}}+\frac{2a^3(13A+B)c^4\cos}{143f(c-c\sin(e+}\\
&= \frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}}+\frac{16a^3(13A+B)c^5\cos}{1287f(c-c\sin(e}\end{aligned}$$

Mathematica [B] time = 6.71864, size = 1351, normalized size = 8.39

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (5*A*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^6) - (5*(4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*
Sin[e + f*x])^(5/2))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*Cos[(5*(e + f*x))/2]*(a + a*S
in[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((5*A + 2*B)*Cos[(
7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(112*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^6) + ((A - 2*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e
+ f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6) - ((2*A + B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[e
 + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(13*(e + f*x))
/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(416*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*A*
Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6) + (5*(4*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(3
*(e + f*x))/2])/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[
e + f*x])^(5/2)*Sin[(5*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((5*A + 2*B)*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((A - 2*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e +
f*x))/2])/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((2*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f
*x])^(5/2)*Sin[(11*(e + f*x))/2])/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*(a + a*Sin[e + f*x])^3*
(c - c*Sin[e + f*x])^(5/2)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [A] time = 0.887, size = 105, normalized size = 0.7

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c^{3}\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(-693B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+\left(-2366A+2590B\right)\sin\left(fx+e\right)\right.}{9009f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -2/9009*(-1+sin(f*x+e))*c^3*(1+sin(f*x+e))^4*a^3*(-693*B*cos(f*x+e)^2*sin(f
*x+e)+(-2366*A+2590*B)*sin(f*x+e)+(-819*A+2016*B)*cos(f*x+e)^2+2782*A-2558*
B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] time = 1.53978, size = 849, normalized size = 5.27

$$-\frac{2\left(693Ba^{3}c^{2}\cos\left(fx+e\right)^{7}+63\left(13A+12B\right)a^{3}c^{2}\cos\left(fx+e\right)^{6}-7\left(13A+B\right)a^{3}c^{2}\cos\left(fx+e\right)^{5}+10\left(13A+B\right)a^{3}c^{2}\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -2/9009*(693*B*a^3*c^2*cos(f*x + e)^7 + 63*(13*A + 12*B)*a^3*c^2*cos(f*x +
e)^6 - 7*(13*A + B)*a^3*c^2*cos(f*x + e)^5 + 10*(13*A + B)*a^3*c^2*cos(f*x
```

```
+ e)^4 - 16*(13*A + B)*a^3*c^2*cos(f*x + e)^3 + 32*(13*A + B)*a^3*c^2*cos(f
*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^2 +
(693*B*a^3*c^2*cos(f*x + e)^6 - 63*(13*A + B)*a^3*c^2*cos(f*x + e)^5 - 70*(
13*A + B)*a^3*c^2*cos(f*x + e)^4 - 80*(13*A + B)*a^3*c^2*cos(f*x + e)^3 - 9
6*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) -
 256*(13*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x
 + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{2a^3c^4(11A+3B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5(11A+3B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (8*a^3*(11*A + 3*B)*c^5*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2))
+ (2*a^3*(11*A + 3*B)*c^4*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2))
 - (2*a^3*B*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.407329, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(11A+3B)\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3c^5(11A+3B)\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*a^3*(11*A + 3*B)*c^5*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2))
+ (2*a^3*(11*A + 3*B)*c^4*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2))
 - (2*a^3*B*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}} + \frac{1}{11}\left(a^3(11A+3B)c^3\right)\int$$
$$= \frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}} - \frac{2a^3Bc^3\cos^7(e+}{11f(c-c\sin(e+}$$
$$= \frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}} + \frac{2a^3(11A+3B)c^4}{99f(c-c\sin(e}$$

Mathematica [B] time = 6.52733, size = 1157, normalized size = 9.33

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/
2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Cos[(7*(e
 + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
 - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e +
f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3
*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((8*A + 3*B)*(
a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e + f*x))/2])/(24*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(5*(e +
f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6) + ((6*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(3/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 3*B)*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(11*(e + f*x))/2])/(1
76*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6)
```

Maple [A] time = 0.946, size = 83, normalized size = 0.7

$$\frac{\left(-2+2\sin\left(fx+e\right)\right)c^{2}\left(1+\sin\left(fx+e\right)\right)^{4}a^{3}\left(\sin\left(fx+e\right)(77A-105B)-63B\left(\cos\left(fx+e\right)\right)^{2}-121A+93B\right)}{693f\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/693*(-1+sin(f*x+e))*c^2*(1+sin(f*x+e))^4*a^3*(sin(f*x+e)*(77*A-105*B)-63*
B*cos(f*x+e)^2-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] time = 1.54666, size = 722, normalized size = 5.82

$$\frac{2\left(63\,Ba^{3}c\cos\left(fx+e\right)^{6}-7\,(11\,A+12\,B)a^{3}c\cos\left(fx+e\right)^{5}-(187\,A+177\,B)a^{3}c\cos\left(fx+e\right)^{4}+2\,(11\,A+3\,B)a^{3}c\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 2/693*(63*B*a^3*c*cos(f*x + e)^6 - 7*(11*A + 12*B)*a^3*c*cos(f*x + e)^5 - (
187*A + 177*B)*a^3*c*cos(f*x + e)^4 + 2*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 -
 4*(11*A + 3*B)*a^3*c*cos(f*x + e)^2 + 16*(11*A + 3*B)*a^3*c*cos(f*x + e) +
 32*(11*A + 3*B)*a^3*c - (63*B*a^3*c*cos(f*x + e)^5 + 7*(11*A + 21*B)*a^3*c
*cos(f*x + e)^4 - 10*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 - 12*(11*A + 3*B)*a^
3*c*cos(f*x + e)^2 - 16*(11*A + 3*B)*a^3*c*cos(f*x + e) - 32*(11*A + 3*B)*a
^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x +
 e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)
^(3/2), x)
```

3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{2a^3c^4(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (2*a^3*(9*A + 5*B)*c^4*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) -
(2*a^3*B*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.304981, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2856, 2673}

$$\frac{2a^3c^4(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*a^3*(9*A + 5*B)*c^4*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) -
(2*a^3*B*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2856

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$
$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}} + \frac{1}{9}\left(a^3(9A+5B)c^3\right)\int \frac{\mathrm{c}}{(c-\mathrm{c}}$$
$$= \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 1.04113, size = 89, normalized size = 1.1

$$\frac{2a^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7(9A+7B\sin(e+fx)-2B)}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(9*A - 2*B + 7*B*Sin[e + f*x
])*Sqrt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [A] time = 0.968, size = 65, normalized size = 0.8

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)c\left(1+\sin\left(fx+e\right)\right)^4a^3\left(7B\sin\left(fx+e\right)+9A-2B\right)}{63f\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/63*(-1+sin(f*x+e))*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [B] time = 1.47145, size = 563, normalized size = 6.95

$$\frac{2\left(7\,Ba^{3}\cos\left(fx+e\right)^{5}+\left(9\,A+26\,B\right)a^{3}\cos\left(fx+e\right)^{4}-\left(27\,A+29\,B\right)a^{3}\cos\left(fx+e\right)^{3}-4\left(18\,A+17\,B\right)a^{3}\cos\left(fx+e\right)^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/63*(7*B*a^3*cos(f*x + e)^5 + (9*A + 26*B)*a^3*cos(f*x + e)^4 - (27*A + 29
*B)*a^3*cos(f*x + e)^3 - 4*(18*A + 17*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)
*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3 + (7*B*a^3*cos(f*x + e)^4 - (9*A + 19
*B)*a^3*cos(f*x + e)^3 - 12*(3*A + 4*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*B)*
a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=200

$$-\frac{2a^3c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+\dots}{\sqrt{2}\sqrt{c-c\sin\dots}}\right.}{\sqrt{c}f}$$

```
[Out] (8*Sqrt[2]*a^3*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[
e + f*x])^(7/2)) - (2*a^3*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f
*x])^(5/2)) - (4*a^3*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (8*a^3*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.521297, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2860, 2679, 2649, 206}

$$-\frac{2a^3c^2(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}-\frac{4a^3c(A+B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}}-\frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}+\frac{8\sqrt{2}a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+\dots}{\sqrt{2}\sqrt{c-c\sin\dots}}\right.}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (8*Sqrt[2]*a^3*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*S
in[e + f*x]])])/(Sqrt[c]*f) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(7*f*(c - c*Sin[
e + f*x])^(7/2)) - (2*a^3*(A + B)*c^2*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f
*x])^(5/2)) - (4*a^3*(A + B)*c*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3
/2)) - (8*a^3*(A + B)*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} + \left(a^3(A+B)c^3\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} + \left(2a^3(A+B)c\right.$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A+B)c}{3f(c-c\sin}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A+B)c}{3f(c-c\sin}$$

$$= -\frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A+B)c}{3f(c-c\sin}$$

$$= \frac{8\sqrt{2}a^3(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - 2a$$

<u>Mathematica</u> [C] time = 1.41376, size = 193, normalized size = 0.96

$$-\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(-(448A+673B)\sin}{420f\sqrt{c-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((6720 + 6
720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B + 6*(7*A +
 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin[3*(e + f*
x)])))/(420*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin[e + f*
x]])
```

Maple [A] time = 1.292, size = 233, normalized size = 1.2

$$-\frac{\left(-2+2\sin\left(fx+e\right)\right)a^{3}}{105\,c^{4}\cos\left(fx+e\right)f}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\left(420\,c^{7/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{c}}\right)A+420\,c^{7/2}\sqrt{2}\mathrm{Arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/105*(-1+sin(f*x+e))*(c*(1+sin(f*x+e)))^(1/2)*a^3*(420*c^(7/2)*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+420*c^(7/2)*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-15*B*(c*(1+sin(f*x+e
)))^(7/2)-21*A*(c*(1+sin(f*x+e)))^(5/2)*c-21*B*(c*(1+sin(f*x+e)))^(5/2)*c-7
0*A*(c*(1+sin(f*x+e)))^(3/2)*c^2-70*B*(c*(1+sin(f*x+e)))^(3/2)*c^2-420*A*c^
3*(c*(1+sin(f*x+e)))^(1/2)-420*B*c^3*(c*(1+sin(f*x+e)))^(1/2))/c^4/cos(f*x+
e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [A] time = 1.48203, size = 944, normalized size = 4.72

$$2\left(\frac{210\sqrt{2}\left((A+B)a^3c\cos\left(fx+e\right)-(A+B)a^3c\sin\left(fx+e\right)+(A+B)a^3c\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos(\cdots}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{\sqrt{c}}\right.\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(210*sqrt(2)*((A + B)*a^3*c*cos(f*x + e) - (A + B)*a^3*c*sin(f*x + e)
 + (A + B)*a^3*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(
c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2))/sqrt(c) - (15*B*a^3*cos(f*x + e)^4 - 3*(7*A + 22*B)*a^
3*cos(f*x + e)^3 - (133*A + 253*B)*a^3*cos(f*x + e)^2 + 4*(133*A + 148*B)*a
^3*cos(f*x + e) + 4*(161*A + 191*B)*a^3 - (15*B*a^3*cos(f*x + e)^3 + 3*(7*A
 + 27*B)*a^3*cos(f*x + e)^2 - 4*(28*A + 43*B)*a^3*cos(f*x + e) - 4*(161*A +
 191*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c
*f*sin(f*x + e) + c*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.04635, size = 938, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/105*(1680*sqrt(2)*(A*a^3 + B*a^3)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*
x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqrt(
-c)*sgn(tan(1/2*f*x + 1/2*e) - 1)) + ((((((((511*A*a^3*c^3*sgn(tan(1/2*f*x
+ 1/2*e) - 1) + 526*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x +
1/2*e)/c^12 + 105*(7*A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 8*B*a^3*c^3*
sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 7*(263*A*a^3*c^
3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 308*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) -
1))/c^12)*tan(1/2*f*x + 1/2*e) + 35*(59*A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e)
- 1) + 74*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*
e) + 35*(59*A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 74*B*a^3*c^3*sgn(tan(
1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 7*(263*A*a^3*c^3*sgn(ta
n(1/2*f*x + 1/2*e) - 1) + 308*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12
)*tan(1/2*f*x + 1/2*e) + 105*(7*A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 8
*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + (511
*A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 526*B*a^3*c^3*sgn(tan(1/2*f*x +
1/2*e) - 1))/c^12)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(7/2) - 4*(420*sqrt(2)*A*
a^3*c^13*arctan(sqrt(c)/sqrt(-c)) + 420*sqrt(2)*B*a^3*c^13*arctan(sqrt(c)/s
qrt(-c)) + 161*sqrt(2)*A*a^3*sqrt(-c)*sqrt(c) + 191*sqrt(2)*B*a^3*sqrt(-c)*
sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(-c)*c^13))/f
```

3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=218

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{2\sqrt{2}a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c(5A+9B)\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}+\frac{a^3(5A+9B)\cos}{3f(c-c\sin(e}$$

```
[Out] (-2*Sqrt[2]*a^3*(5*A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c
- c*Sin[e + f*x]])])/(c^(3/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(2*f*(c
 - c*Sin[e + f*x])^(9/2)) + (a^3*(5*A + 9*B)*c*Cos[e + f*x]^5)/(10*f*(c - c
*Sin[e + f*x])^(5/2)) + (a^3*(5*A + 9*B)*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e
+ f*x])^(3/2)) + (2*a^3*(5*A + 9*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f
*x]])
```

Rubi [A] time = 0.546137, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2679, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{2\sqrt{2}a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^3c(5A+9B)\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}}+\frac{a^3(5A+9B)\cos}{3f(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```



```
[Out] (-2*Sqrt[2]*a^3*(5*A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c
- c*Sin[e + f*x]])])/(c^(3/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(2*f*(c
 - c*Sin[e + f*x])^(9/2)) + (a^3*(5*A + 9*B)*c*Cos[e + f*x]^5)/(10*f*(c - c
*Sin[e + f*x])^(5/2)) + (a^3*(5*A + 9*B)*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e
+ f*x])^(3/2)) + (2*a^3*(5*A + 9*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f
*x]])
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} - \frac{1}{4}\left(a^3(5A+9B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} - \frac{1}{2}\left(a^3(5A+9\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)}{3f(c-c\sin}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)}{3f(c-c\sin}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)}{3f(c-c\sin}$$

$$= -\frac{2\sqrt{2}a^3(5A+9B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx}{2f(c-c\sin(e+fx))^{9/2}}$$

Mathematica [C] time = 1.74262, size = 444, normalized size = 2.04

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(240(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+30(9A+20B)\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A + 9
*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 5*(2*A + 9*B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2] - 3*B*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^2*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/2] + Sin[(
```

```
e + f*x)/2])^6*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] time = 1.217, size = 354, normalized size = 1.6

$$\frac{2a^3}{15f\cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\left(-60A\sqrt{c+c\sin\left(fx+e\right)}c^{5/2}-5A\left(c+c\sin\left(fx+e\right)\right)^{3/2}c^{3/2}-120B\sqrt{c+c\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 2/15*a^3*(sin(f*x+e)*(-60*A*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-5*A*(c+c*sin(f*x
+e))^(3/2)*c^(3/2)-120*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)-15*B*(c+c*sin(f*x+e
))^(3/2)*c^(3/2)-3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)+75*A*2^(1/2)*arctanh(1/
2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3+135*B*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)+90*A*(c+c*sin(f*x+e))^(1/2)*c^(5/
2)+5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+150*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+
15*B*(c+c*sin(f*x+e))^(3/2)*c^(3/2)+3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-75*A
*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3-135*B*2^(1
/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3)*(c*(1+sin(f*x+
e)))^(1/2)/c^(9/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [B] time = 1.53495, size = 1099, normalized size = 5.04

$$\frac{15\sqrt{2}\left((5A+9B)a^3c\cos\left(fx+e\right)^2-(5A+9B)a^3c\cos\left(fx+e\right)-2(5A+9B)a^3c+\left((5A+9B)a^3c\cos\left(fx+e\right)+2(5A+9B)a^3c\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)\cdots\right.}{\cdots}\right.}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*((5*A + 9*B)*a^3*c*cos(f*x + e)^2 - (5*A + 9*B)*a^3*c*cos(
f*x + e) - 2*(5*A + 9*B)*a^3*c + ((5*A + 9*B)*a^3*c*cos(f*x + e) + 2*(5*A +
 9*B)*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*
x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) +
 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2))/sqrt(c) + 2*(3*B*a^3*cos(f*x + e)^4 - (5*A +
18*B)*a^3*cos(f*x + e)^3 - (65*A + 141*B)*a^3*cos(f*x + e)^2 - 30*(3*A + 5*
B)*a^3*cos(f*x + e) - 30*(A + B)*a^3 - (3*B*a^3*cos(f*x + e)^3 + (5*A + 21*
B)*a^3*cos(f*x + e)^2 - 60*(A + 2*B)*a^3*cos(f*x + e) + 30*(A + B)*a^3)*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x
 + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}-\frac{a^3c(3A+11B)\cos\cdots}{8f(c-c\sin(e+\cdots}$$

```
[Out] (5*a^3*(3*A + 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[
e + f*x]])])/(2*Sqrt[2]*c^(5/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(4*f*
(c - c*Sin[e + f*x])^(11/2)) - (a^3*(3*A + 11*B)*c*Cos[e + f*x]^5)/(8*f*(c
- c*Sin[e + f*x])^(7/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x]^3)/(24*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x])/(4*c^2*f*Sqrt[c
 - c*Sin[e + f*x]])
```

Rubi [A] time = 0.548725, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f}-\frac{a^3c(3A+11B)\cos\cdots}{8f(c-c\sin(e+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (5*a^3*(3*A + 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[
e + f*x]])])/(2*Sqrt[2]*c^(5/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(4*f*
(c - c*Sin[e + f*x])^(11/2)) - (a^3*(3*A + 11*B)*c*Cos[e + f*x]^5)/(8*f*(c
- c*Sin[e + f*x])^(7/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x]^3)/(24*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x])/(4*c^2*f*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{1}{8}\left(a^3(3A+11B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} + \frac{1}{16}\left(5a^3(3A\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+1}{24cf(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+1}{24cf(c-c}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+1}{24cf(c-c}$$

$$= \frac{5a^3(3A+11B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} -$$

Mathematica [C] time = 2.30759, size = 434, normalized size = 1.93

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\sin\left(\frac{1}{2}(e+fx)\right)-6(2A+11B)\cos\left(\frac{1}{2}(e+fx)\right)\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(12*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 3*(9*A + 17*B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(3*A + 11*B)*ArcTan[(1/2 + I/
2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - 6*(2*A + 11*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4 + 2*B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 24*(
A + B)*Sin[(e + f*x)/2] - 6*(9*A + 17*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^2*Sin[(e + f*x)/2] - 6*(2*A + 11*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
```

```
])^4*Sin[(e + f*x)/2] - 2*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(3*
(e + f*x))/2]))/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e +
 f*x])^(5/2))
```

Maple [B] time = 1.602, size = 434, normalized size = 1.9

$$-\frac{a^3}{\left(-12+12\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\left(-90\,A\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)c^2+48\,A\sqrt{c+c\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/12/c^(9/2)*a^3*(sin(f*x+e)*(-90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(
1/2)*2^(1/2)/c^(1/2))*c^2+48*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-330*B*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+16*B*(c+c*sin(f*x+
e))^(3/2)*c^(1/2)+240*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2))+(-45*A*2^(1/2)*arct
anh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+24*A*(c+c*sin(f*x+e))^(
1/2)*c^(3/2)-165*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2+8*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+120*B*(c+c*sin(f*x+e))^(1/2)*c^(
3/2))*cos(f*x+e)^2+90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/
c^(1/2))*c^2+54*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-132*A*(c+c*sin(f*x+e))^(1/
2)*c^(3/2)+330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*c^2+86*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-420*B*(c+c*sin(f*x+e))^(1/2)*c^(3
/2))*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [B] time = 1.65029, size = 1285, normalized size = 5.71

$$\frac{15\sqrt{2}\left((3A+11B)a^3\cos\left(fx+e\right)^3+3(3A+11B)a^3\cos\left(fx+e\right)^2-2(3A+11B)a^3\cos\left(fx+e\right)-4(3A+11B)a^3-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(15*sqrt(2)*((3*A + 11*B)*a^3*cos(f*x + e)^3 + 3*(3*A + 11*B)*a^3*cos(
f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B)*a^3 - ((3*A +
 11*B)*a^3*cos(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B
)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
 + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*B*a^3*cos(f*x + e)^4 - 4*(
3*A + 14*B)*a^3*cos(f*x + e)^3 + 3*(13*A + 37*B)*a^3*cos(f*x + e)^2 + 3*(13
*A + 53*B)*a^3*cos(f*x + e) - 12*(A + B)*a^3 - (4*B*a^3*cos(f*x + e)^3 + 12
*(A + 5*B)*a^3*cos(f*x + e)^2 + 3*(17*A + 57*B)*a^3*cos(f*x + e) + 12*(A +
B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*
c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=217

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}-\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}-\frac{a^3c(A+13B)\cos^5(e}{24f(c-c\sin(e+f}$$

```
[Out] (-5*a^3*(A + 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(8*Sqrt[2]*c^(7/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(6*f*(
c - c*Sin[e + f*x])^(13/2)) - (a^3*(A + 13*B)*c*Cos[e + f*x]^5)/(24*f*(c -
c*Sin[e + f*x])^(9/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x]^3)/(48*c*f*(c - c*S
in[e + f*x])^(5/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x])/(16*c^3*f*Sqrt[c - c*
Sin[e + f*x]])
```

Rubi [A] time = 0.549385, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}+\frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}-\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f}-\frac{a^3c(A+13B)\cos^5(e}{24f(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (-5*a^3*(A + 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(8*Sqrt[2]*c^(7/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(6*f*(
c - c*Sin[e + f*x])^(13/2)) - (a^3*(A + 13*B)*c*Cos[e + f*x]^5)/(24*f*(c -
c*Sin[e + f*x])^(9/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x]^3)/(48*c*f*(c - c*S
in[e + f*x])^(5/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x])/(16*c^3*f*Sqrt[c - c*
Sin[e + f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
```

```
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2679

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{1}{12}\left(a^3(A+13B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{1}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{1}{48}\left(5a^3(A+\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B}{48cf(c-c\,\mathrm{s}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B}{48cf(c-c\,\mathrm{s}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} + \frac{5a^3(A+13B}{48cf(c-c\,\mathrm{s}}$$

$$= -\frac{5a^3(A+13B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} -$$

Mathematica [C] time = 3.27079, size = 422, normalized size = 1.94

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(64(A+B)\sin\left(\frac{1}{2}(e+fx)\right)+3(11A+47B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-
1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A + 25*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A + 47*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(Cos[(e +
```

```
f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(24*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/2))
```

Maple [B] time = 1.516, size = 524, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/48/c^(13/2)*a^3*(15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*sin(f*x+e)^3*c^3+195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3-45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-96*B*(c*(1+sin(f*x+e)))^(1/2
)*c^(5/2)*sin(f*x+e)^3-585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2
^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*A
*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c
^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^(1/2)*c^
(5/2)*sin(f*x+e)^2+585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*sin(f*x+e)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*A*2^(1
/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3-928*B*(c*(1+s
in(f*x+e)))^(3/2)*c^(3/2)-288*B*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)
-195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3+12
0*A*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)+888*B*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)
)*(c*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1
/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(7/2), x)
```

Fricas [B] time = 1.69733, size = 1434, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2)*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x
+ e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e) +
8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos(f*
x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)*a^3*cos
(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)*a^3*cos(
f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A + 79*B)*a^3*
cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a^3)*sin(f*x
 + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x +
 e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*co
s(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin
(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}+\frac{a^3c(A-15B)\cos^5}{48f(c-c\sin(e+}$$

```
[Out] (-5*a^3*(A - 15*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(128*Sqrt[2]*c^(9/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(8*f
*(c - c*Sin[e + f*x])^(15/2)) + (a^3*(A - 15*B)*c*Cos[e + f*x]^5)/(48*f*(c
- c*Sin[e + f*x])^(11/2)) - (5*a^3*(A - 15*B)*Cos[e + f*x]^3)/(192*c*f*(c -
 c*Sin[e + f*x])^(7/2)) + (5*a^3*(A - 15*B)*Cos[e + f*x])/(128*c^3*f*(c - c
*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.557293, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2859, 2680, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}+\frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}-\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f}+\frac{a^3c(A-15B)\cos^5}{48f(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2
),x]
```

```
[Out] (-5*a^3*(A - 15*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(128*Sqrt[2]*c^(9/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(8*f
*(c - c*Sin[e + f*x])^(15/2)) + (a^3*(A - 15*B)*c*Cos[e + f*x]^5)/(48*f*(c
- c*Sin[e + f*x])^(11/2)) - (5*a^3*(A - 15*B)*Cos[e + f*x]^3)/(192*c*f*(c -
 c*Sin[e + f*x])^(7/2)) + (5*a^3*(A - 15*B)*Cos[e + f*x])/(128*c^3*f*(c - c
*Sin[e + f*x])^(3/2))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{1}{16}\left(a^3(A-15B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{1}{96}\left(5a^3(A-\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B}{192cf(c-cs}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B}{192cf(c-cs}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B}{192cf(c-cs}$$

$$= -\frac{5a^3(A-15B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} +$$

Mathematica [C] time = 4.67189, size = 355, normalized size = 1.64

$$\frac{a^3(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((120+120i)\sqrt[4]{-1}(A-15B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\frac{1}{4}(e+f\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Cos
[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 2703*
B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e + f*x
))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 + 120
*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2] + 40
5*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e + f*x)
)/2] - 397*A*Sin[(5*(e + f*x))/2] + 579*B*Sin[(5*(e + f*x))/2] - 15*A*Sin[(
7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2]))/(3072*f*(Cos[(e + f*x)/2] +
```

```
Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] time = 1.585, size = 432, normalized size = 2.

$$\frac{a^3}{768\left(-1+\sin\left(fx+e\right)\right)^3\cos\left(fx+e\right)f}\left(60\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)\sqrt{2}c^4\left(A-15\,B\right)\sin\left(fx+e\right)\left(\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/768*a^3*(60*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c
^4*(A-15*B)*sin(f*x+e)*cos(f*x+e)^2-120*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*sin(f*x+e)+15*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)^4-120*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)
^2-240*A*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+440*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2
)-292*A*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-30*A*(c+c*sin(f*x+e))^(7/2)*c^(1/2)+
3600*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)-6600*B*(c+c*sin(f*x+e))^(3/2)*c^(5/2)
+4380*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-1086*B*(c+c*sin(f*x+e))^(7/2)*c^(1/2
)+120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-180
0*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+
sin(f*x+e)))^(1/2)/c^(17/2)/(-1+sin(f*x+e))^3/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^3}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(9/2), x)
```

Fricas [B] time = 1.63184, size = 1646, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/1536*(15*sqrt(2)*((A - 15*B)*a^3*cos(f*x + e)^5 + 5*(A - 15*B)*a^3*cos(f
*x + e)^4 - 8*(A - 15*B)*a^3*cos(f*x + e)^3 - 20*(A - 15*B)*a^3*cos(f*x + e
)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3 - ((A - 15*B)*a^3*c
os(f*x + e)^4 - 4*(A - 15*B)*a^3*cos(f*x + e)^3 - 12*(A - 15*B)*a^3*cos(f*x
 + e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3)*sin(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(5*A + 181*B)*a^3*cos(f*x + e)^4 - (191*A - 561
*B)*a^3*cos(f*x + e)^3 - 2*(169*A + 537*B)*a^3*cos(f*x + e)^2 + 12*(21*A -
59*B)*a^3*cos(f*x + e) + 384*(A + B)*a^3 - (3*(5*A + 181*B)*a^3*cos(f*x + e
)^3 + 2*(103*A - 9*B)*a^3*cos(f*x + e)^2 - 12*(11*A + 91*B)*a^3*cos(f*x + e
) - 384*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^5*f*cos(f*
x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f
*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5
*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5
*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.61863, size = 2140, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] -1/384*(15*sqrt(2)*(A*a^3 - 15*B*a^3)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(sqr
t(-c)*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1)) - 2*(783*(sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^15*A*a^3 - 225*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^15*B*a^3 - 993*(sqrt(c)*
tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^14*A*a^3*sqrt(c)
 + 4911*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
^14*B*a^3*sqrt(c) + 14913*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^13*A*a^3*c - 14031*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^13*B*a^3*c - 11259*(sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^12*A*a^3*c^(3/2) + 77493*(sqrt(c
)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^12*B*a^3*c^(3/
2) - 285*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^11*A*a^3*c^2 - 54861*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
 1/2*e)^2 + c))^11*B*a^3*c^2 + 28715*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c
*tan(1/2*f*x + 1/2*e)^2 + c))^10*A*a^3*c^(5/2) - 124293*(sqrt(c)*tan(1/2*f*
x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^10*B*a^3*c^(5/2) - 17363*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*a^3*
c^3 + 73821*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
 c))^9*B*a^3*c^3 - 37271*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))^8*A*a^3*c^(7/2) + 89817*(sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*B*a^3*c^(7/2) + 8989*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*a^3*c^4 + 10317*(sqr
t(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*B*a^3*c^4
 + 36189*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^6*A*a^3*c^(9/2) - 32115*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^6*B*a^3*c^(9/2) + 6547*(sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*a^3*c^5 - 71325*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*a^3*c^5 - 17777*(sqrt(c
)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*a^3*c^(11/
2) - 7521*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c
))^4*B*a^3*c^(11/2) - 5583*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))^3*A*a^3*c^6 + 35361*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqr
t(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*a^3*c^6 - 5351*(sqrt(c)*tan(1/2*f*x +
1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*a^3*c^(13/2) + 10377*(sqrt
```

```
(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*a^3*c^(1
3/2) - 193*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
c))*A*a^3*c^7 + 2127*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1
/2*e)^2 + c))*B*a^3*c^7 - 61*A*a^3*c^(15/2) + 147*B*a^3*c^(15/2))/(((sqrt(c
)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 - 2*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^8*
c^4*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=266

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(}{\sqrt{2}\sqrt{c-c\sin}}\right.}{512\sqrt{2}c^{11/2}f}$$

```
[Out] -(a^3*(3*A - 17*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(512*Sqrt[2]*c^(11/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(10
*f*(c - c*Sin[e + f*x])^(17/2)) + (a^3*(3*A - 17*B)*c*Cos[e + f*x]^5)/(80*f
*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x]^3)/(96*c*f*(
c - c*Sin[e + f*x])^(9/2)) + (a^3*(3*A - 17*B)*Cos[e + f*x])/(128*c^3*f*(c
- c*Sin[e + f*x])^(5/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x])/(512*c^4*f*(c -
c*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.587054, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2680, 2650, 2649, 206}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(}{\sqrt{2}\sqrt{c-c\sin}}\right.}{512\sqrt{2}c^{11/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/
2),x]
```


```
[Out] -(a^3*(3*A - 17*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(512*Sqrt[2]*c^(11/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(10
*f*(c - c*Sin[e + f*x])^(17/2)) + (a^3*(3*A - 17*B)*c*Cos[e + f*x]^5)/(80*f
*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x]^3)/(96*c*f*(
c - c*Sin[e + f*x])^(9/2)) + (a^3*(3*A - 17*B)*Cos[e + f*x])/(128*c^3*f*(c
- c*Sin[e + f*x])^(5/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x])/(512*c^4*f*(c -
c*Sin[e + f*x])^(3/2))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \left(a^3c^3\right)\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{1}{20}\left(a^3(3A-17B)c^2\right)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{1}{32}\left(a^3(3A-\right.$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)}{96cf(c-c\,\mathrm{si}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)}{96cf(c-c\,\mathrm{si}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)}{96cf(c-c\,\mathrm{si}}$$

$$= \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(3A-17B)}{96cf(c-c\,\mathrm{si}}$$

$$= -\frac{a^3(3A-17B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}} + $$

Mathematica [C] time = 6.86234, size = 485, normalized size = 1.82

$$\frac{(a\sin(e+fx)+a)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(56370A\sin\left(\frac{1}{2}(e+fx)\right)+31140A\sin\left(\frac{3}{2}(e+fx)\right)-10404A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] ((1/512 + I/512)*(-1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec[
(e + f*x)/4]*(Cos[(e + f*x)/4] + Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^11*(a + a*Sin[e + f*x])^3)/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6*(c - c*Sin[e + f*x])^(11/2)) + ((Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(a + a*Sin[e + f*x])^3*(56370*A*Cos[(e + f*x)/2] + 38970*B*Cos[(e + f*
x)/2] - 31140*A*Cos[(3*(e + f*x))/2] - 38580*B*Cos[(3*(e + f*x))/2] - 10404
*A*Cos[(5*(e + f*x))/2] - 12724*B*Cos[(5*(e + f*x))/2] + 435*A*Cos[(7*(e +
```

```
f*x))/2] + 7775*B*Cos[(7*(e + f*x))/2] - 45*A*Cos[(9*(e + f*x))/2] + 255*B*
Cos[(9*(e + f*x))/2] + 56370*A*Sin[(e + f*x)/2] + 38970*B*Sin[(e + f*x)/2]
+ 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(3*(e + f*x))/2] - 10404*A*Sin
[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))/2] - 435*A*Sin[(7*(e + f*x))/
2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[(9*(e + f*x))/2] + 255*B*Sin[(9
*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*S
in[e + f*x])^(11/2))
```

Maple [B] time = 1.792, size = 526, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/15360*a^3*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))
*c^6*(3*A-17*B)*sin(f*x+e)*cos(f*x+e)^4-180*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2*sin(f*x+e)+240*2^(
1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin
(f*x+e)-75*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*
(3*A-17*B)*cos(f*x+e)^4+300*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1
/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2-90*A*(c+c*sin(f*x+e))^(9/2)*c^(3/2
)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x+e))^(5/2)*c^(7/2
)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f*x+e))^(1/2)*c^(11
/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin(f*x+e))^(7/2)*c^(5
/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c*sin(f*x+e))^(3/2)*c
^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^(1/2)*arctanh(1/2*(c+
c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+e)))^(1/2)/c^(23/2)/(-1
+sin(f*x+e))^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{3}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)
^(11/2), x)
```

Fricas [B] time = 1.71799, size = 1971, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3*
cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3*c
os(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3*co
s(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6*(3*
A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32*(3*A
 - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(3*A -
17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c
*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x
+ e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B)*a^3*cos(
f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 389*B)*a^3*co
s(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*A + 43*B)*a^
3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 8
0*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*cos(f*x + e)^2 -
 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin(f*x + e))*sqrt
(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*
c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 -
16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x
+ e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f
*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] sage2
```

3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=200

$$-\frac{12c^2(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}-\frac{32c^3(7A-9B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{128c^4(7A-9B)\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-128*(7*A - 9*B)*c^4*Cos[e + f*x])/(35*a*f*Sqrt[c - c*Sin[e + f*x]]) - (32
*(7*A - 9*B)*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(35*a*f) - (12*(7*A
 - 9*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(35*a*f) - ((7*A - 9*B
)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*a*f) - ((A - B)*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(9/2))/(a*c*f)
```

Rubi [A] time = 0.384575, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2647, 2646}

$$-\frac{12c^2(7A-9B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af}-\frac{32c^3(7A-9B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af}-\frac{128c^4(7A-9B)\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]),
x]
```


```
[Out] (-128*(7*A - 9*B)*c^4*Cos[e + f*x])/(35*a*f*Sqrt[c - c*Sin[e + f*x]]) - (32
*(7*A - 9*B)*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(35*a*f) - (12*(7*A
 - 9*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(35*a*f) - ((7*A - 9*B
)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*a*f) - ((A - B)*Sec[e + f*x
]*(c - c*Sin[e + f*x])^(9/2))/(a*c*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf} - \frac{(7A-9B)\int(c-c\sin(e+}{2a}$$

$$= -\frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} - \frac{(A-B)\sec(e+fx)(c}{acf}$$

$$= -\frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} - \frac{(7A-9B)c\cos(e}{}$$

$$= -\frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af} - \frac{12(7A-9B)c^2\cos(e}{}$$

$$= -\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}} - \frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin}}{35af}$$

Mathematica [A] time = 5.70282, size = 157, normalized size = 0.78

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(196(A-2B)\cos(2(e+fx))+2450A\sin(e+fx)-14A\sin(3}{140af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-s}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4900*
A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] + 2450*A
*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B*Sin[3*(e
 + f*x)]))/(140*a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]
))
```

Maple [A] time = 0.926, size = 111, normalized size = 0.6

$$\frac{2c^4\left(-1+\sin\left(fx+e\right)\right)\left((-7A+29B)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+(308A-436B)\sin\left(fx+e\right)+5B\left(\cos\left(fx+e\right)\right)^4}{35af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/35*c^4/a*(-1+sin(f*x+e))*((-7*A+29*B)*sin(f*x+e)*cos(f*x+e)^2+(308*A-436*
B)*sin(f*x+e)+5*B*cos(f*x+e)^4+(49*A-103*B)*cos(f*x+e)^2+588*A-716*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.55037, size = 645, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7
/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f
*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*x + e)
 + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(7/2) +
407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 20
30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Fricas [A] time = 1.59043, size = 282, normalized size = 1.41

$$-\frac{2\left(5\,Bc^3\cos\left(fx+e\right)^4+(49\,A-103\,B)c^3\cos\left(fx+e\right)^2+4\,(147\,A-179\,B)c^3-\left((7\,A-29\,B)c^3\cos\left(fx+e\right)^2-4\,(77\,A\right.\right.}{35\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147*
A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.08861, size = 1127, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -1/105*(4*(210*sqrt(2)*A*c^(25/2) - 210*sqrt(2)*B*c^(25/2) - 77*sqrt(2)*A*a
^8*sqrt(c) + 109*sqrt(2)*B*a^8*sqrt(c) + 154*A*a^8*sqrt(c) - 218*B*a^8*sqrt
(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(2)*a*c^9 - a*c^9) - 3360*((sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^4*sgn(tan(1
/2*f*x + 1/2*e) - 1) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
 1/2*e)^2 + c))*B*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - A*c^(9/2)*sgn(tan(1/2
*f*x + 1/2*e) - 1) + B*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*ta
n(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan
(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a) - (
((((((3*(119*A*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 178*B*a^7*c^7*sgn(ta
n(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e)/c^12 + 35*(7*A*a^7*c^7*sgn(ta
n(1/2*f*x + 1/2*e) - 1) - 8*B*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*
tan(1/2*f*x + 1/2*e) + 7*(141*A*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 212
*B*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 35*(
25*A*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 34*B*a^7*c^7*sgn(tan(1/2*f*x +
 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 35*(25*A*a^7*c^7*sgn(tan(1/2*f*x
 + 1/2*e) - 1) - 34*B*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*
f*x + 1/2*e) + 7*(141*A*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 212*B*a^7*c
^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 35*(7*A*a^7*
c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 8*B*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) -
1))/c^12)*tan(1/2*f*x + 1/2*e) + 3*(119*A*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e)
- 1) - 178*B*a^7*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)/(c*tan(1/2*f*x +
1/2*e)^2 + c)^(7/2))/f
```

3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=159

$$-\frac{8c^2(5A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}-\frac{32c^3(5A-7B)\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}}-\frac{c(5A-7B)\cos(e+fx)(c-c\sin(e+fx))}{5af}$$

```
[Out] (-32*(5*A - 7*B)*c^3*Cos[e + f*x])/(15*a*f*Sqrt[c - c*Sin[e + f*x]]) - (8*(
5*A - 7*B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a*f) - ((5*A - 7*
B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) - ((A - B)*Sec[e + f*
x]*(c - c*Sin[e + f*x])^(7/2))/(a*c*f)
```

Rubi [A] time = 0.350131, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2647, 2646}

$$-\frac{8c^2(5A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af}-\frac{32c^3(5A-7B)\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}}-\frac{c(5A-7B)\cos(e+fx)(c-c\sin(e+fx))}{5af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-32*(5*A - 7*B)*c^3*Cos[e + f*x])/(15*a*f*Sqrt[c - c*Sin[e + f*x]]) - (8*(
5*A - 7*B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a*f) - ((5*A - 7*
B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*a*f) - ((A - B)*Sec[e + f*
x]*(c - c*Sin[e + f*x])^(7/2))/(a*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf} - \frac{(5A-7B)\int(c-c\sin(e+}{2a}$$

$$= -\frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} - \frac{(A-B)\sec(e+fx)(c-}{acf}$$

$$= -\frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} - \frac{(5A-7B)c\cos(e+fx}{5a}$$

$$= -\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+}}{15af}$$

Mathematica [A] time = 1.77777, size = 134, normalized size = 0.84

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(25(8A-13B)\sin(e+fx)+2(5A-16B)\cos(2(e+fx))+450}{30af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(450*A
 - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e + f*x] +
 3*B*Sin[3*(e + f*x)]))/(30*a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 +
Sin[e + f*x]))
```

Maple [A] time = 0.828, size = 95, normalized size = 0.6

$$-\frac{2c^3\left(-1+\sin\left(fx+e\right)\right)\left(-3B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+\left(-50A+82B\right)\sin\left(fx+e\right)+\left(-5A+16B\right)\left(\cos\left(fx+e\right.\right.\right.}{15af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/15*c^3/a*(-1+sin(f*x+e))*(-3*B*cos(f*x+e)^2*sin(f*x+e)+(-50*A+82*B)*sin(
f*x+e)+(-5*A+16*B)*cos(f*x+e)^2-110*A+142*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1
/2)/f
```

Maxima [B] time = 1.5615, size = 521, normalized size = 3.28

$$\frac{2\left(\frac{5\left(23c^{\frac{5}{2}}+\frac{20c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{65c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{40c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{65c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{20c^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{23c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)A}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}-\frac{2\left(79c^{\frac{5}{2}}+\frac{79c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{205c^{\frac{5}{2}}\sin\ldots}{(\cos(fx\ldots}\right.}{\ldots}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/15*(5*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5/
2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/2)*
```

```
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(5/2)) - 2*(79*c^(5/2) + 79*c^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) + 205*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 170*c^(5/
2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 205*c^(5/2)*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 79*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 79*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Fricas [A] time = 1.49436, size = 230, normalized size = 1.45

$$-\frac{2\left((5\,A-16\,B)c^2\cos\left(fx+e\right)^2+2\,(55\,A-71\,B)c^2+\left(3\,Bc^2\cos\left(fx+e\right)^2+2\,(25\,A-41\,B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(f\right.}}{15\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*cos
(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/
(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.92908, size = 957, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -1/60*(2*(120*sqrt(2)*A*c^(19/2) - 120*sqrt(2)*B*c^(19/2) - 25*sqrt(2)*A*a^
6*sqrt(c) + 41*sqrt(2)*B*a^6*sqrt(c) + 50*A*a^6*sqrt(c) - 82*B*a^6*sqrt(c))
*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqrt(2)*a*c^7 - a*c^7) - 960*((sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^3*sgn(tan(1/2*f*
x + 1/2*e) - 1) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*
e)^2 + c))*B*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - A*c^(7/2)*sgn(tan(1/2*f*x
+ 1/2*e) - 1) + B*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a) - ((((((
55*A*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 98*B*a^5*c^5*sgn(tan(1/2*f*x +
 1/2*e) - 1))*tan(1/2*f*x + 1/2*e)/c^9 + 15*(3*A*a^5*c^5*sgn(tan(1/2*f*x +
1/2*e) - 1) - 4*B*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x +
 1/2*e) + 10*(10*A*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 17*B*a^5*c^5*sgn
(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 10*(10*A*a^5*c^5*sg
n(tan(1/2*f*x + 1/2*e) - 1) - 17*B*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1))/c
^9)*tan(1/2*f*x + 1/2*e) + 15*(3*A*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) -
4*B*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + (55*
A*a^5*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 98*B*a^5*c^5*sgn(tan(1/2*f*x + 1/
2*e) - 1))/c^9)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(5/2))/f
```

3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=118

$$-\frac{4c^2(3A-5B)\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{c(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

```
[Out] (-4*(3*A - 5*B)*c^2*Cos[e + f*x])/(3*a*f*Sqrt[c - c*Sin[e + f*x]]) - ((3*A
- 5*B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) - ((A - B)*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(5/2))/(a*c*f)
```

Rubi [A] time = 0.316923, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2647, 2646}

$$-\frac{4c^2(3A-5B)\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{c(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] (-4*(3*A - 5*B)*c^2*Cos[e + f*x])/(3*a*f*Sqrt[c - c*Sin[e + f*x]]) - ((3*A
- 5*B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) - ((A - B)*Sec[e +
f*x]*(c - c*Sin[e + f*x])^(5/2))/(a*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf} - \frac{(3A-5B)\int(c-c\sin(e+$$

$$= -\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af} - \frac{(A-B)\sec(e+fx)(c-}{acf}$$

$$= -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+f}}{3af}$$

Mathematica [A] time = 0.636489, size = 113, normalized size = 0.96

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((14B-6A)\sin(e+fx)-18A+B\cos(2(e+fx))+27B)}{3af(\sin(e+fx)+1)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x]),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)]
 + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.811, size = 73, normalized size = 0.6

$$\frac{2c^2\left(-1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\left(3A-7B\right)-B\left(\cos\left(fx+e\right)\right)^2+9A-13B\right)}{3af\cos\left(fx+e\right)}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/3*c^2/a*(-1+sin(f*x+e))*(sin(f*x+e)*(3*A-7*B)-B*cos(f*x+e)^2+9*A-13*B)/co
s(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.52605, size = 397, normalized size = 3.36

$$\frac{2\left(\frac{3\left(3c^{\frac{3}{2}}+\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{6c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)A}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}-\frac{2\left(7c^{\frac{3}{2}}+\frac{7c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{12c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{7c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{7c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)B}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*A/((a + a*sin(f*x
+ e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) -
 2*(7*c^(3/2) + 7*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 12*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 7*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*B/((a + a*sin(f*x + e
)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A] time = 1.39912, size = 158, normalized size = 1.34

$$\frac{2\left(Bc\cos\left(fx+e\right)^{2}-(3A-7B)c\sin\left(fx+e\right)-(9A-13B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.59174, size = 786, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -1/3*((6*sqrt(2)*A*c^(13/2) - 6*sqrt(2)*B*c^(13/2) - 3*sqrt(2)*A*a^4*sqrt(c
) + 7*sqrt(2)*B*a^4*sqrt(c) + 6*A*a^4*sqrt(c) - 14*B*a^4*sqrt(c))*sgn(tan(1
/2*f*x + 1/2*e) - 1)/(sqrt(2)*a*c^5 - a*c^5) - ((((3*A*a^3*c^3*sgn(tan(1/2*
f*x + 1/2*e) - 1) - 8*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x
```

```
+ 1/2*e)/c^6 + 3*(A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2*B*a^3*c^3*sgn
(tan(1/2*f*x + 1/2*e) - 1))/c^6)*tan(1/2*f*x + 1/2*e) + 3*(A*a^3*c^3*sgn(ta
n(1/2*f*x + 1/2*e) - 1) - 2*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^6)*t
an(1/2*f*x + 1/2*e) + (3*A*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - 8*B*a^3*
c^3*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^6)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(3/2
) - 24*((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
*A*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - A*c^(
5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + B*c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1
))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2
+ 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqr
t(c) - c)*a))/f
```

3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=73

$$-\frac{c(A-3B)\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

```
[Out] -(((A - 3*B)*c*Cos[e + f*x])/(a*f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*Sec
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a*c*f)
```

Rubi [A] time = 0.274729, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2855, 2646}

$$-\frac{c(A-3B)\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x]
```

```
[Out] -(((A - 3*B)*c*Cos[e + f*x])/(a*f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*Sec
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a*c*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx &= \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\
&= -\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf} - \frac{(A-3B)\int \sqrt{c-c\sin(e+fx}}{2a}\\
&= -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}
\end{aligned}$$

Mathematica [A] time = 0.207427, size = 44, normalized size = 0.6

$$\frac{2\sec(e+fx)\sqrt{c-c\sin(e+fx)}(-A+B\sin(e+fx)+2B)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x]),x]
```

```
[Out] (2*Sec[e + f*x]*(-A + 2*B + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f)
```

Maple [A] time = 0.644, size = 53, normalized size = 0.7

$$2\,{\frac{c\left(-1+\sin\left(fx+e\right)\right)\left(-B\sin\left(fx+e\right)+A-2\,B\right)}{\cos\left(fx+e\right)a\sqrt{c-c\sin\left(fx+e\right)}f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] 2*c/a*(-1+sin(f*x+e))*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Maxima [B] time = 1.50121, size = 235, normalized size = 3.22

$$-\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}-\frac{A\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqr
t(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqrt(
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Fricas [A] time = 1.37219, size = 101, normalized size = 1.38

$$\frac{2\left(B\sin\left(fx+e\right)-A+2B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2*(B*sin(f*x + e) - A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx + \int \frac{B\sqrt{-c\sin\left(e+fx\right)+c}\,\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

```
[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

Giac [B] time = 1.57594, size = 475, normalized size = 6.51

$$-\frac{\frac{\left(\sqrt{2}A\sqrt{c}+\sqrt{2}B\sqrt{c}-4B\sqrt{c}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\sqrt{2}a-a} + \frac{2\left(\frac{Bc\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{Bc\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{a}\right)}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c}} - \frac{4\left(\left(\sqrt{c}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{c\tan\left(\right.}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] -((sqrt(2)*A*sqrt(c) + sqrt(2)*B*sqrt(c) - 4*B*sqrt(c))*sgn(tan(1/2*f*x + 1
/2*e) - 1)/(sqrt(2)*a - a) + 2*(B*c*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f
*x + 1/2*e)/a + B*c*sgn(tan(1/2*f*x + 1/2*e) - 1)/a)/sqrt(c*tan(1/2*f*x + 1
/2*e)^2 + c) - 4*((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*
e)^2 + c))*A*c*sgn(tan(1/2*f*x + 1/2*e) - 1) - (sqrt(c)*tan(1/2*f*x + 1/2*e
) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c*sgn(tan(1/2*f*x + 1/2*e) - 1) -
 A*c^(3/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + B*c^(3/2)*sgn(tan(1/2*f*x + 1/2*
e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c
))*sqrt(c) - c)*a))/f
```

3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(Sqrt[2]*a*Sqrt[c]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*c*f)
```

Rubi [A] time = 0.283368, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2649, 206}

$$\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]
```


```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(Sqrt[2]*a*Sqrt[c]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
a*c*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} + \frac{(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a}$$

$$= -\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf} - \frac{(A+B)\,\mathrm{Subst}\left(\int \frac{1}{2c-x^2}\,dx, x, -\ldots\right.}{af}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

Mathematica [C] time = 0.460433, size = 140, normalized size = 1.54

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-(1+i)\sqrt[4]{-1}(A+B)\tan^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(\tan\left(\ldots\right.\right.\right.\right.}{af(\sin(e+fx)+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[e +
 f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 1.197, size = 130, normalized size = 1.4

$$-\frac{-1+\sin\left(fx+e\right)}{2\,af\cos\left(fx+e\right)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\left(1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{c}}\right)\sqrt{c\left(1+\sin\left(fx+e\right)\right)}A+\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{c\left(1+\sin\left(\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/a*(-1+sin(f*x+e))*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e))
)^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)*B
)/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Fricas [B] time = 1.39279, size = 447, normalized size = 4.91

$$\frac{\sqrt{2}(A+B)\sqrt{c}\cos\left(fx+e\right)\log\left(-\frac{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)}{\sqrt{c}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)-4\sqrt{-c\,s}}{4\,acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x +
 e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) +
 sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x
 + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c)*
(A - B))/(a*c*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+fx\right)}{\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)+\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
+ c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + sqrt(-c*sin(e + f*x) + c)), x))/a
```

Giac [B] time = 1.75312, size = 536, normalized size = 5.89

$$\frac{\left(2\sqrt{2}Ac\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+2\sqrt{2}Bc\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)-2\,Ac\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)-2\,Bc\arctan\left(\frac{\sqrt{c}}{\sqrt{-c}}\right)+A\sqrt{-c}\sqrt{c}-B\sqrt{-c}\sqrt{c}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}{\sqrt{2}a\sqrt{-c}c-2\,a\sqrt{-c}c}+\frac{\sqrt{2}(A+B)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{\cdots}\right.}{\cdots}\right)}{a\sqrt{-c}s\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] ((2*sqrt(2)*A*c*arctan(sqrt(c)/sqrt(-c)) + 2*sqrt(2)*B*c*arctan(sqrt(c)/sqr
t(-c)) - 2*A*c*arctan(sqrt(c)/sqrt(-c)) - 2*B*c*arctan(sqrt(c)/sqrt(-c)) +
A*sqrt(-c)*sqrt(c) - B*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(sqr
t(2)*a*sqrt(-c)*c - 2*a*sqrt(-c)*c) + sqrt(2)*(A + B)*arctan(-1/2*sqrt(2)*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c)
)/sqrt(-c))/(a*sqrt(-c)*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 2*((sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A - (sqrt(c)*tan(1/2*f
*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B - A*sqrt(c) + B*sqrt(c)
)/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 +
 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt
(c) - c)*a*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] ((3*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(4*Sqrt[2]*a*c^(3/2)*f) + ((3*A - B)*Cos[e + f*x])/(4*a*f*(c - c*Sin[e
+ f*x])^(3/2)) - ((A - B)*Sec[e + f*x])/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.330427, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2855, 2650, 2649, 206}

$$\frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] ((3*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(4*Sqrt[2]*a*c^(3/2)*f) + ((3*A - B)*Cos[e + f*x])/(4*a*f*(c - c*Sin[e
+ f*x])^(3/2)) - ((A - B)*Sec[e + f*x])/(a*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}\,dx}{2a}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} + \frac{(3A-B)\int \frac{1}{\sqrt{c-c\sin(e}}}{8ac}$$

$$= \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}} - \frac{(3A-B)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(3A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} + \frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}} - \frac{(A-E}{acf\sqrt{c-}}$$

Mathematica [C] time = 0.561778, size = 284, normalized size = 2.09

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (1 + I)*
(-1)^(1/4)*(3*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]) + 2*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(4*
a*f*(1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] time = 0.98, size = 225, normalized size = 1.7

$$-\frac{1}{8\,af\cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\left(3\,A\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)c-B\sqrt{c+c\sin\left(fx+e\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*
arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-6*A*c^(3/2)+2*B*c^(3/
2))-3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)/c^(1/2))*c+B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x
+e))^(1/2)*2^(1/2)/c^(1/2))*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-c*sin(
f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

Fricas [A] time = 1.55292, size = 608, normalized size = 4.47

$$-\frac{\sqrt{2}\left((3A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-(3A-B)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(\right.\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+\right.}\right.}{16\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x + e
))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqr
```

```
t(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e)
 - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c*sin(
f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{A}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx+\int \frac{B\sin\left(e+fx\right)}{-c\sqrt{-c\sin\left(e+fx\right)+c}\sin^2\left(e+fx\right)+c\sqrt{-c\sin\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] (Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e
 + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*s
in(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a
```

Giac [B] time = 2.40715, size = 878, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/4*(sqrt(2)*(3*A - B)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a*sqrt(-c)*c*sgn(t
an(1/2*f*x + 1/2*e) - 1)) + 4*((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1
/2*f*x + 1/2*e)^2 + c))*A - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*
f*x + 1/2*e)^2 + c))*B - A*sqrt(c) + B*sqrt(c))/(((sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)*a*c*sgn(tan(1/2*f*x
+ 1/2*e) - 1)) + 2*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^3*A + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
 1/2*e)^2 + c))^3*B - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^2*A*sqrt(c) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2
```

```
*f*x + 1/2*e)^2 + c))^2*B*sqrt(c) - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*
tan(1/2*f*x + 1/2*e)^2 + c))*A*c - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))*B*c - A*c^(3/2) - B*c^(3/2))/(((sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 - 2*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^2*a*c*sgn(t
an(1/2*f*x + 1/2*e) - 1)))/f
```

3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (3*(5*A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(32*Sqrt[2]*a*c^(5/2)*f) + (3*(5*A - 3*B)*Cos[e + f*x])/(32*a*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((A + B)*Sec[e + f*x])/(4*a*c*f*(c - c*Sin[e +
 f*x])^(3/2)) - ((5*A - 3*B)*Sec[e + f*x])/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*
x]])
```

Rubi [A] time = 0.424661, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2859, 2687, 2650, 2649, 206}

$$-\frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}+\frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (3*(5*A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])])/(32*Sqrt[2]*a*c^(5/2)*f) + (3*(5*A - 3*B)*Cos[e + f*x])/(32*a*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((A + B)*Sec[e + f*x])/(4*a*c*f*(c - c*Sin[e +
 f*x])^(3/2)) - ((5*A - 3*B)*Sec[e + f*x])/(8*a*c^2*f*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} + \frac{(5A-3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,dx}{8ac^2}$$

$$= \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}} + \frac{(3(5A-3B))\int}{16}$$

$$= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\operatorname{se}}{8ac^2f\sqrt{c-c\operatorname{s}}}$$

$$= \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}} - \frac{(5A-3B)\operatorname{se}}{8ac^2f\sqrt{c-c\operatorname{s}}}$$

$$= \frac{3(5A-3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} + \frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}} + \frac{}{4a}$$

Mathematica [C] time = 0.884677, size = 404, normalized size = 2.24

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4 -\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*A -
 B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
+ Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]) + 8*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]) + 2*(7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(
e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(32*a*f*(1 + Sin[e + f*
x])*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] time = 1.285, size = 350, normalized size = 1.9

$$-\frac{1}{64\,a\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\left(-30\,A\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{}}{\sqrt{c}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/64/c^(9/2)/a*(sin(f*x+e)*(-30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*A*c^(5/2)+18*B*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^
2-24*B*c^(5/2))+(-15*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+30*A*c^(5/2)+9*B*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-18*B*c^(5/2
))*cos(f*x+e)^2+30*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*
x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-24*A*c^(5/2)-18*B*(c+c*sin(f*x+e))^(1/2)*2
^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+40*B*c^(5/2)
)/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

Fricas [A] time = 1.51291, size = 753, normalized size = 4.18

$$-\frac{3\sqrt{2}\left((5A-3B)\cos\left(fx+e\right)^3+2\,(5A-3B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(5A-3B)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)}{}\right.}{128\left(ac^3f\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/128*(3*sqrt(2)*((5*A - 3*B)*cos(f*x + e)^3 + 2*(5*A - 3*B)*cos(f*x + e)*
sin(f*x + e) - 2*(5*A - 3*B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 -
 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) +
 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*
x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(5*A
 - 3*B)*cos(f*x + e)^2 + 4*(5*A - 3*B)*sin(f*x + e) - 12*A + 20*B)*sqrt(-c*
sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x
 + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] sage2
```

3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=242

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}-\frac{64c^2(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f}-\frac{512c^3(7A-13B)\sec(e}{10}$$

```
[Out] (2048*(7*A - 13*B)*c^4*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*a^2*f) -
 (512*(7*A - 13*B)*c^3*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(105*a^2*f)
 - (64*(7*A - 13*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(105*a^2*f
) - (16*(7*A - 13*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*a^2*f)
 - ((7*A - 13*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(21*a^2*f) - ((A
- B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.65076, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}-\frac{64c^2(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f}-\frac{512c^3(7A-13B)\sec(e}{10}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] (2048*(7*A - 13*B)*c^4*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*a^2*f) -
 (512*(7*A - 13*B)*c^3*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(105*a^2*f)
 - (64*(7*A - 13*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(105*a^2*f
) - (16*(7*A - 13*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*a^2*f)
 - ((7*A - 13*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(21*a^2*f) - ((A
- B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/(3*a^2*c^2*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f} - \frac{(7A-13B)\int \sec^2(e+f}{6}$$

$$= -\frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e+fx)(c}{3a^2c^2}$$

$$= -\frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)\sec(e+}{2}$$

$$= -\frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-13B)c\sec}{}$$

$$= -\frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \frac{64(7A-13B)c^2 s}{}$$

$$= \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f} - \frac{512(7A-13B)c^3 se}{}$$

Mathematica [B] time = 6.8705, size = 953, normalized size = 3.94

$$-\frac{(26A-83B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\sin\left(\frac{3}{2}(e+fx)\right)(c-c\sin(e+fx))^{9/2}}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(\sin(e+fx)a+a)^2} - \frac{(2A-13B)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{20f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (-32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/
2))/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) +
(32*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x]
)^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2)
 + ((164*A - 351*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*
(a + a*Sin[e + f*x])^2) + ((26*A - 83*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(12*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((2*A - 13*B)*Cos[(5*(e
 + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9
/2))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2)
+ (B*Cos[(7*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Si
```

```
n[e + f*x])^(9/2))/(28*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin
[e + f*x])^2) + ((164*A - 351*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((26*A - 83*B)*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2)*Sin[(3*(e + f*x))/2])/(12*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((2*A - 13
*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2)*Sin[
(5*(e + f*x))/2])/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[
e + f*x])^2) - (B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*
x])^(9/2)*Sin[(7*(e + f*x))/2])/(28*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a + a*Sin[e + f*x])^2)
```

Maple [A] time = 1.054, size = 143, normalized size = 0.6

$$-\frac{2\,c^5\left(-1+\sin\left(fx+e\right)\right)\left(15\,B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+(196\,A-544\,B)\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+(7448\,A-\right.}{105\,a^2\left(1+\sin\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/105*c^5/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(15*B*sin(f*x+e)*cos(f*x+e)^4
+(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(7448*A-13592*B)*sin(f*x+e)+(21*A-11
4*B)*cos(f*x+e)^4+(-1848*A+3732*B)*cos(f*x+e)^2+6888*A-13032*B)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.60448, size = 1029, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 537
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
```

```
+ 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(9/2)*s
in(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 723*c^(
9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^2 + 3*a^2*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2
)) - 2*(4707*c^(9/2) + 14121*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3525
0*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 68549*c^(9/2)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 99549*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 134802*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 138012*c^(9/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 134802*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 99549*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 68549*c^(9/2
)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 35250*c^(9/2)*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 14121*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 47
07*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*B/((a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)^(9/2)))/f
```

Fricas [A] time = 1.57781, size = 396, normalized size = 1.64

$$\frac{2\left(3\,(7\,A-38\,B)c^4\cos\left(fx+e\right)^4-12\,(154\,A-311\,B)c^4\cos\left(fx+e\right)^2+24\,(287\,A-543\,B)c^4+\left(15\,Bc^4\cos\left(fx+e\right)^4+4\right.\right.}{105\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x +
 e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136*B
)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*
x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.53848, size = 1754, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/105*(4*(3577*sqrt(2)*A*a^16*sqrt(c) - 7483*sqrt(2)*B*a^16*sqrt(c) - 5110
*A*a^16*sqrt(c) + 10690*B*a^16*sqrt(c) - 1610*sqrt(2)*A*c^(25/2) + 2870*sqr
t(2)*B*c^(25/2) + 2100*A*c^(25/2) - 3780*B*c^(25/2))*sgn(tan(1/2*f*x + 1/2*
e) - 1)/(5*sqrt(2)*a^2*c^8 - 7*a^2*c^8) + ((((((((2261*A*a^14*c^8*sgn(tan(1
/2*f*x + 1/2*e) - 1) - 4934*B*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1
/2*f*x + 1/2*e)/c^12 + 105*(17*A*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) - 3
2*B*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 7*
(913*A*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1972*B*a^14*c^8*sgn(tan(1/2
*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 35*(169*A*a^14*c^8*sgn(tan
(1/2*f*x + 1/2*e) - 1) - 346*B*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12
)*tan(1/2*f*x + 1/2*e) + 35*(169*A*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) -
 346*B*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) +
 7*(913*A*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1972*B*a^14*c^8*sgn(tan(
1/2*f*x + 1/2*e) - 1))/c^12)*tan(1/2*f*x + 1/2*e) + 105*(17*A*a^14*c^8*sgn(
tan(1/2*f*x + 1/2*e) - 1) - 32*B*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^
12)*tan(1/2*f*x + 1/2*e) + (2261*A*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) -
 4934*B*a^14*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^12)/(c*tan(1/2*f*x + 1/2*
e)^2 + c)^(7/2) + 2240*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^5*A*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 6*(sqrt(c)*tan(1
/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*c^5*sgn(tan(1/2*f
*x + 1/2*e) - 1) + 15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^4*A*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 24*(sqrt(c)*tan
(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(11/2)*sgn(ta
n(1/2*f*x + 1/2*e) - 1) - 10*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2
*f*x + 1/2*e)^2 + c))^3*A*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) + 16*(sqrt(c)*t
an(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^6*sgn(tan(1
/2*f*x + 1/2*e) - 1) - 30*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
```

```
x + 1/2*e)^2 + c))^2*A*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 48*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(13/2)*sg
n(tan(1/2*f*x + 1/2*e) - 1) + 27*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan
(1/2*f*x + 1/2*e)^2 + c))*A*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 42*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^7*sgn(tan(1
/2*f*x + 1/2*e) - 1) - 5*A*c^(15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 8*B*c^(
15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=201

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}-\frac{32c^2(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}+\frac{128c^3(5A-11B)\sec(}{1}$$

```
[Out] (128*(5*A - 11*B)*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a^2*f) - (
32*(5*A - 11*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (
4*(5*A - 11*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*a^2*f) - ((5*
A - 11*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*a^2*f) - ((A - B)*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.558201, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}-\frac{32c^2(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f}+\frac{128c^3(5A-11B)\sec(}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
2,x]
```


```
[Out] (128*(5*A - 11*B)*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a^2*f) - (
32*(5*A - 11*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(15*a^2*f) - (
4*(5*A - 11*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*a^2*f) - ((5*
A - 11*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(15*a^2*f) - ((A - B)*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(3*a^2*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{(5A-11B)\int \sec^2(e+f$$

$$= -\frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c}{3a^2c^2}$$

$$= -\frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)\sec(e+f}{15}$$

$$= -\frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-11B)c\sec(e}{}$$

$$= \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(5A-11B)c^2\sec(e}{}$$

Mathematica [A] time = 2.96615, size = 159, normalized size = 0.79

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(12(25A-62B)\cos(2(e+fx))-2730A\sin(e+fx)-10A\sin}{60a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-2100
*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)] - 27
30*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 46*B*Sin[
3*(e + f*x)]))/(60*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e +
 f*x])^2)
```

Maple [A] time = 0.915, size = 121, normalized size = 0.6

$$\frac{2c^4\left(-1+\sin\left(fx+e\right)\right)\left((-5A+23B)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+(-340A+724B)\sin\left(fx+e\right)+3B\left(\cos\left(fx+e\right.\right.\right.}{15a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/15*c^4/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*((-5*A+23*B)*sin(f*x+e)*cos(f*x
+e)^2+(-340*A+724*B)*sin(f*x+e)+3*B*cos(f*x+e)^4+(75*A-189*B)*cos(f*x+e)^2-
300*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.56883, size = 905, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^
(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^
(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^
(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2896*c
^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 361
2*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/2)*sin(f*x + e)^
10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Fricas [A] time = 1.51195, size = 331, normalized size = 1.65

$$-\frac{2\left(3\,Bc^3\cos\left(fx+e\right)^4+3\,(25\,A-63\,B)c^3\cos\left(fx+e\right)^2-12\,(25\,A-57\,B)c^3-\left((5\,A-23\,B)c^3\cos\left(fx+e\right)^2+4\,(85\,A\right.\right.}{15\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(25
*A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a
^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.36949, size = 1589, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/60*(4*(140*sqrt(2)*A*a^12*sqrt(c) - 371*sqrt(2)*B*a^12*sqrt(c) - 200*A*a
^12*sqrt(c) + 530*B*a^12*sqrt(c) - 280*sqrt(2)*A*c^(19/2) + 640*sqrt(2)*B*c
^(19/2) + 360*A*c^(19/2) - 840*B*c^(19/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5
*sqrt(2)*a^2*c^6 - 7*a^2*c^6) + (((((17*(5*A*a^10*c^6*sgn(tan(1/2*f*x + 1/2
*e) - 1) - 14*B*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e
)/c^9 + 15*(5*A*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 12*B*a^10*c^6*sgn(
tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 10*(16*A*a^10*c^6*sg
n(tan(1/2*f*x + 1/2*e) - 1) - 43*B*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1))/
c^9)*tan(1/2*f*x + 1/2*e) + 10*(16*A*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1)
 - 43*B*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) +
 15*(5*A*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 12*B*a^10*c^6*sgn(tan(1/2
*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 17*(5*A*a^10*c^6*sgn(tan(1/
2*f*x + 1/2*e) - 1) - 14*B*a^10*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)/(c*
tan(1/2*f*x + 1/2*e)^2 + c)^(5/2) + 320*(3*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) -
 9*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*
c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + 21*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) -
 39*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B
*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 14*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*A*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1)
+ 26*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*
B*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 42*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sq
rt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1
) + 78*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^
2*B*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 39*(sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1
) - 69*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*
B*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 7*A*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e)
```

```
 - 1) + 13*B*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2} dx$

Optimal. Leaf size=154

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}+\frac{32c^2(A-3B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-3B)\sec(e+fx)(c-c}{3a^2f}$$

```
[Out] (32*(A - 3*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - (8*(A
- 3*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - 3*B)*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(9/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.479186, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}+\frac{32c^2(A-3B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}-\frac{(A-3B)\sec(e+fx)(c-c}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
2,x]
```


```
[Out] (32*(A - 3*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - (8*(A
- 3*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - 3*B)*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(9/2))/(3*a^2*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx &= \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{a^2c^2}\\
&= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f} - \frac{(A-3B)\int \sec^2(e+fx)(c}{2a^2c}\\
&= -\frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-}{3a^2c^2f}\\
&= -\frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3B)\sec(e+fx)(c}{3a^2f}\\
&= \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3B)c\sec(e+fx)(c}{3a^2f}
\end{aligned}$$

Mathematica [A] time = 1.21923, size = 130, normalized size = 0.84

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((201B-72A)\sin(e+fx)+6(A-4B)\cos(2(e+fx))-50A}{6a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-50*A
 + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x] + B*
Sin[3*(e + f*x)]))/(6*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[
e + f*x])^2)
```

Maple [A] time = 0.905, size = 105, normalized size = 0.7

$$-\frac{2c^3\left(-1+\sin\left(fx+e\right)\right)\left(-B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+\left(18A-50B\right)\sin\left(fx+e\right)+\left(-3A+12B\right)\left(\cos\left(fx+e\right)\right)^2\right.}{3a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^3/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(-B*cos(f*x+e)^2*sin(f*x+e)+(18
*A-50*B)*sin(f*x+e)+(-3*A+12*B)*cos(f*x+e)^2+14*A-46*B)/cos(f*x+e)/(c-c*sin
(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.5569, size = 779, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e
```

```
) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(17
*c^(5/2) + 51*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 92*c^(5/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 149*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 150*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 149*c^(5/2)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 92*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 + 51*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(5/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Fricas [A] time = 1.49868, size = 270, normalized size = 1.75

$$-\frac{2\left(3\,(A-4\,B)c^2\cos\left(fx+e\right)^2-2\,(7\,A-23\,B)c^2+\left(Bc^2\cos\left(fx+e\right)^2-2\,(9\,A-25\,B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x
+ e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f
*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.16073, size = 1339, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/3*((21*sqrt(2)*A*a^8*sqrt(c) - 91*sqrt(2)*B*a^8*sqrt(c) - 30*A*a^8*sqrt(
c) + 130*B*a^8*sqrt(c) - 10*sqrt(2)*A*c^(13/2) + 46*sqrt(2)*B*c^(13/2) + 12
*A*c^(13/2) - 60*B*c^(13/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sqrt(2)*a^2*c
^4 - 7*a^2*c^4) + ((((3*A*a^6*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 14*B*a^6*
c^4*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e)/c^6 + 3*(A*a^6*c^4*
sgn(tan(1/2*f*x + 1/2*e) - 1) - 4*B*a^6*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1))/
c^6)*tan(1/2*f*x + 1/2*e) + 3*(A*a^6*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 4*
B*a^6*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^6)*tan(1/2*f*x + 1/2*e) + (3*A*a
^6*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 14*B*a^6*c^4*sgn(tan(1/2*f*x + 1/2*e
) - 1))/c^6)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(3/2) - 16*(3*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*c^3*sgn(tan(1/2*f*x
+ 1/2*e) - 1) - 6*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*
e)^2 + c))^4*A*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 15*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(7/2)*sgn(tan(1/2*
f*x + 1/2*e) - 1) + 4*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^3*A*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 10*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^4*sgn(tan(1/2*f*x
+ 1/2*e) - 1) + 12*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2
*e)^2 + c))^2*A*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 30*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(9/2)*sgn(tan(1/2
*f*x + 1/2*e) - 1) - 12*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))*A*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) + 27*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^5*sgn(tan(1/2*f*x +
1/2*e) - 1) + 2*A*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 5*B*c^(11/2)*sgn
(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2
*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{4c(A-7B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

```
[Out] (4*(A - 7*B)*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - 7*B
)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x
]^3*(c - c*Sin[e + f*x])^(7/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.408315, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}-\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f}+\frac{4c(A-7B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
2,x]
```


```
[Out] (4*(A - 7*B)*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - 7*B
)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x
]^3*(c - c*Sin[e + f*x])^(7/2))/(3*a^2*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
```

```
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^2c^2}$$
$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f} - \frac{(A-7B)\int \sec^2(e+fx)}{6a}$$
$$= -\frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-}{3a^2c^2f}$$
$$= \frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-7B)\sec(e+fx)(c-}{3a^2f}$$

Mathematica [A] time = 0.691932, size = 113, normalized size = 0.98

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(6(A-5B)\sin(e+fx)+2A+3B\cos(2(e+fx))-23B)}{3a^2f(\sin(e+fx)+1)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^2,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)]
 + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

Maple [A] time = 0.84, size = 81, normalized size = 0.7

$$-\frac{2\,c^{2}\left(-1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)\left(3\,A-15\,B\right)+3\,B\left(\cos\left(fx+e\right)\right)^{2}+A-13\,B\right)}{3\,a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/3*c^2/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(sin(f*x+e)*(3*A-15*B)+3*B*cos(
f*x+e)^2+A-13*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.54192, size = 651, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] -2/3*((c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*A
/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*sin(f*x + e)/(
cos(f*x + e) + 1) + 21*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*c^(
3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 21*c^(3/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^(3/2)
```

```
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A] time = 1.5179, size = 207, normalized size = 1.8

$$\frac{2\left(3\,Bc\cos\left(fx+e\right)^{2}+3\,(A-5\,B)c\sin\left(fx+e\right)+(A-13\,B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] 2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqrt
(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.86378, size = 1110, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -2/3*((2*sqrt(2)*A*c^(3/2) - 14*sqrt(2)*B*c^(3/2) - 3*A*c^(3/2) + 21*B*c^(3
/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sqrt(2)*a^2 - 7*a^2) - 3*(B*c^2*sgn(t
an(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*e)/a^2 + B*c^2*sgn(tan(1/2*f*x +
 1/2*e) - 1)/a^2)/sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - 2*(3*(sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c^2*sgn(tan(1/2*f*
x + 1/2*e) - 1) + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))^5*B*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - 3*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*c^(5/2)*sgn(tan(1/2*f*x
 + 1/2*e) - 1) + 21*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))^4*B*c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 2*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*A*c^3*sgn(tan(1/2*f*x
 + 1/2*e) - 1) - 14*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))^3*B*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 6*(sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*c^(7/2)*sgn(tan(1/2*f*x
 + 1/2*e) - 1) - 42*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))^2*B*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 9*(sqrt(c)*tan(1/2
*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^4*sgn(tan(1/2*f*x +
 1/2*e) - 1) + 39*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*
e)^2 + c))*B*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + A*c^(9/2)*sgn(tan(1/2*f*x
+ 1/2*e) - 1) - 7*B*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1
/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=78

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}-\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

```
[Out] -((A + 5*B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - B)*Sec
[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*c^2*f)
```

Rubi [A] time = 0.312557, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2855, 2673}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}-\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2,
x]
```

```
[Out] -((A + 5*B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - B)*Sec
[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*c^2*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f} + \frac{(A+5B)\int \sec^2(e+fx)(c-}{6a^2c}$$

$$= -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin}{3a^2c^2f}$$

Mathematica [A] time = 0.280934, size = 87, normalized size = 1.12

$$-\frac{2\sqrt{c-c\sin(e+fx)}(A+3B\sin(e+fx)+2B)}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^2,x]
```

```
[Out] (-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 1.041, size = 63, normalized size = 0.8

$$\frac{2c\left(-1+\sin\left(fx+e\right)\right)\left(3B\sin\left(fx+e\right)+A+2B\right)}{3a^2\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/3*c/a^2*(-1+sin(f*x+e))/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.52507, size = 463, normalized size = 5.94

$$\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{A\left(\sqrt{c}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] 2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sq
rt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Fricas [A] time = 1.59285, size = 157, normalized size = 2.01

$$-\frac{2\left(3B\sin\left(fx+e\right)+A+2B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x
+ e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70291, size = 934, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] -1/6*((13*sqrt(2)*A*sqrt(c) + 5*sqrt(2)*B*sqrt(c) - 18*A*sqrt(c) - 6*B*sqrt
(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(5*sqrt(2)*a^2 - 7*a^2) - 8*(3*(sqrt(c)*
tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c*sgn(tan(1/
2*f*x + 1/2*e) - 1) + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^4*A*c^(3/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 6*(sqrt(c)*tan
(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(3/2)*sgn(tan
(1/2*f*x + 1/2*e) - 1) - 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))^3*A*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - 4*(sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^2*sgn(tan(1/2*
f*x + 1/2*e) - 1) - 6*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))^2*A*c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 12*(sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(5/2)*sgn(tan(
1/2*f*x + 1/2*e) - 1) + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))*A*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 12*(sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^3*sgn(tan(1/2*f*x
+ 1/2*e) - 1) - A*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2*B*c^(7/2)*sgn(t
an(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*
```

```
f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f
*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2))/f
```

3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(2*Sqrt[2]*a^2*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(2*a^2*c*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2
*c^2*f)
```

Rubi [A] time = 0.353657, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2855, 2675, 2649, 206}

$$-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf}+\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]),
x]
```


```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(2*Sqrt[2]*a^2*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(2*a^2*c*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*a^2
*c^2*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f} + \frac{(A+B)\int \sec^2(e+fx)\sqrt{c-}}{2a^2c}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(}{3a^2c^2f}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(}{3a^2c^2f}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx}}{2a^2cf}$$

Mathematica [C] time = 0.533808, size = 176, normalized size = 1.3

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\right.}{6a^2f(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 + 3*
I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(6*a^2*f*(1 + Sin[e + f*x])^2*Sq
rt[c - c*Sin[e + f*x]])
```

Maple [A] time = 1.105, size = 168, normalized size = 1.2

$$-\frac{-1+\sin(fx+e)}{12\,a^2(1+\sin(fx+e))\cos(fx+e)f}\left(3\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c(1+\sin(fx+e))}\sqrt{2}}{\sqrt{c}}\right)(c(1+\sin(fx+e)))^{3/2}cA-6\,Ac^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12*(-1+sin(f*x+e))*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A-6*A*c^(5/2)*sin(f*x+e)+3*2^(1/2)*a
rctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/
2)*c*B-6*B*c^(5/2)*sin(f*x+e)-10*A*c^(5/2)-2*B*c^(5/2))/a^2/c^(5/2)/(1+sin(
f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.77789, size = 603, normalized size = 4.47

$$\frac{3\sqrt{2}\left((A+B)\cos\left(fx+e\right)\sin\left(fx+e\right)+(A+B)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx-\right.\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)s}\right.}{24\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e)
- cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.87945, size = 1046, normalized size = 7.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/6*((30*sqrt(2)*A*c*arctan(sqrt(c)/sqrt(-c)) + 30*sqrt(2)*B*c*arctan(sqrt(
c)/sqrt(-c)) - 42*A*c*arctan(sqrt(c)/sqrt(-c)) - 42*B*c*arctan(sqrt(c)/sqrt
(-c)) - 15*sqrt(2)*A*sqrt(-c)*sqrt(c) + 3*sqrt(2)*B*sqrt(-c)*sqrt(c) + 22*A
*sqrt(-c)*sqrt(c) - 4*B*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(7*
sqrt(2)*a^2*sqrt(-c)*c - 10*a^2*sqrt(-c)*c) + 3*sqrt(2)*(A + B)*arctan(-1/2
*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
 - sqrt(c))/sqrt(-c))/(a^2*sqrt(-c)*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 2*(9*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A - 3*
(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B + 1
5*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*s
qrt(c) + 3*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 +
c))^4*B*sqrt(c) - 10*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1
/2*e)^2 + c))^3*A*c - 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^3*B*c - 30*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*
f*x + 1/2*e)^2 + c))^2*A*c^(3/2) - 6*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c
*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(3/2) + 21*(sqrt(c)*tan(1/2*f*x + 1/2*e
) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^2 + 9*(sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^2 - 5*A*c^(5/2) - B*c^(5/2))
/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 +
2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(
c) - c)^3*a^2*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+B}{6a^2cf\sqrt{c}}$$

```
[Out] ((5*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(8*Sqrt[2]*a^2*c^(3/2)*f) + ((5*A + B)*Cos[e + f*x])/(8*a^2*f*(c - c*Si
n[e + f*x])^(3/2)) - ((5*A + B)*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e +
 f*x]]) - ((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rubi [A] time = 0.391888, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2967, 2855, 2687, 2650, 2649, 206}

$$-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}+\frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}+\frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}}-\frac{(5A+B}{6a^2cf\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((5*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]]
)])/(8*Sqrt[2]*a^2*c^(3/2)*f) + ((5*A + B)*Cos[e + f*x])/(8*a^2*f*(c - c*Si
n[e + f*x])^(3/2)) - ((5*A + B)*Sec[e + f*x])/(6*a^2*c*f*Sqrt[c - c*Sin[e +
 f*x]]) - ((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*c^2*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} + \frac{(5A+B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,d}{6a^2c}$$

$$= -\frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f} +$$

$$= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e}{}$$

$$= \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e}{}$$

$$= \frac{(5A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5}{6a^2}$$

Mathematica [C] time = 0.881429, size = 300, normalized size = 1.71

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-1)^(
1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^3))/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(3/2))
```

Maple [A] time = 1.145, size = 258, normalized size = 1.5

$$-\frac{1}{48\,a^{2}\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\left(15\,\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{c+c\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{c}}\right)\left(c+c\sin\left(fx+e\right)\right)^{3/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/48/c^(7/2)/a^2*(sin(f*x+e)*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A+3*2^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*B-20*A*c^(5/2)-4*
B*c^(5/2))+(30*A*c^(5/2)+6*B*c^(5/2))*cos(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A-3*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)
*c*B-4*A*c^(5/2)-20*B*c^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.81056, size = 560, normalized size = 3.2

$$\frac{3\,\sqrt{2}(5\,A+B)\sqrt{c}\cos\left(fx+e\right)^{3}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\,\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3\,c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)-2\,c\right)\sin\left(fx\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{96\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 + 2
*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1
) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x
+ e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A +
 B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*sin(f*x
 + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=225

$$-\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}+\frac{5(7A-B)\cos(e+f}{64a^2cf(c-c\sin(e+f}$$

```
[Out] (5*(7*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(64*Sqrt[2]*a^2*c^(5/2)*f) + (5*(7*A - B)*Cos[e + f*x])/(64*a^2*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((7*A - B)*Sec[e + f*x])/(24*a^2*c*f*(c - c*Si
n[e + f*x])^(3/2)) - (5*(7*A - B)*Sec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Si
n[e + f*x]]) - ((A - B)*Sec[e + f*x]^3)/(3*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```

Rubi [A] time = 0.483725, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2967, 2855, 2681, 2687, 2650, 2649, 206}

$$-\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}+\frac{5(7A-B)\cos(e+f}{64a^2cf(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)
),x]
```


```
[Out] (5*(7*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(64*Sqrt[2]*a^2*c^(5/2)*f) + (5*(7*A - B)*Cos[e + f*x])/(64*a^2*c*f*(
c - c*Sin[e + f*x])^(3/2)) + ((7*A - B)*Sec[e + f*x])/(24*a^2*c*f*(c - c*Si
n[e + f*x])^(3/2)) - (5*(7*A - B)*Sec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Si
n[e + f*x]]) - ((A - B)*Sec[e + f*x]^3)/(3*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x
]])
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx}{a^2c^2}$$

$$= -\frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(7A-B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}\,dx}{6a^2c}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}} + \frac{(5(7A-B))}{4}$$

$$= \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\mathrm{s}}{3a^2c^2f\sqrt{c-}}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-}{48a^2c^2f\sqrt{}}$$

$$= \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-}{48a^2c^2f\sqrt{}}$$

$$= \frac{5(7A-B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} + \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{}{24}$$

Mathematica [C] time = 1.46215, size = 430, normalized size = 1.91

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(11A+3B)\cos^3(e+fx)+24(B-3A)\left(\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e +
```

```
f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(
7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(A
 + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(11*A +
3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3))/(192*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e +
 f*x])^(5/2))
```

Maple [B] time = 1.369, size = 426, normalized size = 1.9

$$-\frac{1}{384\,a^{2}\left(1+\sin\left(fx+e\right)\right)\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(-210\,Ac^{7/2}\left(\sin\left(fx+e\right)\right)^{3}+30\,Bc^{7/2}\left(\sin\left(fx+e\right)\right)^{3}+7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/384/c^(11/2)/a^2*(-210*A*c^(7/2)*sin(f*x+e)^3+30*B*c^(7/2)*sin(f*x+e)^3+
70*A*c^(7/2)*sin(f*x+e)^2-10*B*c^(7/2)*sin(f*x+e)^2+322*A*c^(7/2)*sin(f*x+e
)-46*B*c^(7/2)*sin(f*x+e)+105*A*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/
2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-15*B*(c*(1+sin
(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2
))*sin(f*x+e)^2*c^2-86*A*c^(7/2)+122*B*c^(7/2)+105*A*(c*(1+sin(f*x+e)))^(3/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^2-15*B*(
c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*c^2-210*A*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*arctanh(1/2*(c*(1+sin
(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^2+30*B*(c*(1+sin(f*x+e)))^(3/
2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)
*c^2)/(1+sin(f*x+e))/(-1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.85341, size = 717, normalized size = 3.19

$$-\frac{15\sqrt{2}\left((7A-B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-(7A-B)\cos\left(fx+e\right)^{3}\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^{2}-2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\cdots}{\cos(fx+e)^{2}+(\cos(fx\cdots}\right.}{768}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/768*(15*sqrt(2)*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(f
*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e)
+ c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(
f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*(7*A
- B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*sin(f*
x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e
)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.83316, size = 1805, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/192*(15*sqrt(2)*(7*A - B)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a^2*sqrt(-c)*
c^2*sgn(tan(1/2*f*x + 1/2*e) - 1)) + 16*(15*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A - 9*(sqrt(c)*tan(1/2*f*x + 1/2*e)
- sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B + 33*(sqrt(c)*tan(1/2*f*x + 1/2*e
) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*sqrt(c) - 15*(sqrt(c)*tan(1/2*f
*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*sqrt(c) - 22*(sqrt(c)
*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*A*c + 10*(sqr
t(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c - 66*
(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*c^(
3/2) + 30*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c
))^2*B*c^(3/2) + 51*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))*A*c^2 - 21*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))*B*c^2 - 11*A*c^(5/2) + 5*B*c^(5/2))/(((sqrt(c)*tan(1/2*f*x
 + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^3*a^2*c^2*sgn(t
an(1/2*f*x + 1/2*e) - 1)) + 6*(53*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*ta
n(1/2*f*x + 1/2*e)^2 + c))^7*A + 29*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*
tan(1/2*f*x + 1/2*e)^2 + c))^7*B - 179*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*A*sqrt(c) - 75*(sqrt(c)*tan(1/2*f*x + 1/2
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*B*sqrt(c) + 127*(sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c + 55*(sqrt(c)*ta
n(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*c + 195*(sqrt(
c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*c^(3/2) +
 91*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B
*c^(3/2) + 7*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^3*A*c^2 - (sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)
^2 + c))^3*B*c^2 - 121*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
 1/2*e)^2 + c))^2*A*c^(5/2) - 65*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan
(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(5/2) - 67*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^3 - 27*(sqrt(c)*tan(1/2*f*x + 1/2*e
) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^3 - 15*A*c^(7/2) - 7*B*c^(7/2))
/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 -
2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(
c) - c)^4*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1)))/f
```

3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=242

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}+\frac{512c^2(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-\frac{2048c^3(A-3B)\sec^3(e}{1}$$

```
[Out] (-2048*(A - 3*B)*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f)
+ (512*(A - 3*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) -
 (64*(A - 3*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (16
*(A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*f) - ((A - 3*
B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c*f) - ((A - B)*Sec[e
 + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.64716, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}+\frac{512c^2(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f}-\frac{2048c^3(A-3B)\sec^3(e}{1}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
3,x]
```


```
[Out] (-2048*(A - 3*B)*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f)
+ (512*(A - 3*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) -
 (64*(A - 3*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (16
*(A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*f) - ((A - 3*
B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c*f) - ((A - B)*Sec[e
 + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/(5*a^3*c^3*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
```

```
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{(A-3B)\int \sec^4(e+fx)(\ldots}{2a^3\ldots}$$

$$= -\frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c\ldots}{5a^3c^3\ldots}$$

$$= -\frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec^3(e+fx)\ldots}{5a^3\ldots}$$

$$= -\frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A-3B)\sec^3(e+\ldots}{15\ldots}$$

$$= \frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{64(A-3B)c\sec^3(e\ldots}{\ldots}$$

$$= -\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{512(A-3B)c^2\,\mathrm{se}\ldots}{\ldots}$$

Mathematica [A] time = 4.42569, size = 176, normalized size = 0.73

$$-\frac{c^4(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}(-40(137A-402B)\cos(2(e+fx))-10(A-6B)\cos(4(e+fx))+15600A\sin(e+\ldots}{120a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 33516*B - 4
0*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)] + 15600*
A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1335*B*Sin
[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(120*a^3*f*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.92, size = 143, normalized size = 0.6

$$-\frac{2c^5\left(-1+\sin\left(fx+e\right)\right)\left(3B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4+(100A-336B)\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+(-1000A+3\ldots}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\mathrm{c}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^5/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(3*B*sin(f*x+e)*cos(f*x+e)^4
+(100*A-336*B)*cos(f*x+e)^2*sin(f*x+e)+(-1000*A+3048*B)*sin(f*x+e)+(5*A-30*
B)*cos(f*x+e)^4+(680*A-1980*B)*cos(f*x+e)^2-1048*A+3096*B)/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.65223, size = 1276, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c^
(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 30
200*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e
) + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5301*c^(9
/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x + e)^13/(c
os(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*A/
((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2)) - 6*(181*c^(9/2) + 905*c^(9/
2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2627*c^(9/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 5870*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10521*c^(9/
2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15351*c^(9/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 19695*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 20772
*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 19695*c^(9/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8 + 15351*c^(9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 +
 10521*c^(9/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 5870*c^(9/2)*sin(f*x
 + e)^11/(cos(f*x + e) + 1)^11 + 2627*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e)
 + 1)^12 + 905*c^(9/2)*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 181*c^(9/2)*
sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*
```

```
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(9/2)))/f
```

Fricas [A] time = 1.90222, size = 420, normalized size = 1.74

$$-\frac{2\left(5\,(A-6\,B)c^4\cos\left(fx+e\right)^4+20\,(34\,A-99\,B)c^4\cos\left(fx+e\right)^2-8\,(131\,A-387\,B)c^4+\left(3\,Bc^4\cos\left(fx+e\right)^4+4\,(25\,A\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^2
 - 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*co
s(f*x + e)^2 - 8*(125*A - 381*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e) +
c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(
f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.9761, size = 2191, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/60*(2*(2255*sqrt(2)*A*a^18*sqrt(c) - 8241*sqrt(2)*B*a^18*sqrt(c) - 3190*A
*a^18*sqrt(c) + 11658*B*a^18*sqrt(c) - 4824*sqrt(2)*A*c^(19/2) + 15144*sqrt
(2)*B*c^(19/2) + 6800*A*c^(19/2) - 21360*B*c^(19/2))*sgn(tan(1/2*f*x + 1/2*
e) - 1)/(29*sqrt(2)*a^3*c^5 - 41*a^3*c^5) + ((((((115*A*a^15*c^7*sgn(tan(1/
2*f*x + 1/2*e) - 1) - 438*B*a^15*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2
*f*x + 1/2*e)/c^9 + 15*(7*A*a^15*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 24*B*a
^15*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 10*(22*A
*a^15*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 81*B*a^15*c^7*sgn(tan(1/2*f*x + 1
/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + 10*(22*A*a^15*c^7*sgn(tan(1/2*f*x +
 1/2*e) - 1) - 81*B*a^15*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*
x + 1/2*e) + 15*(7*A*a^15*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 24*B*a^15*c^7
*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^9)*tan(1/2*f*x + 1/2*e) + (115*A*a^15*c^7
*sgn(tan(1/2*f*x + 1/2*e) - 1) - 438*B*a^15*c^7*sgn(tan(1/2*f*x + 1/2*e) -
1))/c^9)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(5/2) + 128*(15*(sqrt(c)*tan(1/2*f*
x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*c^5*sgn(tan(1/2*f*x +
1/2*e) - 1) - 45*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))^9*B*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) + 105*(sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*A*c^(11/2)*sgn(tan(1/2*f*x
 + 1/2*e) - 1) - 375*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1
/2*e)^2 + c))^8*B*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 340*(sqrt(c)*tan
(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*c^6*sgn(tan(1/2
*f*x + 1/2*e) - 1) - 900*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))^7*B*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 260*(sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*A*c^(13/2)*sgn(tan
(1/2*f*x + 1/2*e) - 1) + 780*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2
*f*x + 1/2*e)^2 + c))^6*B*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1054*(sq
rt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c^7*sg
n(tan(1/2*f*x + 1/2*e) - 1) + 2754*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))^5*B*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) + 670*(sq
rt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*c^(15/
2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1650*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqr
t(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1)
 + 900*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^
3*A*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2340*(sqrt(c)*tan(1/2*f*x + 1/2*e)
- sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1)
 - 980*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^
2*A*c^(17/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 2460*(sqrt(c)*tan(1/2*f*x + 1/
2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(17/2)*sgn(tan(1/2*f*x + 1
/2*e) - 1) + 295*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))*A*c^9*sgn(tan(1/2*f*x + 1/2*e) - 1) - 765*(sqrt(c)*tan(1/2*f*x +
1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^9*sgn(tan(1/2*f*x + 1/2*e)
```

```
 - 1) - 31*A*c^(19/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 81*B*c^(19/2)*sgn(tan
(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*
x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3} dx$

Optimal. Leaf size=209

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}-\frac{128c^2(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(3A-13B)\sec^3(e-$$

```
[Out] (-128*(3*A - 13*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f
) + (32*(3*A - 13*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f)
 - (4*(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (
(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*c*f) - ((A
- B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.567435, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}-\frac{128c^2(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f}-\frac{(3A-13B)\sec^3(e-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
3,x]
```


```
[Out] (-128*(3*A - 13*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f
) + (32*(3*A - 13*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f)
 - (4*(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (
(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*c*f) - ((A
- B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/(5*a^3*c^3*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f} - \frac{(3A-13B)\int \sec^4(e+f}{10}$$

$$= -\frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c}{5a^3c}$$

$$= -\frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A-13B)\sec^3(e+}{15}$$

$$= \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3A-13B)\sec^3(e}{}$$

$$= -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{32(3A-13B)c\,s}{}$$

Mathematica [A] time = 2.76265, size = 158, normalized size = 0.76

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)((2200B-540A)\cos(2(e+fx))+1410A\sin(e+fx)-30A}{60a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(1092*
A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] + 14
10*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 170*B*Sin
[3*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x])^3)
```

Maple [A] time = 1.097, size = 121, normalized size = 0.6

$$\frac{2c^4\left(-1+\sin\left(fx+e\right)\right)\left((-15A+85B)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+(180A-820B)\sin\left(fx+e\right)+5B\left(\cos\left(fx+e\right)\right.\right.}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c^4/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*((-15*A+85*B)*sin(f*x+e)*cos(
f*x+e)^2+(180*A-820*B)*sin(f*x+e)+5*B*cos(f*x+e)^4+(-135*A+545*B)*cos(f*x+e
)^2+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.6508, size = 1153, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(3*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^(
7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1220*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 106
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
110*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(f*x + e)
^12/(cos(f*x + e) + 1)^12)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1)
+ 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))
 - 2*(147*c^(7/2) + 735*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1992*c^(7
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4015*c^(7/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 6605*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8370*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9520*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 8370*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 660
5*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4015*c^(7/2)*sin(f*x + e)^9
/(cos(f*x + e) + 1)^9 + 1992*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10
+ 735*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 147*c^(7/2)*sin(f*x +
 e)^12/(cos(f*x + e) + 1)^12)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/
2)))/f
```

Fricas [A] time = 1.81164, size = 367, normalized size = 1.76

$$\frac{2\left(5\,Bc^3\cos\left(fx+e\right)^4-5\,(27\,A-109\,B)c^3\cos\left(fx+e\right)^2+4\,(51\,A-211\,B)c^3-5\left((3\,A-17\,B)c^3\cos\left(fx+e\right)^2-4\,(9\,A-\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51*
A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)*s
in(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.61968, size = 2021, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] 1/15*((615*sqrt(2)*A*a^12*sqrt(c) - 3895*sqrt(2)*B*a^12*sqrt(c) - 870*A*a^1
2*sqrt(c) + 5510*B*a^12*sqrt(c) - 426*sqrt(2)*A*c^(13/2) + 1986*sqrt(2)*B*c
^(13/2) + 600*A*c^(13/2) - 2800*B*c^(13/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(
29*sqrt(2)*a^3*c^3 - 41*a^3*c^3) + 5*((((3*A*a^9*c^5*sgn(tan(1/2*f*x + 1/2*
e) - 1) - 20*B*a^9*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1))*tan(1/2*f*x + 1/2*e)/
c^6 + 3*(A*a^9*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 6*B*a^9*c^5*sgn(tan(1/2*
f*x + 1/2*e) - 1))/c^6)*tan(1/2*f*x + 1/2*e) + 3*(A*a^9*c^5*sgn(tan(1/2*f*x
 + 1/2*e) - 1) - 6*B*a^9*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1))/c^6)*tan(1/2*f*
x + 1/2*e) + (3*A*a^9*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*B*a^9*c^5*sgn(
tan(1/2*f*x + 1/2*e) - 1))/c^6)/(c*tan(1/2*f*x + 1/2*e)^2 + c)^(3/2) + 8*(1
5*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*c
^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 45*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^9*B*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + 45*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*A*c^(9
/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 375*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqr
t(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*B*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1)
+ 300*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7
*A*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1060*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*B*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1)
- 180*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6
*A*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 860*(sqrt(c)*tan(1/2*f*x + 1/2*
```

```
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*B*c^(11/2)*sgn(tan(1/2*f*x + 1/2
*e) - 1) - 918*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 + c))^5*A*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) + 3298*(sqrt(c)*tan(1/2*f*x +
 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*c^6*sgn(tan(1/2*f*x + 1/2
*e) - 1) + 630*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 + c))^4*A*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2050*(sqrt(c)*tan(1/2*
f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(13/2)*sgn(tan(1/2
*f*x + 1/2*e) - 1) + 780*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))^3*A*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2820*(sqrt(c)*tan
(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^7*sgn(tan(1/2
*f*x + 1/2*e) - 1) - 900*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
 + 1/2*e)^2 + c))^2*A*c^(15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 3020*(sqrt(c
)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(15/2)*s
gn(tan(1/2*f*x + 1/2*e) - 1) + 255*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))*A*c^8*sgn(tan(1/2*f*x + 1/2*e) - 1) - 925*(sqrt
(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^8*sgn(ta
n(1/2*f*x + 1/2*e) - 1) - 27*A*c^(17/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 97*
B*c^(17/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=160

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}-\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}+\frac{8(A-11B)\sec^3(e+fx)(c}{5a^3f}$$

```
[Out] (-32*(A - 11*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) + (
8*(A - 11*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) - ((A - 1
1*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.479844, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}-\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf}+\frac{8(A-11B)\sec^3(e+fx)(c}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] (-32*(A - 11*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) + (
8*(A - 11*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) - ((A - 1
1*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c^3*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}\,dx}{a^3c^3}$$
$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f} - \frac{(A-11B)\int \sec^4(e+fx)}{10a}$$
$$= -\frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c}{5a^3c^3}$$
$$= \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-11B)\sec^3(e+fx)(}{5a^3c}$$
$$= -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + 8(A-11B)\sec^3(e+$$

Mathematica [A] time = 1.26005, size = 132, normalized size = 0.82

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(5(8A-133B)\sin(e+fx)-30(A-8B)\cos(2(e+fx))+58A}{30a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(58*A
- 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*x] + 15
*B*Sin[3*(e + f*x)]))/(30*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 +
Sin[e + f*x])^3)
```

Maple [A] time = 1.176, size = 105, normalized size = 0.7

$$\frac{2c^3\left(-1+\sin\left(fx+e\right)\right)\left(15B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+\left(10A-170B\right)\sin\left(fx+e\right)+\left(-15A+120B\right)\left(\cos\left(fx+e\right.\right.\right.}{15a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c^3/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)
+(10*A-170*B)*sin(f*x+e)+(-15*A+120*B)*cos(f*x+e)^2+22*A-182*B)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.60501, size = 1027, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2)*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2)*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e)
```

```
+ 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(31*c^(5/2) + 155*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 395*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 680*
c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1030*c^(5/2)*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 1050*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 10
30*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 680*c^(5/2)*sin(f*x + e)^7
/(cos(f*x + e) + 1)^7 + 395*c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1
55*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 31*c^(5/2)*sin(f*x + e)^10
/(cos(f*x + e) + 1)^10)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 1
0*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Fricas [A] time = 1.72745, size = 313, normalized size = 1.96

$$-\frac{2\left(15\,(A-8\,B)c^{2}\cos\left(fx+e\right)^{2}-2\,(11\,A-91\,B)c^{2}-5\left(3\,Bc^{2}\cos\left(fx+e\right)^{2}+2\,(A-17\,B)c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx\right.}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*c
os(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(
a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x
+ e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.53042, size = 1716, normalized size = 10.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -2/15*((39*sqrt(2)*A*c^(5/2) + 441*sqrt(2)*B*c^(5/2) - 55*A*c^(5/2) - 625*B
*c^(5/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29*sqrt(2)*a^3 - 41*a^3) + 15*(B*c
^3*sgn(tan(1/2*f*x + 1/2*e) - 1)*tan(1/2*f*x + 1/2*e)/a^3 + B*c^3*sgn(tan(1
/2*f*x + 1/2*e) - 1)/a^3)/sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c) - 2*(15*(sqrt(
c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*c^3*sgn(t
an(1/2*f*x + 1/2*e) - 1) - 15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))^9*B*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - 15*(sqrt(c)*
tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*A*c^(7/2)*sgn(
tan(1/2*f*x + 1/2*e) - 1) - 105*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(
1/2*f*x + 1/2*e)^2 + c))^8*B*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 100*(s
qrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*c^4*s
gn(tan(1/2*f*x + 1/2*e) - 1) - 500*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))^7*B*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqr
t(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*A*c^(9/2)
*sgn(tan(1/2*f*x + 1/2*e) - 1) + 340*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c
*tan(1/2*f*x + 1/2*e)^2 + c))^6*B*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 2
38*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*
c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) + 1598*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sq
rt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) + 1
90*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*
c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1070*(sqrt(c)*tan(1/2*f*x + 1/2*e)
 - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e
) - 1) + 180*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^3*A*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - 1380*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^6*sgn(tan(1/2*f*x + 1/2*e
) - 1) - 260*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^2*A*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 1540*(sqrt(c)*tan(1/2*f*
x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(13/2)*sgn(tan(1/2*f
*x + 1/2*e) - 1) + 55*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
1/2*e)^2 + c))*A*c^7*sgn(tan(1/2*f*x + 1/2*e) - 1) - 455*(sqrt(c)*tan(1/2*f
*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^7*sgn(tan(1/2*f*x + 1
```

```
/2*e) - 1) - 7*A*c^(15/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 47*B*c^(15/2)*sgn
(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2
*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}+\frac{4(A+9B)\sec^3(e+fx)(c-}{15a^3f}$$

```
[Out] (4*(A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) - ((A +
9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.412688, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}-\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf}+\frac{4(A+9B)\sec^3(e+fx)(c-}{15a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
3,x]
```


```
[Out] (4*(A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) - ((A +
9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c*f) - ((A - B)*Sec[
e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/(5*a^3*c^3*f)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
```

```
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2674

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ
[m + p, 0]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f} + \frac{(A+9B)\int \sec^4(e+fx)(\cdots}{10a^3\cdots}$$

$$= -\frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-\cdots}{5a^3c^3f\cdots}$$

$$= \frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(A+9B)\sec^3(e+fx)(c\cdots}{5a^3cf\cdots}$$

Mathematica [A] time = 0.713139, size = 113, normalized size = 0.93

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(10(A+3B)\sin(e+fx)-2A-15B\cos(2(e+fx))+27B)}{15a^3f(\sin(e+fx)+1)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^3,x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*x
)] + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.993, size = 83, normalized size = 0.7

$$-{\frac{2\,{c}^{2} \left(-1+\sin \left(fx+e \right) \right) \left(\sin \left(fx+e \right) \left(5\,A+15\,B \right) -15\,B \left(\cos \left(fx+e \right) \right) ^{2}-A+21\,B \right) }{15\,{a}^{3} \left(1+\sin \left(fx+e \right) \right) ^{2}\cos \left(fx+e \right) f}{\frac{1}{\sqrt{c-c\sin \left(fx+e \right) }}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/15*c^2/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(sin(f*x+e)*(5*A+15*B)-15*B*
cos(f*x+e)^2-A+21*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.58665, size = 895, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*((c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 6
*(c^(3/2) + 5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 14*c^(3/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 15*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 26*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 14*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
```

```
 5*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A] time = 1.77413, size = 246, normalized size = 2.03

$$\frac{2\left(15\,Bc\cos\left(fx+e\right)^{2}-5\,(A+3\,B)c\sin\left(fx+e\right)+(A-21\,B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*sq
rt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*
x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.99072, size = 1391, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1/15*((99*sqrt(2)*A*c^(3/2) + 21*sqrt(2)*B*c^(3/2) - 140*A*c^(3/2) - 30*B*
c^(3/2))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29*sqrt(2)*a^3 - 41*a^3) - 4*(15*(s
qrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*c^2*s
gn(tan(1/2*f*x + 1/2*e) - 1) + 15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*ta
n(1/2*f*x + 1/2*e)^2 + c))^8*A*c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 30*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*B*c^(5
/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 40*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt
(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) - 60*
(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*B*c^3
*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*
tan(1/2*f*x + 1/2*e)^2 + c))^6*A*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 34
*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c^
4*sgn(tan(1/2*f*x + 1/2*e) - 1) + 204*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^5*B*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) + 10*(
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*c^(9
/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 180*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqr
t(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*B*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1)
- 180*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3
*B*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) -
1) + 240*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c)
)^2*B*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 5*(sqrt(c)*tan(1/2*f*x + 1/2
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^6*sgn(tan(1/2*f*x + 1/2*e) -
1) - 60*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
*B*c^6*sgn(tan(1/2*f*x + 1/2*e) - 1) - A*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e)
- 1) + 6*B*c^(13/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x +
 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x +
1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=85

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}-\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}$$

```
[Out] -((3*A + 7*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*c*f) - ((A
 - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.315198, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2967, 2855, 2673}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}-\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3,
x]
```

```
[Out] -((3*A + 7*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*c*f) - ((A
 - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*c^3*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx}{a^3c^3}$$
$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}+\frac{(3A+7B)\int \sec^4(e+fx)(}{10a^3c}$$
$$= -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf}-\frac{(A-B)\sec^5(e+fx)(c-}{5a^3c^3f}$$

Mathematica [A] time = 0.306657, size = 89, normalized size = 1.05

$$-\frac{2\sqrt{c-c\sin(e+fx)}(3A+5B\sin(e+fx)+2B)}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^3,x]
```

```
[Out] (-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 1.001, size = 65, normalized size = 0.8

$$\frac{2\,c\left(-1+\sin\left(fx+e\right)\right)\left(5\,B\sin\left(fx+e\right)+3\,A+2\,B\right)}{15\,a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/15*c/a^3*(-1+sin(f*x+e))/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.56314, size = 682, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] 2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5*a^3*sin(f
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)))/f
```

Fricas [A] time = 1.62582, size = 196, normalized size = 2.31

$$\frac{2\left(5\,B\sin\left(fx+e\right)+3\,A+2\,B\right)\sqrt{-c\sin\left(fx+e\right)+c}}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*
x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72661, size = 1539, normalized size = 18.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] -1/60*((573*sqrt(2)*A*sqrt(c) + 177*sqrt(2)*B*sqrt(c) - 810*A*sqrt(c) - 250
*B*sqrt(c))*sgn(tan(1/2*f*x + 1/2*e) - 1)/(29*sqrt(2)*a^3 - 41*a^3) - 16*(1
5*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^9*A*c
*sgn(tan(1/2*f*x + 1/2*e) - 1) + 45*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*
tan(1/2*f*x + 1/2*e)^2 + c))^8*A*c^(3/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 30
*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^8*B*c^
(3/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 60*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sq
rt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*c^2*sgn(tan(1/2*f*x + 1/2*e) - 1) + 2
0*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*B*c
^2*sgn(tan(1/2*f*x + 1/2*e) - 1) - 60*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(
c*tan(1/2*f*x + 1/2*e)^2 + c))^6*A*c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) -
40*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*B*
c^(5/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) - 102*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
 sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1)
- 68*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*
B*c^3*sgn(tan(1/2*f*x + 1/2*e) - 1) + 30*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sq
rt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4*A*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1)
```

```
 + 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^4
*B*c^(7/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 60*(sqrt(c)*tan(1/2*f*x + 1/2*e)
 - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*A*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1
) + 60*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^
3*B*c^4*sgn(tan(1/2*f*x + 1/2*e) - 1) - 60*(sqrt(c)*tan(1/2*f*x + 1/2*e) -
sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) -
1) - 40*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))
^2*B*c^(9/2)*sgn(tan(1/2*f*x + 1/2*e) - 1) + 15*(sqrt(c)*tan(1/2*f*x + 1/2*
e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1
) + 20*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*
B*c^5*sgn(tan(1/2*f*x + 1/2*e) - 1) - 3*A*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e)
 - 1) - 2*B*c^(11/2)*sgn(tan(1/2*f*x + 1/2*e) - 1))/(((sqrt(c)*tan(1/2*f*x
+ 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x +
 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3))/f
```

3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=174

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^2c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin}}{4a^3cf}$$

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(4*Sqrt[2]*a^3*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(4*a^3*c*f) - ((A + B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3
*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rubi [A] time = 0.437845, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2967, 2855, 2675, 2649, 206}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}-\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f}-\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin}}{4a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]),
x]
```

```
[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]
)/(4*Sqrt[2]*a^3*Sqrt[c]*f) - ((A + B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]
])/(4*a^3*c*f) - ((A + B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(6*a^3
*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c^3*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f} + \frac{(A+B)\int \sec^4(e+fx)(c-}{2a^3c^2}$$

$$= -\frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c}{5a^3c^3f}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(c-c\sin}{6a^3c^2f}$$

$$= -\frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(c-c\sin}{6a^3c^2f}$$

$$= \frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+f}{4a^3cf}$$

Mathematica [C] time = 0.785356, size = 204, normalized size = 1.17

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-15(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(A
 + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(A +
 B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^5))/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e +
f*x]])
```

Maple [A] time = 1.209, size = 200, normalized size = 1.2

$$\frac{-1+\sin\left(fx+e\right)}{120\,a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\left(74\,c^{9/2}A+80\,Ac^{9/2}\sin\left(fx+e\right)+30\,Ac^{9/2}\left(\sin\left(fx+e\right)\right)^2-15\,\sqrt{2}\mathrm{Artanh}\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/120/a^3*(-1+sin(f*x+e))/c^(9/2)/(1+sin(f*x+e))^2*(74*c^(9/2)*A+80*A*c^(9/
2)*sin(f*x+e)+30*A*c^(9/2)*sin(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*
x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*A+26*c^(9/2)*B+8
0*B*c^(9/2)*sin(f*x+e)+30*B*c^(9/2)*sin(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c*
(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*B)/cos(
f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.76415, size = 729, normalized size = 4.19

$$15\sqrt{2}\left((A+B)\cos\left(fx+e\right)^{3}-2\,(A+B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A+B)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\,s}\cdots}{\cdots}\right.\cdots$$

$$240\left(a^{3}cf\cos\left(fx-\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/240*(15*sqrt(2)*((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x
+ e) - 2*(A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*s
qrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*co
s(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (
cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(A + B)*cos(f*x
```

```
 + e)^2 - 40*(A + B)*sin(f*x + e) - 52*A - 28*B)*sqrt(-c*sin(f*x + e) + c))
/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*
cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.14438, size = 1511, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/60*((870*sqrt(2)*A*c*arctan(sqrt(c)/sqrt(-c)) + 870*sqrt(2)*B*c*arctan(sq
rt(c)/sqrt(-c)) - 1230*A*c*arctan(sqrt(c)/sqrt(-c)) - 1230*B*c*arctan(sqrt(
c)/sqrt(-c)) - 850*sqrt(2)*A*sqrt(-c)*sqrt(c) - 40*sqrt(2)*B*sqrt(-c)*sqrt(
c) + 1203*A*sqrt(-c)*sqrt(c) + 57*B*sqrt(-c)*sqrt(c))*sgn(tan(1/2*f*x + 1/2
*e) - 1)/(41*sqrt(2)*a^3*sqrt(-c)*c - 58*a^3*sqrt(-c)*c) + 15*sqrt(2)*(A +
B)*arctan(-1/2*sqrt(2)*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x +
 1/2*e)^2 + c) - sqrt(c))/sqrt(-c))/(a^3*sqrt(-c)*sgn(tan(1/2*f*x + 1/2*e)
- 1)) + 2*(105*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 + c))^9*A - 15*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e
)^2 + c))^9*B + 435*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/
2*e)^2 + c))^8*A*sqrt(c) + 75*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/
2*f*x + 1/2*e)^2 + c))^8*B*sqrt(c) + 580*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sq
rt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*A*c + 100*(sqrt(c)*tan(1/2*f*x + 1/2*e)
 - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^7*B*c - 620*(sqrt(c)*tan(1/2*f*x + 1
/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*A*c^(3/2) - 140*(sqrt(c)*tan(
1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^6*B*c^(3/2) - 1258*(
```

```
sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5*A*c^2
- 442*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^5
*B*c^2 + 490*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2
+ c))^4*A*c^(5/2) + 250*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 + c))^4*B*c^(5/2) + 900*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*t
an(1/2*f*x + 1/2*e)^2 + c))^3*A*c^3 + 420*(sqrt(c)*tan(1/2*f*x + 1/2*e) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^3*B*c^3 - 860*(sqrt(c)*tan(1/2*f*x + 1/2
*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*A*c^(7/2) - 380*(sqrt(c)*tan(1/
2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))^2*B*c^(7/2) + 265*(sqr
t(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*A*c^4 + 145
*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 + c))*B*c^4
- 37*A*c^(9/2) - 13*B*c^(9/2))/(((sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan
(1/2*f*x + 1/2*e)^2 + c))^2 + 2*(sqrt(c)*tan(1/2*f*x + 1/2*e) - sqrt(c*tan(
1/2*f*x + 1/2*e)^2 + c))*sqrt(c) - c)^5*a^3*sgn(tan(1/2*f*x + 1/2*e) - 1)))
/f
```

3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e\cdots}{\sqrt{2}\sqrt{c-c\sin\cdots}}\right.}{16\sqrt{2}a^3c^{3/2}f}$$

```
[Out] ((7*A + 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + ((7*A + 3*B)*Cos[e + f*x])/(16*a^3*f*(c
- c*Sin[e + f*x])^(3/2)) - ((7*A + 3*B)*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c -
c*Sin[e + f*x]]) - ((7*A + 3*B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3
0*a^3*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c
^3*f)
```

Rubi [A] time = 0.479694, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2967, 2855, 2675, 2687, 2650, 2649, 206}

$$-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}-\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f}+\frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e\cdots}{\sqrt{2}\sqrt{c-c\sin\cdots}}\right.}{16\sqrt{2}a^3c^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)
),x]
```

```
[Out] ((7*A + 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + ((7*A + 3*B)*Cos[e + f*x])/(16*a^3*f*(c
- c*Sin[e + f*x])^(3/2)) - ((7*A + 3*B)*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c -
c*Sin[e + f*x]]) - ((7*A + 3*B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(3
0*a^3*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c
^3*f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
```

```
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f} + \frac{(7A+3B)\int \sec^4(e+f}{10a}$$

$$= -\frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-}{5a^3c^3f}$$

$$= -\frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+f}}{30a^3c^2f}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\mathrm{s}}{}$$

$$= \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\mathrm{s}}{}$$

$$= \frac{(7A+3B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} + \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{}{1}$$

Mathematica [C] time = 1.42393, size = 357, normalized size = 1.59

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(15(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-40*A*Cos[e + f*x]^2 + 24*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2 - 30*(3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2]
```

```
 + Sin[(e + f*x)/2])^4 + 15*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (15 + 15*I)*(-1)^(1/4)*(7*A + 3*B)
*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(A + B)*Si
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(240*a^3*f*(1 + Si
n[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))
```

Maple [A] time = 1.18, size = 308, normalized size = 1.4

$$-\frac{1}{480\,a^3\left(1+\sin\left(fx+e\right)\right)^2\cos\left(fx+e\right)f}\left(-210\,Ac^{7/2}\left(\sin\left(fx+e\right)\right)^3-90\,Bc^{7/2}\left(\sin\left(fx+e\right)\right)^3-350\,Ac^{7/2}\left(\sin\left(fx+e\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/480/c^(9/2)/a^3*(-210*A*c^(7/2)*sin(f*x+e)^3-90*B*c^(7/2)*sin(f*x+e)^3-3
50*A*c^(7/2)*sin(f*x+e)^2-150*B*c^(7/2)*sin(f*x+e)^2+42*A*c^(7/2)*sin(f*x+e
)+18*B*c^(7/2)*sin(f*x+e)+278*A*c^(7/2)-18*B*c^(7/2)+105*A*(c*(1+sin(f*x+e)
))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(
f*x+e)*c+45*B*(c*(1+sin(f*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-105*A*(c*(1+sin(f*x+e)))^(5/2)*2^(1/
2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-45*B*(c*(1+sin(f
*x+e)))^(5/2)*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*c)/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.89959, size = 732, normalized size = 3.27

$$\frac{15\sqrt{2}\left((7A+3B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+(7A+3B)\cos\left(fx+e\right)^{3}\right)\sqrt{c}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx\right.\right.}{\cos\left(fx+e\right)^{2}+\left(\cos\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/960*(15*sqrt(2)*((7*A + 3*B)*cos(f*x + e)^3*sin(f*x + e) + (7*A + 3*B)*co
s(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x +
e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*c
os(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(25*(7*A + 3*B)*cos(f*x + e)^2 + 3*(
5*(7*A + 3*B)*cos(f*x + e)^2 - 28*A - 12*B)*sin(f*x + e) - 36*A - 84*B)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*
cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{7(9A+B)\tanh}{128\sqrt{}}$$

```
[Out] (7*(9*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(128*Sqrt[2]*a^3*c^(5/2)*f) + (7*(9*A + B)*Cos[e + f*x])/(128*a^3*c*f
*(c - c*Sin[e + f*x])^(3/2)) + (7*(9*A + B)*Sec[e + f*x])/(240*a^3*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (7*(9*A + B)*Sec[e + f*x])/(96*a^3*c^2*f*Sqrt[c -
 c*Sin[e + f*x]]) - ((9*A + B)*Sec[e + f*x]^3)/(30*a^3*c^2*f*Sqrt[c - c*Sin
[e + f*x]]) - ((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*c^3*
f)
```

Rubi [A] time = 0.55501, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {2967, 2855, 2687, 2681, 2650, 2649, 206}

$$-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}-\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{7(9A+B)\tanh}{128\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)
),x]
```

```
[Out] (7*(9*A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x
]])])/(128*Sqrt[2]*a^3*c^(5/2)*f) + (7*(9*A + B)*Cos[e + f*x])/(128*a^3*c*f
*(c - c*Sin[e + f*x])^(3/2)) + (7*(9*A + B)*Sec[e + f*x])/(240*a^3*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (7*(9*A + B)*Sec[e + f*x])/(96*a^3*c^2*f*Sqrt[c -
 c*Sin[e + f*x]]) - ((9*A + B)*Sec[e + f*x]^3)/(30*a^3*c^2*f*Sqrt[c - c*Sin
[e + f*x]]) - ((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(5*a^3*c^3*
f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
```

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx}{a^3c^3}$$

$$= -\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f} + \frac{(9A+B)\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}\,d}{10a^3c^2}$$

$$= -\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)}{}$$

$$= \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(9A+}{30a^3c^2f}$$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9}{96a^3c}$$

$$= \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} + \frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}} - \frac{7(9}{96a^3c}$$

$$= \frac{7(9A+B)\tanh^{-1}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} + \frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}} +$$

Mathematica [C] time = 2.3776, size = 479, normalized size = 1.86

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(15(15A+7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
```

```
^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (105 + 10
5*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/
4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(1920*a^3*f*(1 + Sin[e + f*
x])^3*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A] time = 1.475, size = 410, normalized size = 1.6

$$-\frac{1}{3840\,a^{3}\left(1+\sin\left(fx+e\right)\right)^{2}\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\left(1260\,c^{9/2}A+140\,c^{9/2}B\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3840/c^(13/2)/a^3*((1260*c^(9/2)*A+140*c^(9/2)*B)*sin(f*x+e)*cos(f*x+e)^
2+(-1890*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*s
in(f*x+e))^(5/2)*c^2*A+864*c^(9/2)*A-210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e
))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B+96*c^(9/2)*B)*sin(f*
x+e)+(-1890*c^(9/2)*A-210*c^(9/2)*B)*cos(f*x+e)^4+(-945*2^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+252*c
^(9/2)*A-105*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c
+c*sin(f*x+e))^(5/2)*c^2*B+28*c^(9/2)*B)*cos(f*x+e)^2+1890*2^(1/2)*arctanh(
1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A+96
*c^(9/2)*A+210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
(c+c*sin(f*x+e))^(5/2)*c^2*B+864*c^(9/2)*B)/(1+sin(f*x+e))^2/(-1+sin(f*x+e)
)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.8395, size = 659, normalized size = 2.55

$$105\sqrt{2}(9A+B)\sqrt{c}\cos\left(fx+e\right)^{5}\log\left(-\frac{c\cos\left(fx+e\right)^{2}+2\sqrt{2}\sqrt{-c\sin\left(fx+e\right)+c}\sqrt{c}\left(\cos\left(fx+e\right)+\sin\left(fx+e\right)+1\right)+3c\cos\left(fx+e\right)+\left(c\cos\left(fx+e\right)-2c\right)\text{si}}{\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^2
 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e)
 + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(
f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(105*
(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A + B)*co
s(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin(f*x +
 e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.131 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=94

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rubi [A] time = 0.339329, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}\,dx$$
$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos}{5}$$

Mathematica [A] time = 0.995429, size = 118, normalized size = 1.26

$$-\frac{c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(4(23B-60A)\sin(e+fx)+4\cos(2(e+fx))(4(5A-6B)\sin(e+f}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] -(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(
-60*A + 23*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-35*A + 25*B + 4*(5*A - 6*
B)*Sin[e + f*x]) + Cos[4*(e + f*x)]*(5*A - 15*B + 4*B*Sin[e + f*x])))/(160*
f)
```

Maple [B] time = 0.398, size = 174, normalized size = 1.9

$$\frac{\left(-4B\left(\cos\left(fx+e\right)\right)^4+5A\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-15B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-20A\left(\cos\left(fx+e\right)\right)^2+28}{20f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-3\left(\cos\left(fx+e\right)\right)^2-4\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*sin
(f*x+e)-20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)+25*B*sin(f*x+e)
```

```
+40*A-24*B)*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/
(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(7/2), x)
```

Fricas [A] time = 1.81031, size = 342, normalized size = 3.64

$$-\frac{\left(5\,(A-3\,B)c^3\cos\left(fx+e\right)^4-40\,(A-B)c^3\cos\left(fx+e\right)^2+5\,(7\,A-5\,B)c^3+4\left(Bc^3\cos\left(fx+e\right)^4+(5\,A-7\,B)c^3\cos\right.\right.}{20\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/20*(5*(A - 3*B)*c^3*cos(f*x + e)^4 - 40*(A - B)*c^3*cos(f*x + e)^2 + 5*(
7*A - 5*B)*c^3 + 4*(B*c^3*cos(f*x + e)^4 + (5*A - 7*B)*c^3*cos(f*x + e)^2 -
 2*(5*A - 3*B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

3.132 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=94

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rubi [A] time = 0.337209, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a + a\sin(e+fx)}(A + B\sin(e+fx))(c - c\sin(e+fx))^{5/2}\,dx = (A+B)\int \sqrt{a + a\sin(e+fx)}(c - c\sin(e+fx))^{5/2}\,dx$$
$$= -\frac{a(A+B)\cos(e+fx)(c - c\sin(e+fx))^{5/2}}{3f\sqrt{a + a\sin(e+fx)}} + \frac{aB\cos}{4}$$

Mathematica [A] time = 0.841178, size = 102, normalized size = 1.09

$$\frac{c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c - c\sin(e+fx)}(16(7A-2B)\sin(e+fx) - 4\cos(2(e+fx))(4(A-2B)\sin(e+fx) -}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A - 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(-12*A
+ 9*B + 4*(A - 2*B)*Sin[e + f*x])))/(96*f)
```

Maple [A] time = 0.378, size = 129, normalized size = 1.4

$$\frac{\left(3\,B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right) + 4\,A\left(\cos\left(fx+e\right)\right)^2 - 8\,B\left(\cos\left(fx+e\right)\right)^2 + 12\,A\sin\left(fx+e\right) - 9\,B\sin\left(fx+e\right) - 16\,A}{12\,f\left(\left(\cos\left(fx+e\right)\right)^2 + 2\,\sin\left(fx+e\right) - 2\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2-8*B*cos(f*x+e)^2+12*A*
sin(f*x+e)-9*B*sin(f*x+e)-16*A+8*B)*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(
a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(5/2), x)
```

Fricas [A] time = 1.71983, size = 293, normalized size = 3.12

$$\frac{\left(3\,Bc^2\cos\left(fx+e\right)^4+12\,(A-B)c^2\cos\left(fx+e\right)^2-3\,(4\,A-3\,B)c^2-4\left((A-2\,B)c^2\cos\left(fx+e\right)^2-2\,(2\,A-B)c^2\right)\sin\left(\right.}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*c^2*cos(f*x + e)^4 + 12*(A - B)*c^2*cos(f*x + e)^2 - 3*(4*A - 3*B
)*c^2 - 4*((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(2*A - B)*c^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.133 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=94

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rubi [A] time = 0.333473, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e
+ f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}\,dx$$

$$= -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos}{3}$$

Mathematica [A] time = 0.582407, size = 84, normalized size = 0.89

$$\frac{c\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(2(6A-B)\sin(e+fx)+\cos(2(e+fx))(3A+2B\sin(e+fx)-3B))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2*(6*A
 - B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*A - 3*B + 2*B*Sin[e + f*x])))/(12*
f)
```

Maple [A] time = 0.366, size = 91, normalized size = 1.

$$\frac{\left(-2B\left(\cos\left(fx+e\right)\right)^{2}+3A\sin\left(fx+e\right)-3B\sin\left(fx+e\right)-6A+2B\right)\sin\left(fx+e\right)}{6f\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{a\left(1+\mathrm{s}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(-2*B*cos(f*x+e)^2+3*A*sin(f*x+e)-3*B*sin(f*x+e)-6*A+2*B)*(-c*(-1+sin
(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(-1+sin(f*x+e))/cos(f*x
+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2), x)
```

Fricas [A] time = 1.7092, size = 227, normalized size = 2.41

$$\frac{\left(3\,(A-B)c\cos\left(fx+e\right)^{2}-3\,(A-B)c+2\left(Bc\cos\left(fx+e\right)^{2}+(3\,A-B)c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(f\right.}}{6\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(3*(A - B)*c*cos(f*x + e)^2 - 3*(A - B)*c + 2*(B*c*cos(f*x + e)^2 + (3*
A - B)*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/
(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.134 $\int \sqrt{a + a\sin(e + fx)}(A+B\sin(e+fx))\sqrt{c - c\sin(e + fx)}\, d$

Optimal. Leaf size=92

$$\frac{aB\cos(e + fx)(c - c\sin(e + fx))^{3/2}}{2cf\sqrt{a\sin(e + fx) + a}} - \frac{a(A + B)\cos(e + fx)\sqrt{c - c\sin(e + fx)}}{f\sqrt{a\sin(e + fx) + a}}$$

```
[Out] -((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Si
n[e + f*x]])
```

Rubi [A] time = 0.308625, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{aB\cos(e + fx)(c - c\sin(e + fx))^{3/2}}{2cf\sqrt{a\sin(e + fx) + a}} - \frac{a(A + B)\cos(e + fx)\sqrt{c - c\sin(e + fx)}}{f\sqrt{a\sin(e + fx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*Si
n[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = (A+B)\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx - \frac{B}{}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos(e+}{2cf\sqrt{}}$$

Mathematica [A] time = 0.206581, size = 63, normalized size = 0.68

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(4A\sin(e+fx)-B\cos(2(e+fx)))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(-(B*Cos[2*(e + f*x)]) + 4*A*Sin[e
 + f*x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)
```

Maple [A] time = 0.355, size = 57, normalized size = 0.6

$$\frac{\left(B\sin\left(fx+e\right)+2A\right)\sin\left(fx+e\right)}{2f\cos\left(fx+e\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\sqrt{a\left(1+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(B*sin(f*x+e)+2*A)*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*
x+e)))^(1/2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c), x)
```

Fricas [A] time = 1.65383, size = 157, normalized size = 1.71

$$-\frac{\left(B\cos\left(fx+e\right)^{2}-2\,A\sin\left(fx+e\right)-B\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(
e + f*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=100

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((a*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.337057, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2738, 2737, 2667, 31}

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -((a*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(c*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx &= (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx - \frac{B\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}{c}\\ &= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} + \frac{(a(A+B)c\cos(e+fx))\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+f}}\\ &= \frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}} - \frac{(a(A+B)\cos(e+fx))\,\text{Subst}\left(\int \frac{1}{c+x}\right.}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c}}\\ &= -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{aB\cos(e+fx)\sqrt{c-c\sin(e}}{cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [C] time = 1.17749, size = 120, normalized size = 1.2

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(B\sin(e+fx)+(A+B)\left(2\log\left(i-e^{i(e+fx)}\right)-ifx\right)\right)}{f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*((A + B
)*((-I)*f*x + 2*Log[I - E^(I*(e + f*x))]) + B*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [B] time = 0.367, size = 394, normalized size = 3.9

$$-\frac{1}{f\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)}\left(A\cos\left(fx+e\right)\ln\left(2\left(\cos\left(fx+e\right)+1\right)^{-1}\right)-2A\cos\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx\right.}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x
+e)+B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*B*sin(f*x+e)*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln(2/(cos(f*x+e)+1))+2*A*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)-B*ln(2/(cos(f*x+e)+1))+2*B*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B)*(a*(1+sin(f*x+e)))^(1/2)/(-1+co
s(f*x+e)-sin(f*x+e))/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [A] time = 1.57322, size = 236, normalized size = 2.36

$$\frac{B\left(\frac{2\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)}{\sqrt{c}}+\frac{2\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\left(c+\frac{c\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}\right)\left(\cos\left(fx+e\right)+1\right)}\right)+A\left(\frac{2\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{\sqrt{c}}-\frac{\sqrt{a}\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right.}\right.}{\sqrt{c}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
+ A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e
) + c), x)
```

3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + (a*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.357862, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2737, 2667, 31, 2738}

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + (a*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} - \frac{(aB\cos(e+fx))\int \frac{\cos(e+}{c-c\sin(}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\,\mathrm{si}}}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(aB\cos(e+fx))\,\mathrm{Subst}\left(\int\right.}{cf\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(1-\mathrm{si}}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c}}$$

Mathematica [C] time = 1.16847, size = 147, normalized size = 1.48

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(A+2B\log\left(i-e^{i(e+fx)}\right)+iB\left(fx+2i\log\left(i-e^{i(e+fx)}\right)\right)\sin(e+\right.}{f(c-c\sin(e+fx))^{3/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + B -
I*B*f*x + 2*B*Log[I - E^(I*(e + f*x))] + I*B*(f*x + (2*I)*Log[I - E^(I*(e +
 f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin
[e + f*x])^(3/2))
```

Maple [B] time = 0.351, size = 403, normalized size = 4.1

$$\frac{1}{f\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)}\left(B\left(\cos\left(fx+e\right)\right)^{2}\ln\left(2\left(\cos\left(fx+e\right)+1\right)^{-1}\right)-2B\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-\frac{-1+\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*B*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+
e)^2-A*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+B*cos(f
*x+e)*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+2*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-4*B*sin(f*x+e)*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+e)+B*sin(f*x+e)-2*B*ln(2/(cos(f*x+e)
+1))+4*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A-B)*(a*(1+sin(f*x+e)))
^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^2\cos\left(fx+e\right)^2+2\,c^2\sin\left(fx+e\right)-2\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(3/2), x)
```

3.137 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=92

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

```
[Out] (a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))
```

Rubi [A] time = 0.344732, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$
$$= \frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}(c-c\sin$$

Mathematica [A] time = 0.550104, size = 101, normalized size = 1.1

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(A+2B\sin(e+fx)-B)}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))
```

Maple [A] time = 0.352, size = 137, normalized size = 1.5

$$-\frac{\left(A\left(\cos\left(fx+e\right)\right)^2 - A\sin\left(fx+e\right)\cos\left(fx+e\right) - B\left(\cos\left(fx+e\right)\right)^2 + B\sin\left(fx+e\right)\cos\left(fx+e\right) + 2A\cos\left(fx+e\right) +}{2f\left(-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(A*cos(f*x+e)^2-A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2+B*sin(f*x+e)*
cos(f*x+e)+2*A*cos(f*x+e)+3*A*sin(f*x+e)-B*sin(f*x+e)-3*A+B)*sin(f*x+e)*(a*
(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(5/2), x)
```

Fricas [A] time = 1.70649, size = 224, normalized size = 2.43

$$-\frac{\left(2\,B\sin\left(fx+e\right)+A-B\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(c^{3}f\cos\left(fx+e\right)^{3}+2\,c^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,c^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f
*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(5/2), x)
```

3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

```
[Out] (a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))
```

Rubi [A] time = 0.33662, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = (A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx - \frac{B\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$
$$= \frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx}{2cf\sqrt{a+a\sin(e+fx)}(c-c}$$

Mathematica [A] time = 0.627621, size = 103, normalized size = 1.1

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(2A+3B\sin(e+fx)-B)}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
 f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]))
```

Maple [B] time = 0.364, size = 205, normalized size = 2.2

$$-\frac{\left(2A\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+2A\left(\cos\left(fx+e\right)\right)^{3}-B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-B\left(\cos\left(fx+e\right)\right)^{3}+6A\sin\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/6/f*(2*A*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^3-B*cos(f*x+e)^2*sin(f*x
+e)-B*cos(f*x+e)^3+6*A*sin(f*x+e)*cos(f*x+e)-8*A*cos(f*x+e)^2-3*B*sin(f*x+e
)*cos(f*x+e)+4*B*cos(f*x+e)^2-14*A*sin(f*x+e)-8*A*cos(f*x+e)+4*B*sin(f*x+e)
```

```
+B*cos(f*x+e)+14*A-4*B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+s
in(f*x+e))/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(7/2), x)
```

Fricas [A] time = 1.75888, size = 263, normalized size = 2.8

$$-\frac{\left(3\,B\sin\left(fx+e\right)+2\,A-B\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x +
 e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
c)^(7/2), x)
```

3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(3A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{15f}-\frac{B\cos(}{}$$

```
[Out] -(a^2*(3*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(30*f*Sqrt[a + a*S
in[e + f*x]]) - (a*(3*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(7/2))/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(7/2))/(6*f)
```

Rubi [A] time = 0.358177, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(3A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{15f}-\frac{B\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(a^2*(3*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(30*f*Sqrt[a + a*S
in[e + f*x]]) - (a*(3*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(7/2))/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(7/2))/(6*f)
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+f}{6f}$$

$$= -\frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin}{15f}$$

$$= -\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a+a\sin(e+fx)}} - \frac{a(3}{}$$

Mathematica [A] time = 1.59013, size = 205, normalized size = 1.4

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(15(16A-11B)\cos(2(e+fx))+30(2A-B)\cos(4(e+fx)}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\text{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]]*(15*(16*A - 11*B)*Cos[2*(e + f*x)] + 30*(2*A - B)*Cos[4*(e + f*x)] +
 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] - 240*B*Sin[e + f*x] + 60*A*Sin[
3*(e + f*x)] + 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x)] + 24*B*Sin[5*(
```

```
e + f*x)]))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3)
```

Maple [A] time = 0.324, size = 185, normalized size = 1.3

$$\frac{\left(5\,B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+6\,A\left(\cos\left(fx+e\right)\right)^{4}-12\,B\left(\cos\left(fx+e\right)\right)^{4}+15\,A\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-10\,B}{30\,f\left(\left(\cos\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4-12*B*cos(f*x+e)^4+15*A
*cos(f*x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2+4*B
*cos(f*x+e)^2+15*A*sin(f*x+e)-10*B*sin(f*x+e)-24*A+8*B)*(-c*(-1+sin(f*x+e))
)^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/c
os(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A] time = 1.87883, size = 356, normalized size = 2.44

$$\frac{\left(5\,Bac^{3}\cos\left(fx+e\right)^{6}+15\,(A-B)ac^{3}\cos\left(fx+e\right)^{4}-5\,(3\,A-2\,B)ac^{3}-2\left(3\,(A-2\,B)ac^{3}\cos\left(fx+e\right)^{4}-2\,(3\,A-B)ac\right.\right.}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a*c^3*cos(f*x + e)^6 + 15*(A - B)*a*c^3*cos(f*x + e)^4 - 5*(3*A -
 2*B)*a*c^3 - 2*(3*(A - 2*B)*a*c^3*cos(f*x + e)^4 - 2*(3*A - B)*a*c^3*cos(f
*x + e)^2 - 4*(3*A - B)*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=146

$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(5A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{20f}-\frac{B\cos(\ldots}{\ldots}$$

```
[Out] -(a^2*(5*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(30*f*Sqrt[a + a*S
in[e + f*x]]) - (a*(5*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2))/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(5/2))/(5*f)
```

Rubi [A] time = 0.361314, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a\sin(e+fx)+a}}-\frac{a(5A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{20f}-\frac{B\cos(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -(a^2*(5*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(30*f*Sqrt[a + a*S
in[e + f*x]]) - (a*(5*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2))/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c
- c*Sin[e + f*x])^(5/2))/(5*f)
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+f}{5f}$$

$$= -\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin}{20f}$$

$$= -\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a+a\sin(e+fx)}} - \frac{a(5}{}$$

Mathematica [A] time = 1.69345, size = 172, normalized size = 1.18

$$\frac{c^2(\sin(e+fx)-1)^2(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(4(100A-11B)\sin(e+fx)+3\cos(4(e+fx))(5A+4B\sin(}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(4*(100*A - 11*B)*Sin[e + f*x] + 3*Cos[4*(e + f*x)]*(5*A - 5*B + 4*B*
Sin[e + f*x]) + 4*Cos[2*(e + f*x)]*(15*(A - B) + 4*(5*A + 2*B)*Sin[e + f*x]
)))/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[
```

```
(e + f*x)/2])^3)
```

Maple [A] time = 0.303, size = 147, normalized size = 1.

$$\frac{\left(-12\,B\left(\cos\left(fx+e\right)\right)^{4}+15\,A\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-15\,B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-20\,A\left(\cos\left(fx+e\right)\right)^{2}+\right.}{60\,f\left(-1+\sin\left(fx+e\right)\right)\left(\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/60/f*(-12*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*s
in(f*x+e)-20*A*cos(f*x+e)^2+4*B*cos(f*x+e)^2+15*A*sin(f*x+e)-15*B*sin(f*x+e
)-40*A+8*B)*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/
(-1+sin(f*x+e))/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [A] time = 1.81677, size = 302, normalized size = 2.07

$$\frac{\left(15\,(A-B)ac^{2}\cos\left(fx+e\right)^{4}-15\,(A-B)ac^{2}+4\left(3\,Bac^{2}\cos\left(fx+e\right)^{4}+(5\,A-B)ac^{2}\cos\left(fx+e\right)^{2}+2\,(5\,A-B)ac^{2}\right)\mathrm{s}\right.}{60\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/60*(15*(A - B)*a*c^2*cos(f*x + e)^4 - 15*(A - B)*a*c^2 + 4*(3*B*a*c^2*cos
(f*x + e)^4 + (5*A - B)*a*c^2*cos(f*x + e)^2 + 2*(5*A - B)*a*c^2)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] sage0*x
```

3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{aA\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}-\frac{B\cos(e+fx)(a\sin(e$$

```
[Out] -(a^2*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]]) - (a*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2
))/(3*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2))/(4*f)
```

Rubi [A] time = 0.348195, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{aA\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}-\frac{B\cos(e+fx)(a\sin(e$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```


```
[Out] -(a^2*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*f*Sqrt[a + a*Sin[e + f*
x]]) - (a*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2
))/(3*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2))/(4*f)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+f}{4f}$$

$$= -\frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx}{3f}$$

$$= -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}} - \frac{aA\cos(e}{}$$

Mathematica [A] time = 0.780104, size = 96, normalized size = 0.72

$$-\frac{c(\sin(e+fx)-1)\sec^3(e+fx)(a(\sin(e+fx)+1))^{3/2}\sqrt{c-c\sin(e+fx)}(8A(9\sin(e+fx)+\sin(3(e+fx)))-12B\cos(2}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c
- c*Sin[e + f*x]]*(-12*B*Cos[2*(e + f*x)] - 3*B*Cos[4*(e + f*x)] + 8*A*(9*S
in[e + f*x] + Sin[3*(e + f*x)])))/(96*f)
```

Maple [A] time = 0.27, size = 86, normalized size = 0.6

$$\frac{\left(3B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+4A\left(\cos\left(fx+e\right)\right)^{2}+3B\sin\left(fx+e\right)+8A\right)\sin\left(fx+e\right)}{12f\left(\cos\left(fx+e\right)\right)^{3}}\left(-c\left(-1+\sin\left(fx+e\right)\right)\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+3*B*sin(f*x+e)+8*A)*(-
c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [A] time = 1.73974, size = 216, normalized size = 1.61

$$-\frac{\left(3Bac\cos\left(fx+e\right)^{4}-3Bac-4\left(Aac\cos\left(fx+e\right)^{2}+2Aac\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{12f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/12*(3*B*a*c*cos(f*x + e)^4 - 3*B*a*c - 4*(A*a*c*cos(f*x + e)^2 + 2*A*a*c
)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f
*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rubi [A] time = 0.324691, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e}{3a}$$

Mathematica [A] time = 0.577818, size = 81, normalized size = 0.84

$$-\frac{a\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(\cos(2(e+fx))(3(A+B)+2B\sin(e+fx))-2(6A+B)\sin(e+fx)}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] -(a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-2*(6
*A + B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*(A + B) + 2*B*Sin[e + f*x])))/(1
2*f)
```

Maple [A] time = 0.326, size = 91, normalized size = 1.

$$\frac{\left(-2B\left(\cos\left(fx+e\right)\right)^2+3A\sin\left(fx+e\right)+3B\sin\left(fx+e\right)+6A+2B\right)\sin\left(fx+e\right)}{6f\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)}\sqrt{-c\left(-1+\sin\left(fx+e\right)\right)}\left(a\left(1+\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(-2*B*cos(f*x+e)^2+3*A*sin(f*x+e)+3*B*sin(f*x+e)+6*A+2*B)*(-c*(-1+sin
(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(1+sin(f*x+e))/cos(f*x+
e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [A] time = 1.67991, size = 228, normalized size = 2.38

$$-\frac{\left(3\left(A+B\right)a\cos\left(fx+e\right)^{2}-3\left(A+B\right)a+2\left(Ba\cos\left(fx+e\right)^{2}-\left(3A+B\right)a\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*(A + B)*a*cos(f*x + e)^2 - 3*(A + B)*a + 2*(B*a*cos(f*x + e)^2 - (3
*A + B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=145

$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e+fx)}{2f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*a^2*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.381569, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$-\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)(a\sin(e+fx)}{2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```



```
[Out] (-2*a^2*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))}{\sqrt{c-c\sin(e+fx}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e}{2f\sqrt{c-c\sin(e+}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e}{2f\sqrt{c-c\sin(e+}}$$

$$= -\frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e}{2f\sqrt{c-c\sin(e+}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx}{f\sqrt{c-c\sin}}$$

Mathematica [A] time = 0.709697, size = 136, normalized size = 0.94

$$-\frac{(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+2B)\sin(e+fx)+16(A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.}{4f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-(B*C
os[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*
(A + 2*B)*Sin[e + f*x]))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[
c - c*Sin[e + f*x]])
```

Maple [B] time = 0.342, size = 495, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(-B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e)^3+2*A*sin(f*x+e)*cos(f*x+e)-
4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2-4*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A
*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*sin(f*x+e)*cos(f
*x+e)-4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^2-4*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1
))+8*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)
+4*A*ln(2/(cos(f*x+e)+1))-8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*
B*sin(f*x+e)+B*cos(f*x+e)+4*B*ln(2/(cos(f*x+e)+1))-8*B*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+2*A+3*B)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*
x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+}{2f(c-c\sin(e+fx))}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (a^2*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 3*B)*Cos[e + f*x]*Sqrt[
a + a*Sin[e + f*x]])/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.385151, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+}{2f(c-c\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (a^2*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 3*B)*Cos[e + f*x]*Sqrt[
a + a*Sin[e + f*x]])/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+3B)\int\frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a}}{2cf\sqrt{c-c\sin(e}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a}}{2cf\sqrt{c-c\sin(e}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a}}{2cf\sqrt{c-c\sin(e}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B)\cos(e+fx)\log}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

Mathematica [A] time = 0.905223, size = 210, normalized size = 1.33

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2\sin(e+fx)\left(2(A+3B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.\right.}{2cf(\sin(e+fx)-1)\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4*A +
 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] +
12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.288, size = 749, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(2*A+4*B-2*A*sin(f*x+e)-2*A*cos(f*x+e)^2-A*cos(f*x+e)*ln(2/(cos(f*x+e)
+1))-3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+B*cos(f*x+e)^2*sin(f*x+e)-2*A*co
s(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)
*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))+6*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
+B*cos(f*x+e)^3-B*cos(f*x+e)+3*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)
-6*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-2*A*s
in(f*x+e)*ln(2/(cos(f*x+e)+1))+4*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))+3*B*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))
+6*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*B*sin(f*x+e)*l
n(2/(cos(f*x+e)+1))+12*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+2*A*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))-A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-4*B*cos(f*x+e)^2+2*A*ln(
2/(cos(f*x+e)+1))-4*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*B*ln(2/(
cos(f*x+e)+1))-12*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*sin(f*x+
e))*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+
e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [B] time = 1.57983, size = 494, normalized size = 3.13

$$-\frac{B\left(\frac{6a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}} - \frac{3a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}} + \frac{2\left(\frac{3a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c^{\frac{3}{2}} - \frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{2c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{2c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right) + A\left(\frac{2a^{\frac{3}{2}}\log\left(\frac{\sin}{\cos}\right.}{c^{\frac{3}{2}}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] -(B*(6*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 3*a^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 2*(3*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) - 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c^(3/2) - 2*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) + A*(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/c^(3/2) - a^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 4
```

```
*a^(3/2)*sqrt(c)*sin(f*x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(
f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)\sqrt{a\sin(e+fx)+}}{cf(c-c\sin(e+fx))^{3/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f
*x])^(3/2)) - (a^2*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.387005, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)\sqrt{a\sin(e+fx)+}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f
*x])^(3/2)) - (a^2*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{B\int\frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e}{cf(c-c\sin(e+fx))^{3}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e}{cf(c-c\sin(e+fx))^{3}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e}{cf(c-c\sin(e+fx))^{3}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e}{cf(c-c\sin(e+fx))^{3}}$$

Mathematica [A] time = 0.978476, size = 198, normalized size = 1.33

$$\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\left(A+4B\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\right.\right.}{c^2f(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos[
2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.28, size = 594, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(-A+3*B+A*sin(f*x+e)+A*cos(f*x+e)^2-3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1
))+2*B*cos(f*x+e)^2*sin(f*x+e)+6*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))+2*B*cos(f*x+e)^3-2*B*cos(f*x+e)+2*B*ln(2/(cos(f*x+e)+1))*si
n(f*x+e)*cos(f*x+e)-4*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+
e)*cos(f*x+e)+B*sin(f*x+e)*cos(f*x+e)-2*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+4
*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*sin(f*x+e)*ln(
2/(cos(f*x+e)+1))+8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-A*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)^2+4*B*ln(2/(cos(f*x+e)+1))-8*B*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+
e)+1))+B*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)^3*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-3*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(3/2)/(sin(f
*x+e)*cos(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)
))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 -
(c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.274596, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2972, 2742}

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(5/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
```

```
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\cos(e+fx)(a+a}{24cf(c-c\sin(e+}$$

Mathematica [A] time = 1.03821, size = 125, normalized size = 1.3

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3(A-B)\sin(e+fx)+A-3B\cos(2(e+fx))+4B)}{6c^3f(\sin(e+fx)-1)^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + 4
*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(6*c^3*f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.273, size = 223, normalized size = 2.3

$$\frac{\left(A\left(\cos\left(fx+e\right)\right)^3 + A\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right) + B\left(\cos\left(fx+e\right)\right)^3 + B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right) - 4A\left(\cos\left(fx-\right.\right.\right.}{6f\left(\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/6/f*(A*cos(f*x+e)^3+A*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^3+B*cos(f*x+e)
^2*sin(f*x+e)-4*A*cos(f*x+e)^2+3*A*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^2-3
*B*sin(f*x+e)*cos(f*x+e)-7*A*cos(f*x+e)-10*A*sin(f*x+e)-B*cos(f*x+e)+2*B*si
n(f*x+e)+10*A-2*B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+
e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A] time = 2.02924, size = 309, normalized size = 3.22

$$\frac{\left(6\,Ba\cos\left(fx+e\right)^{2}-3\,(A-B)a\sin\left(fx+e\right)-(A+7\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(3\,c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)-\left(c^{4}f\cos\left(fx+e\right)^{3}-4\,c^{4}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4
*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e-}{8f(c-c\sin(e+fx)}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(7/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)
)/(96*c^2*f*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.375503, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e-}{8f(c-c\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c -
c*Sin[e + f*x])^(7/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)
)/(96*c^2*f*(c - c*Sin[e + f*x])^(5/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a}{24cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a}{24cf(c-c\sin(e+}$$

Mathematica [A] time = 1.39668, size = 123, normalized size = 0.84

$$\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(4A\sin(e+fx)+2A-3B\cos(2(e+fx))+3B)}{12c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A +
3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2]
```

```
 + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.279, size = 217, normalized size = 1.5

$$\frac{\left(A\left(\cos\left(fx+e\right)\right)^{4}-A\left(\cos\left(fx+e\right)\right)^{3}\sin\left(fx+e\right)+4\,A\left(\cos\left(fx+e\right)\right)^{3}+5\,A\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-12\,A\left(\cos\right.\right.}{6\,f\left(\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/6/f*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*sin(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos(f*
x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2+7*A*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e
)^2-3*B*sin(f*x+e)*cos(f*x+e)-10*A*cos(f*x+e)-17*A*sin(f*x+e)+3*B*sin(f*x+e
)+17*A-3*B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(
f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [A] time = 2.09412, size = 336, normalized size = 2.3

$$-\frac{\left(3\,Ba\cos\left(fx+e\right)^{2}-2\,Aa\sin\left(fx+e\right)-(A+3\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(
f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f
*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a}{10f(c-c\sin(e}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + (a*(3*A - 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(40*c*f*(
c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 7*B)*Cos[e + f*x])/(120*c^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.372795, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2739, 2738}

$$-\frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}+\frac{a(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a}{10f(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + (a*(3*A - 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(40*c*f*(
c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 7*B)*Cos[e + f*x])/(120*c^2*f*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-7B)\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}}{10c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e+fx)\sqrt{a}}{40cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e+fx)\sqrt{a}}{40cf(c-c\sin(e+}$$

Mathematica [A] time = 1.96955, size = 126, normalized size = 0.82

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(5(3A+B)\sin(e+fx)+9(A+B)-10B\cos(2(e+fx)))}{60c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(9*(A
+ B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(60*c^5*f*(Cos[(e
```

```
 + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [B] time = 0.299, size = 339, normalized size = 2.2

$$-\frac{\left(9 A \left(\cos \left(f x+e\right)\right)^{5}+9 A \left(\cos \left(f x+e\right)\right)^{4} \sin \left(f x+e\right)-B \left(\cos \left(f x+e\right)\right)^{5}-B \sin \left(f x+e\right) \left(\cos \left(f x+e\right)\right)^{4}-54 A \left(c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] -1/60/f*(9*A*cos(f*x+e)^5+9*A*cos(f*x+e)^4*sin(f*x+e)-B*cos(f*x+e)^5-B*sin(
f*x+e)*cos(f*x+e)^4-54*A*cos(f*x+e)^4+45*A*cos(f*x+e)^3*sin(f*x+e)+6*B*cos(
f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-108*A*cos(f*x+e)^3-153*A*cos(f*x+e)^2*
sin(f*x+e)+12*B*cos(f*x+e)^3+17*B*cos(f*x+e)^2*sin(f*x+e)+288*A*cos(f*x+e)^
2-135*A*sin(f*x+e)*cos(f*x+e)-52*B*cos(f*x+e)^2+35*B*sin(f*x+e)*cos(f*x+e)+
159*A*cos(f*x+e)+294*A*sin(f*x+e)-11*B*cos(f*x+e)-46*B*sin(f*x+e)-294*A+46*
B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)+cos(f*x+e)^2-
2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \sin \left(f x+e\right)+A\right) \left(a \sin \left(f x+e\right)+a\right)^{\frac{3}{2}}}{\left(-c \sin \left(f x+e\right)+c\right)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

Fricas [A] time = 2.09092, size = 393, normalized size = 2.55

$$-\frac{\left(20\,Ba\cos\left(fx+e\right)^{2}-5\,(3\,A+B)a\sin\left(fx+e\right)-(9\,A+19\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx\cdots\right.}}{60\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a)
*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5
 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5
- 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

3.149 $\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx$

Optimal. Leaf size=198

$$-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{105f}-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a\sin(e+fx)+a}}-\frac{a(7A$$

```
[Out] -(a^3*(7*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a^2*(7*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(7/2))/(105*f) - (a*(7*A - B)*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Si
n[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(7*f)
```

Rubi [A] time = 0.476191, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{105f}-\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a\sin(e+fx)+a}}-\frac{a(7A$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -(a^3*(7*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a^2*(7*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c
- c*Sin[e + f*x])^(7/2))/(105*f) - (a*(7*A - B)*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Si
n[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(7*f)
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
```

```
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx &= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+\\ &\quad 7f}\\ &= -\frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c}{42f}\\ &= -\frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c}{105f}\\ &= -\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a+a\sin(e+fx)}}-2\end{aligned}$$

Mathematica [A] time = 2.85785, size = 223, normalized size = 1.13

$$-\frac{c^3(\sin(e+fx)-1)^3(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(525(A-B)\cos(2(e+fx))+210(A-B)\cos(4(e+fx))}{67}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] -(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e +
 f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)] + 35*A*
Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] - 525*B*Sin[
e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A*Sin[5*(e +
 f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(6720*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.368, size = 203, normalized size = 1.

$$\frac{\left(-30\,B\left(\cos\left(fx+e\right)\right)^{6}+35\,A\left(\cos\left(fx+e\right)\right)^{4}\sin\left(fx+e\right)-35\,B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}-42\,A\left(\cos\left(fx+e\right)\right)^{4}+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/210/f*(-30*B*cos(f*x+e)^6+35*A*cos(f*x+e)^4*sin(f*x+e)-35*B*sin(f*x+e)*co
s(f*x+e)^4-42*A*cos(f*x+e)^4+6*B*cos(f*x+e)^4+35*A*cos(f*x+e)^2*sin(f*x+e)-
35*B*cos(f*x+e)^2*sin(f*x+e)-56*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2+35*A*sin(f*
x+e)-35*B*sin(f*x+e)-112*A+16*B)*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(
1+sin(f*x+e)))^(5/2)/(-1+sin(f*x+e))/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A] time = 2.34748, size = 371, normalized size = 1.87

$$\frac{\left(35\,(A-B)a^2c^3\cos\left(fx+e\right)^6-35\,(A-B)a^2c^3+2\left(15\,Ba^2c^3\cos\left(fx+e\right)^6+3\,(7\,A-B)a^2c^3\cos\left(fx+e\right)^4+4\,(7\,A-B\right.\right.}{210\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] 1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^2
*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^2*
c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=180

$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2A\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{5f}-\frac{aA\cos(e+fx)(a\sin}{}$$

```
[Out] (-2*a^3*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a^2*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2))/(5*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2))/(5*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(5/2))/(6*f)
```

Rubi [A] time = 0.465897, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a^2A\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{5f}-\frac{aA\cos(e+fx)(a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```


```
[Out] (-2*a^3*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a^2*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(5/2))/(5*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2))/(5*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(5/2))/(6*f)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
```

```
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+}{6f}$$
$$= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e}{5f}$$
$$= -\frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+}{5f}$$
$$= -\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} - \frac{a^2A\cos}{}$$

Mathematica [A] time = 0.841482, size = 113, normalized size = 0.63

$$\frac{a^2c^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(600A\sin(e+fx)+100A\sin(3(e+fx))+12A\sin(5(e+fx))-}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-75*B*Cos[2*(e + f*x)] - 30*B*Cos[4*(e + f*x)] - 5*B*Cos[6*(e + f*x)] + 600
*A*Sin[e + f*x] + 100*A*Sin[3*(e + f*x)] + 12*A*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A] time = 0.285, size = 114, normalized size = 0.6

$$\frac{\left(5\,B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+6\,A\left(\cos\left(fx+e\right)\right)^{4}+5\,B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+8\,A\left(\cos\left(fx+e\right)\right)^{2}+5\,B\sin}{30\,f\left(\cos\left(fx+e\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4+5*B*cos(f*x+e)^2*sin(f
*x+e)+8*A*cos(f*x+e)^2+5*B*sin(f*x+e)+16*A)*(-c*(-1+sin(f*x+e)))^(5/2)*sin(
f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [A] time = 2.18402, size = 279, normalized size = 1.55

$$-\frac{\left(5\,Ba^{2}c^{2}\cos\left(fx+e\right)^{6}-5\,Ba^{2}c^{2}-2\left(3\,Aa^{2}c^{2}\cos\left(fx+e\right)^{4}+4\,Aa^{2}c^{2}\cos\left(fx+e\right)^{2}+8\,Aa^{2}c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx\right.}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/30*(5*B*a^2*c^2*cos(f*x + e)^6 - 5*B*a^2*c^2 - 2*(3*A*a^2*c^2*cos(f*x +
e)^4 + 4*A*a^2*c^2*cos(f*x + e)^2 + 8*A*a^2*c^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

3.151 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=142

$$\frac{c^2(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{c(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{20f}-\frac{B\cos(e+$$

```
[Out] ((5*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((5*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c
- c*Sin[e + f*x]])/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(5*f)
```

Rubi [A] time = 0.362424, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$\frac{c^2(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{c(5A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{20f}-\frac{B\cos(e+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```



```
[Out] ((5*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((5*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c
- c*Sin[e + f*x]])/(20*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(5*f)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+}{5f}$$

$$= \frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c}}{20f}$$

$$= \frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{(5}{}$$

Mathematica [A] time = 1.8909, size = 165, normalized size = 1.16

$$-\frac{c(\sin(e+fx)-1)(a(\sin(e+fx)+1))^{5/2}\sqrt{c-c\sin(e+fx)}(4(100A+11B)\sin(e+fx)+4\cos(2(e+fx))(4(5A-2B)s}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]]*(4*(100*A + 11*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-15*(A + B) + 4*(5*
A - 2*B)*Sin[e + f*x]) - 3*Cos[4*(e + f*x)]*(5*(A + B) + 4*B*Sin[e + f*x]))
)/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
```

```
 + f*x)/2])^5)
```

Maple [A] time = 0.302, size = 147, normalized size = 1.

$$\frac{\left(-12 B \left(\cos \left(f x+e\right)\right)^{4}+15 A \left(\cos \left(f x+e\right)\right)^{2} \sin \left(f x+e\right)+15 B \left(\cos \left(f x+e\right)\right)^{2} \sin \left(f x+e\right)+20 A \left(\cos \left(f x+e\right)\right)^{2}+4}{60 f \left(1+\sin \left(f x+e\right)\right) \left(\cos \left(f x+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/60/f*(-12*B*cos(f*x+e)^4+15*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*s
in(f*x+e)+20*A*cos(f*x+e)^2+4*B*cos(f*x+e)^2+15*A*sin(f*x+e)+15*B*sin(f*x+e
)+40*A+8*B)*(-c*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/
(1+sin(f*x+e))/cos(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B \sin \left(f x+e\right)+A\right) \left(a \sin \left(f x+e\right)+a\right)^{\frac{5}{2}} \left(-c \sin \left(f x+e\right)+c\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [A] time = 2.091, size = 304, normalized size = 2.14

$$-\frac{\left(15 \left(A+B\right) a^{2} c \cos \left(f x+e\right)^{4}-15 \left(A+B\right) a^{2} c+4 \left(3 B a^{2} c \cos \left(f x+e\right)^{4}-\left(5 A+B\right) a^{2} c \cos \left(f x+e\right)^{2}-2 \left(5 A+B\right) a^{2} c\right) s}{60 f \cos \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] -1/60*(15*(A + B)*a^2*c*cos(f*x + e)^4 - 15*(A + B)*a^2*c + 4*(3*B*a^2*c*co
s(f*x + e)^4 - (5*A + B)*a^2*c*cos(f*x + e)^2 - 2*(5*A + B)*a^2*c)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] sage2
```

3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rubi [A] time = 0.318236, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(}{}$$

Mathematica [A] time = 0.874536, size = 102, normalized size = 1.06

$$\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(16(7A+2B)\sin(e+fx)-4\cos(2(e+fx))(4(A+2B)\sin(e+fx)}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B*
Cos[4*(e + f*x)] + 16*(7*A + 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(12*A +
 9*B + 4*(A + 2*B)*Sin[e + f*x])))/(96*f)
```

Maple [A] time = 0.339, size = 129, normalized size = 1.3

$$\frac{\left(3\,B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)+4\,A\left(\cos\left(fx+e\right)\right)^{2}+8\,B\left(\cos\left(fx+e\right)\right)^{2}-12\,A\sin\left(fx+e\right)-9\,B\sin\left(fx+e\right)-16}{12\,f\left(\left(\cos\left(fx+e\right)\right)^{2}-2\,\sin\left(fx+e\right)-2\right)\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2-12*A*
sin(f*x+e)-9*B*sin(f*x+e)-16*A-8*B)*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(
a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [A] time = 1.95623, size = 293, normalized size = 3.05

$$\frac{\left(3\,Ba^2\cos\left(fx+e\right)^4-12\,(A+B)a^2\cos\left(fx+e\right)^2+3\,(4\,A+3\,B)a^2-4\left((A+2\,B)a^2\cos\left(fx+e\right)^2-2\,(2\,A+B)a^2\right)\sin\left(\right.}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*(3*B*a^2*cos(f*x + e)^4 - 12*(A + B)*a^2*cos(f*x + e)^2 + 3*(4*A + 3*B
)*a^2 - 4*((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(2*A + B)*a^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=193

$$-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)(a\sin}{2f\sqrt{c-c\sin(e+}$$

```
[Out] (-4*a^3*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*(a + a*S
in[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.462769, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$-\frac{2a^2(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+B)\cos(e+fx)(a\sin}{2f\sqrt{c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-4*a^3*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x]*(a + a*S
in[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(}{3f\sqrt{c-c\sin(e+}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a}{2f\sqrt{c-c\sin(}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a}{2f\sqrt{c-c\sin(}}$$

$$= -\frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a}{2f\sqrt{c-c\sin(}}$$

$$= -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A+B)\cos(e+fx}{f\sqrt{c-c\sin}}$$

Mathematica [A] time = 1.56903, size = 177, normalized size = 0.92

$$-\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((36A+51B)\sin(e+fx)-3(A+3B)\cos(2(e+fx))+96A\right.}{12f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-3*(A
 + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] +
96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e + f*x]
- B*Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c
 - c*Sin[e + f*x]])
```

Maple [B] time = 0.356, size = 591, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/6/f*(-15*A-17*B+15*A*sin(f*x+e)+15*A*cos(f*x+e)^2+3*A*cos(f*x+e)^2*sin(f
*x+e)+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+7*B*cos(f*x+e)^2*sin(f*x+e)-3*A*
cos(f*x+e)+2*B*cos(f*x+e)^3*sin(f*x+e)-48*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)^3+9*B*cos(f*x+e)^3-9*B*cos(f*x+e)+24*
A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))-26*B*sin(f*x+e)*cos(f*x+e)+24*B*cos(f*x+e)*ln(2/(cos(f*x+
e)+1))-48*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*B*sin(
f*x+e)*ln(2/(cos(f*x+e)+1))-48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))-18*A*sin(f*x+e)*cos(f*x+e)-2*B*cos(f*x+e)^4+19*B*cos(f*x+e)^2-2
4*A*ln(2/(cos(f*x+e)+1))+48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24
*B*ln(2/(cos(f*x+e)+1))+48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+17*
B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^
3+2*sin(f*x+e)*cos(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(
-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left((A+2\,B)a^2\cos\left(fx+e\right)^2-2\,(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2\,(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+}}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a(A+2B)\cos(e+fx)(a}{2cf\sqrt{c-c\sin}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^3*(A + 2*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(A + 2*B)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 2*B)*Co
s[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.484763, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a(A+2B)\cos(e+fx)(a}{2cf\sqrt{c-c\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^3*(A + 2*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a
+ a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(A + 2*B)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 2*B)*Co
s[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(A+2B)\int \frac{(a+a\sin(e+fx))^{5/}}{\sqrt{c-c\sin(e+fx)}}}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(A+2B)\cos(e+fx)(a}{2cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e+fx)}{cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e+fx)}{cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^2(A+2B)\cos(e+fx)}{cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^3(A+2B)\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

Mathematica [A] time = 1.75204, size = 231, normalized size = 1.1

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(2A+7B)\cos(2(e+fx))+\sin(e+fx)\left(-64(A+2B)\log\right.\right.}{8cf(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(28*
A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8*A + 31
*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] +
B*Sin[3*(e + f*x)]))/(8*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin
[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.272, size = 845, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(12*A+22*B-12*A*sin(f*x+e)-12*A*cos(f*x+e)^2+2*A*cos(f*x+e)^2*sin(f*x
+e)-8*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-16*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+
1))+6*B*cos(f*x+e)^2*sin(f*x+e)-2*A*cos(f*x+e)-16*A*cos(f*x+e)*sin(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(c
os(f*x+e)+1))+B*cos(f*x+e)^3*sin(f*x+e)+16*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))+32*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+2*A*cos(f*x+e)^3+7*B*cos(f*x+e)^3-7*B*cos(f*x+e)+16*B*ln(2/(cos(f
*x+e)+1))*sin(f*x+e)*cos(f*x+e)-32*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))*sin(f*x+e)*cos(f*x+e)-16*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+32*A*sin(f*
x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+15*B*sin(f*x+e)*cos(f*x+e)-
16*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+32*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-32*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+64*B*sin(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+10*A*sin(f*x+e)*cos(f*x+e)+16*A*c
os(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*A*cos(f*x+e)^2*ln(
2/(cos(f*x+e)+1))-B*cos(f*x+e)^4-21*B*cos(f*x+e)^2+16*A*ln(2/(cos(f*x+e)+1)
)-32*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+32*B*ln(2/(cos(f*x+e)+1))
-64*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-22*B*sin(f*x+e))*(a*(1+sin
(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(
f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+}}{c^2\cos\left(fx+e\right)^2+2c^2\sin\left(fx+e\right)-2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+5B)\cos(e+fx)(a\sin}{4cf(c-c\sin(e+}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (a^3*(A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]]
)/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + 5*B
)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.489921, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$-\frac{a^2(A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^2f\sqrt{c-c\sin(e+fx)}}-\frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a(A+5B)\cos(e+fx)(a\sin}{4cf(c-c\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (a^3*(A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]]
)/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + 5*B
)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+5B)\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+}{4cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+}{4cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+}{4cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+}{4cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+}{4cf(c-c\sin(e+}$$

Mathematica [A] time = 1.22688, size = 207, normalized size = 0.98

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2-2(A+5B}{f(c-c\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A +
 B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] time = 0.265, size = 1093, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(-2*A-14*B+2*A*sin(f*x+e)+2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)*cos(f*x+e)^2*sin(f*x+e)-A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+2*
A*cos(f*x+e)^2-2*A*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)*ln(2/(cos(f*x+e)+
1))+15*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-9*B*cos(f*x+e)^2*sin(f*x+e)+2*A*
cos(f*x+e)+4*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))-2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+B*cos(f*x+e)^3*sin(f*x+
e)-4*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-30*B*cos(f*x+e
)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^3-8*B*cos(f*x
+e)^3+8*B*cos(f*x+e)-10*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+20*B*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+4*A*sin(f*x
+e)*ln(2/(cos(f*x+e)+1))-8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))-6*B*sin(f*x+e)*cos(f*x+e)+10*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-20*B
*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+20*B*sin(f*x+e)*ln(2
/(cos(f*x+e)+1))-40*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-6*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)
^2*ln(2/(cos(f*x+e)+1))+2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(
f*x+e)^3-A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-B*cos(f*x+e)^4+15*B*cos(f*x+e)
^2-4*A*ln(2/(cos(f*x+e)+1))+8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
20*B*ln(2/(cos(f*x+e)+1))+40*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1
0*B*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-5*B*
cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-5*B*cos(f*x+e)^3*ln(2/(cos(f*x
+e)+1))+10*B*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+14*B*s
in(f*x+e))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3
*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(-1+
sin(f*x+e)))^(5/2)
```

Maxima [B] time = 1.64921, size = 683, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] -((8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e
```

```
) + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) + a^
(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))*A - B*(10*a^(5/
2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 5*a^(5/2)*log(sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + 2*(5*a^(5/2)*sin(f*x + e)/(cos(
f*x + e) + 1) - 16*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*a^(5/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 16*a^(5/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 5*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(5/2) - 4*c^
(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 8*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*c^(5/2)*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) +
1)^5 + c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{3c^3\cos\left(fx+e\right)^2-4c^3-\left(c^3\cos\left(fx+e\right)^2-4c^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e
)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin
(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=196

$$\frac{a^2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^5}{6f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e
 + f*x])^(5/2)) + (a^2*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c -
 c*Sin[e + f*x])^(3/2)) + (a^3*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.48772, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$\frac{a^2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^5}{6f(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e
 + f*x])^(5/2)) + (a^2*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c -
 c*Sin[e + f*x])^(3/2)) + (a^3*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{B\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e}{2cf(c-c\sin(e+fx)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e}{2cf(c-c\sin(e+fx)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e}{2cf(c-c\sin(e+fx)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e}{2cf(c-c\sin(e+fx)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e}{2cf(c-c\sin(e+fx)}$$

Mathematica [A] time = 1.23685, size = 204, normalized size = 1.04

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-6(A+2B)\right.}{3f(c-c\sin(e+fx))^{7/2}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] ((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.299, size = 832, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/3/f*(4*A+16*B-4*A*sin(f*x+e)-5*A*cos(f*x+e)^2+A*cos(f*x+e)^2*sin(f*x+e)-2
4*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-A*cos(f*x+e)^3*sin(f*x+e)+13*B*cos(f*
x+e)^2*sin(f*x+e)-7*B*cos(f*x+e)^3*sin(f*x+e)+48*B*cos(f*x+e)^2*ln(-(-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*B*cos(f*x+e)^3-6*B*cos(f*x+e)+12*B*ln(2/(
cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))*sin(f*x+e)*cos(f*x+e)+10*B*sin(f*x+e)*cos(f*x+e)-12*B*cos(f*x+e)*
ln(2/(cos(f*x+e)+1))+24*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))-24*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))+4*A*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^4*ln(2/(
cos(f*x+e)+1))-6*B*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
6*B*cos(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*
cos(f*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)^4+7*B*cos(f*x+e)^
4-23*B*cos(f*x+e)^2+24*B*ln(2/(cos(f*x+e)+1))-48*B*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-24*B*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))+12*B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+9*B*cos(f
*x+e)^3*ln(2/(cos(f*x+e)+1))-18*B*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))-16*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(
f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*s
in(f*x+e)+4)/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left((A+2B)a^2\cos\left(fx+e\right)^2-2(A+B)a^2+\left(Ba^2\cos\left(fx+e\right)^2-2(A+B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+}}{c^4\cos\left(fx+e\right)^4-8c^4\cos\left(fx+e\right)^2+8c^4+4\left(c^4\cos\left(fx+e\right)^2-2c^4\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos
(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c -
c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.275696, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2972, 2742}

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c -
c*Sin[e + f*x])^(7/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
```

```
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\int \frac{(a+a\sin(e+fx))^{5/}}{(c-c\sin(e+fx))^{7/}}}{8c}$$
$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\cos(e+fx)(a+}{48cf(c-c\sin(e}$$

Mathematica [A] time = 3.0597, size = 145, normalized size = 1.51

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((4A+17B)\sin(e+fx)-3(A-B)\cos(2(e+fx))+5A-3}{12c^4f(\sin(e+fx)-1)^4\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A
- 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[3*
(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.283, size = 309, normalized size = 3.2

$$-\frac{\left(A\left(\cos\left(fx+e\right)\right)^4 - A\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right) - B\left(\cos\left(fx+e\right)\right)^4 + B\left(\cos\left(fx+e\right)\right)^3\sin\left(fx+e\right) + 4A\left(\cos\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/6/f*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*sin(f*x+e)-B*cos(f*x+e)^4+B*cos(f*x+e
)^3*sin(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos(f*x+e)^2*sin(f*x+e)+2*B*cos(f*x+e)^
3+B*cos(f*x+e)^2*sin(f*x+e)-9*A*cos(f*x+e)^2+4*A*sin(f*x+e)*cos(f*x+e)+3*B*
cos(f*x+e)^2-4*B*sin(f*x+e)*cos(f*x+e)-10*A*cos(f*x+e)-14*A*sin(f*x+e)-2*B*
cos(f*x+e)+2*B*sin(f*x+e)+14*A-2*B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(co
s(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2
*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.50729, size = 400, normalized size = 4.17

$$-\frac{\left(3\,(A-B)a^2\cos\left(fx+e\right)^2-4\,(A-B)a^2+2\left(3\,Ba^2\cos\left(fx+e\right)^2-(A+5\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\cdots}}{6\left(c^5f\cos\left(fx+e\right)^5-8\,c^5f\cos\left(fx+e\right)^3+8\,c^5f\cos\left(fx+e\right)+4\left(c^5f\cos\left(fx+e\right)^3-2\,c^5f\cos\left(fx+e\right)\right)\sin\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] -1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x + e
)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x
 + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+ \ldots}{10f(c-c\sin(e+fx))^{1 \ldots}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(9/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2))/(240*c^2*f*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.376271, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}+\frac{(A-4B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+ \ldots}{10f(c-c\sin(e+fx))^{1 \ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(9/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2))/(240*c^2*f*(c - c*Sin[e + f*x])^(7/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\int \frac{(a+a\sin(e+fx))^{5/}}{(c-c\sin(e+fx))^{9/}}}{5c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+}{40cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+}{40cf(c-c\sin(e}$$

Mathematica [A] time = 4.20428, size = 146, normalized size = 1.

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(-5(8A+13B)\sin(e+fx)+10(2A+B)\cos(2(e+fx))-36}{120c^5f(\sin(e+fx)-1)^5\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36*
A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 15*
B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
```

```
Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.305, size = 368, normalized size = 2.5

$$\frac{\left(4A\left(\cos\left(fx+e\right)\right)^{5}+4A\left(\cos\left(fx+e\right)\right)^{4}\sin\left(fx+e\right)-B\left(\cos\left(fx+e\right)\right)^{5}-B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}-24A\left(\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/30/f*(4*A*cos(f*x+e)^5+4*A*cos(f*x+e)^4*sin(f*x+e)-B*cos(f*x+e)^5-B*sin(f
*x+e)*cos(f*x+e)^4-24*A*cos(f*x+e)^4+20*A*cos(f*x+e)^3*sin(f*x+e)+6*B*cos(f
*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-48*A*cos(f*x+e)^3-68*A*cos(f*x+e)^2*sin
(f*x+e)-3*B*cos(f*x+e)^3+2*B*cos(f*x+e)^2*sin(f*x+e)+118*A*cos(f*x+e)^2-50*
A*sin(f*x+e)*cos(f*x+e)-22*B*cos(f*x+e)^2+20*B*sin(f*x+e)*cos(f*x+e)+74*A*c
os(f*x+e)+124*A*sin(f*x+e)+4*B*cos(f*x+e)-16*B*sin(f*x+e)-124*A+16*B)*sin(f
*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(
f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(-1+sin(f
*x+e)))^(11/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.5766, size = 451, normalized size = 3.09

$$-\frac{\left(5\left(2A+B\right)a^{2}\cos\left(fx+e\right)^{2}-2\left(7A+2B\right)a^{2}+5\left(3Ba^{2}\cos\left(fx+e\right)^{2}-2\left(A+2B\right)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}}{30\left(5c^{6}f\cos\left(fx+e\right)^{5}-20c^{6}f\cos\left(fx+e\right)^{3}+16c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12c^{6}f\cos\left(fx+e\right)^{3}+16c^{6}f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos(
f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*
c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c
^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=196

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5
/2))/(160*c^2*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
a*Sin[e + f*x])^(5/2))/(960*c^3*f*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] time = 0.483459, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5
/2))/(160*c^2*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
a*Sin[e + f*x])^(5/2))/(960*c^3*f*(c - c*Sin[e + f*x])^(7/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\int \frac{(a+a\sin(e+fx))^{5/}}{(c-c\sin(e+fx))^{11}}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+}{40cf(c-c\sin(e-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+}{40cf(c-c\sin(e-}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+}{40cf(c-c\sin(e-}$$

Mathematica [A] time = 5.74742, size = 144, normalized size = 0.73

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(6(6A+7B)\sin(e+fx)-15(A+B)\cos(2(e+fx))+29A-}{120c^6f(\sin(e+fx)-1)^6\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*A
 + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B*S
in[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.332, size = 423, normalized size = 2.2

$$\frac{\left(-444\,A+52\,B+444\,A\sin\left(fx+e\right)+545\,A\left(\cos\left(fx+e\right)\right)^{2}+49\,A\left(\cos\left(fx+e\right)\right)^{4}\sin\left(fx+e\right)-343\,A\left(\cos\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/60/f*(-444*A+52*B+444*A*sin(f*x+e)+545*A*cos(f*x+e)^2+49*A*cos(f*x+e)^4*s
in(f*x+e)-343*A*cos(f*x+e)^2*sin(f*x+e)-7*A*cos(f*x+e)^5*sin(f*x+e)-7*B*sin
(f*x+e)*cos(f*x+e)^4+119*A*cos(f*x+e)^3*sin(f*x+e)+29*B*cos(f*x+e)^2*sin(f*
x+e)+242*A*cos(f*x+e)-17*B*cos(f*x+e)^3*sin(f*x+e)+B*cos(f*x+e)^5*sin(f*x+e
)-224*A*cos(f*x+e)^3+12*B*cos(f*x+e)^3-6*B*cos(f*x+e)+7*A*cos(f*x+e)^6-B*co
s(f*x+e)^6+46*B*sin(f*x+e)*cos(f*x+e)-202*A*sin(f*x+e)*cos(f*x+e)+42*A*cos(
f*x+e)^5-6*B*cos(f*x+e)^5-168*A*cos(f*x+e)^4+24*B*cos(f*x+e)^4-75*B*cos(f*x
+e)^2-52*B*sin(f*x+e))*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-co
s(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4
*sin(f*x+e)+4)/(-c*(-1+sin(f*x+e)))^(13/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.56969, size = 490, normalized size = 2.5

$$\frac{\left(15\,(A+B)a^2\cos\left(fx+e\right)^2-2\,(11\,A+7\,B)a^2+2\left(10\,Ba^2\cos\left(fx+e\right)^2-(9\,A+13\,B)a^2\right)\sin\left(fx+e\right)\right)\sqrt{}}{60\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\left(fx+e\right)^5-16\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] 1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos(
f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c
^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 1
6*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(13/2), x)
```

3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=250

$$-\frac{a^2(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{84f}-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\,\mathrm{si}}{126f}$$

```
[Out] -(a^4*(9*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*f*Sqrt[a + a*
Sin[e + f*x]]) - (a^3*(9*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(9/2))/(126*f) - (a^2*(9*A - B)*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(84*f) - (a*(9*A - B)*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(9/2))/(72*f) - (B*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(9*f)
```

Rubi [A] time = 0.566807, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{a^2(9A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{84f}-\frac{a^3(9A-B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\,\mathrm{si}}{126f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] -(a^4*(9*A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*f*Sqrt[a + a*
Sin[e + f*x]]) - (a^3*(9*A - B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(9/2))/(126*f) - (a^2*(9*A - B)*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(84*f) - (a*(9*A - B)*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(9/2))/(72*f) - (B*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(9*f)
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
```

```
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+f}{9f}$$

$$= -\frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\,\mathrm{s}}{72f}$$

$$= -\frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c}{84f}$$

$$= -\frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\,\mathrm{si}}{126f}$$

$$= -\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a+a\sin(e+fx)}} - \frac{a^3(}{}$$

Mathematica [B] time = 7.13863, size = 870, normalized size = 3.48

$$\frac{7(10A-B)\sin(e+fx)(a(\sin(e+fx)+1))^{7/2}(c-c\sin(e+fx))^{9/2}}{128f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{7(A-B)\cos(2(e+fx))(a(\sin(e}{128f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(9/2),x]
```

```
[Out] (7*(A - B)*Cos[2*(e + f*x)]*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*x
])^(9/2))/(128*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^7) + (7*(A - B)*Cos[4*(e + f*x)]*(a*(1 + Sin[e + f*x]))
^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(256*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + ((A - B)*Cos[6*(e + f*x)]
*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(128*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + ((
A - B)*Cos[8*(e + f*x)]*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(
9/2))/(1024*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7) + (7*(10*A - B)*Sin[e + f*x]*(a*(1 + Sin[e + f*x]))^(7/
2)*(c - c*Sin[e + f*x])^(9/2))/(128*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + (7*A*(a*(1 + Sin[e + f*x]))^(
7/2)*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)])/(64*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + ((7*A + 2*B
)*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2)*Sin[5*(e + f*x)])
/(320*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^7) + ((4*A + 5*B)*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*
x])^(9/2)*Sin[7*(e + f*x)])/(1792*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + (B*(a*(1 + Sin[e + f*x]))^(7/2)
*(c - c*Sin[e + f*x])^(9/2)*Sin[9*(e + f*x)])/(2304*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A] time = 0.306, size = 259, normalized size = 1.

$$\frac{\left(-280\,B\left(\cos\left(fx+e\right)\right)^{8}+315\,A\left(\cos\left(fx+e\right)\right)^{6}\sin\left(fx+e\right)-315\,B\left(\cos\left(fx+e\right)\right)^{6}\sin\left(fx+e\right)-360\,A\left(\cos\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/2520/f*(-280*B*cos(f*x+e)^8+315*A*cos(f*x+e)^6*sin(f*x+e)-315*B*cos(f*x+e
)^6*sin(f*x+e)-360*A*cos(f*x+e)^6+40*B*cos(f*x+e)^6+315*A*cos(f*x+e)^4*sin(
f*x+e)-315*B*sin(f*x+e)*cos(f*x+e)^4-432*A*cos(f*x+e)^4+48*B*cos(f*x+e)^4+3
15*A*cos(f*x+e)^2*sin(f*x+e)-315*B*cos(f*x+e)^2*sin(f*x+e)-576*A*cos(f*x+e)
^2+64*B*cos(f*x+e)^2+315*A*sin(f*x+e)-315*B*sin(f*x+e)-1152*A+128*B)*(-c*(-
1+sin(f*x+e)))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(-1+sin(f*x+e))/co
```

```
s(f*x+e)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.89381, size = 427, normalized size = 1.71

$$\frac{\left(315\,(A-B)a^3c^4\cos\left(fx+e\right)^8-315\,(A-B)a^3c^4+8\left(35\,Ba^3c^4\cos\left(fx+e\right)^8+5\,(9\,A-B)a^3c^4\cos\left(fx+e\right)^6+6\,(9\,A-\right.\right.}{2520\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] 1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B*
a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)*a
^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B)*a
^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] sage2
```

3.161 $\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx$

Optimal. Leaf size=226

$$-\frac{2a^4 A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3 A\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2 A\cos(e+fx)(a$$

```
[Out] (-2*a^4*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^3*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(7/2))/(35*f) - (a^2*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(7/2))/(7*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
*(c - c*Sin[e + f*x])^(7/2))/(8*f)
```

Rubi [A] time = 0.559084, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$-\frac{2a^4 A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a\sin(e+fx)+a}}-\frac{4a^3 A\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{35f}-\frac{a^2 A\cos(e+fx)(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (-2*a^4*A*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^3*A*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(7/2))/(35*f) - (a^2*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*f) - (a*A*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(7/2))/(7*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
*(c - c*Sin[e + f*x])^(7/2))/(8*f)
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
```

```
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+}{8f}$$

$$= -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e}{7f}$$

$$= -\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e}{7f}$$

$$= -\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e}{35f}$$

$$= -\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} - \frac{4a^3A\,c}{}$$

Mathematica [A] time = 1.673, size = 135, normalized size = 0.6

$$\frac{a^3c^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(19600A\sin(e+fx)+3920A\sin(3(e+fx))+784A\sin(5(e+f}{3584}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-1960*B*Cos[2*(e + f*x)] - 980*B*Cos[4*(e + f*x)] - 280*B*Cos[6*(e + f*x)]
- 35*B*Cos[8*(e + f*x)] + 19600*A*Sin[e + f*x] + 3920*A*Sin[3*(e + f*x)] +
784*A*Sin[5*(e + f*x)] + 80*A*Sin[7*(e + f*x)]))/(35840*f)
```

Maple [A] time = 0.33, size = 142, normalized size = 0.6

$$\frac{\left(35\,B\left(\cos\left(fx+e\right)\right)^{6}\sin\left(fx+e\right)+40\,A\left(\cos\left(fx+e\right)\right)^{6}+35\,B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+48\,A\left(\cos\left(fx+e\right)\right)^{4}+35}{280\,f\left(\cos\left(fx+e\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/280/f*(35*B*cos(f*x+e)^6*sin(f*x+e)+40*A*cos(f*x+e)^6+35*B*sin(f*x+e)*cos
(f*x+e)^4+48*A*cos(f*x+e)^4+35*B*cos(f*x+e)^2*sin(f*x+e)+64*A*cos(f*x+e)^2+
35*B*sin(f*x+e)+128*A)*(-c*(-1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+
e)))^(7/2)/cos(f*x+e)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [A] time = 1.81399, size = 324, normalized size = 1.43

$$-\frac{\left(35\,Ba^3c^3\cos\left(fx+e\right)^8-35\,Ba^3c^3-8\left(5\,Aa^3c^3\cos\left(fx+e\right)^6+6\,Aa^3c^3\cos\left(fx+e\right)^4+8\,Aa^3c^3\cos\left(fx+e\right)^2+16\,A\right.\right.}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] -1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*x
 + e)^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*a^
3*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*
cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

3.162 $\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx$

Optimal. Leaf size=192

$$\frac{2c^2(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}+\frac{c^3(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}+\frac{c(7A+$$

```
[Out] ((7*A + B)*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(105*f*Sqrt[c - c*S
in[e + f*x]]) + (2*(7*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sq
rt[c - c*Sin[e + f*x]])/(105*f) + ((7*A + B)*c*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2))/(7*f)
```

Rubi [A] time = 0.459797, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$\frac{2c^2(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}+\frac{c^3(7A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}+\frac{c(7A+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5
/2),x]
```


```
[Out] ((7*A + B)*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(105*f*Sqrt[c - c*S
in[e + f*x]]) + (2*(7*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sq
rt[c - c*Sin[e + f*x]])/(105*f) + ((7*A + B)*c*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2))/(7*f)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
```

```
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+}{7f}$$

$$= \frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\,s}{42f}$$

$$= \frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-}}{105f}$$

$$= \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{105f\sqrt{c-c\sin(e+fx)}} + \frac{2(}{}$$

Mathematica [A] time = 2.19684, size = 232, normalized size = 1.21

$$\frac{a^3c^2(\sin(e+fx)-1)^2(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(-525(A+B)\cos(2(e+fx))-210(A+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] (a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)*C
os[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Si
n[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(e +
f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e + f*x
)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^7)
```

Maple [A] time = 0.359, size = 203, normalized size = 1.1

$$\frac{\left(-30\,B\left(\cos\left(fx+e\right)\right)^{6}+35\,A\left(\cos\left(fx+e\right)\right)^{4}\sin\left(fx+e\right)+35\,B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+42\,A\left(\cos\left(fx+e\right)\right)^{4}+6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/210/f*(-30*B*cos(f*x+e)^6+35*A*cos(f*x+e)^4*sin(f*x+e)+35*B*sin(f*x+e)*co
s(f*x+e)^4+42*A*cos(f*x+e)^4+6*B*cos(f*x+e)^4+35*A*cos(f*x+e)^2*sin(f*x+e)+
35*B*cos(f*x+e)^2*sin(f*x+e)+56*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2+35*A*sin(f*
x+e)+35*B*sin(f*x+e)+112*A+16*B)*(-c*(-1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(a*(
1+sin(f*x+e)))^(7/2)/(1+sin(f*x+e))/cos(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [A] time = 1.73989, size = 373, normalized size = 1.94

$$-\frac{\left(35\,(A+B)a^3c^2\cos\left(fx+e\right)^6-35\,(A+B)a^3c^2+2\left(15\,Ba^3c^2\cos\left(fx+e\right)^6-3\,(7\,A+B)a^3c^2\cos\left(fx+e\right)^4-4\,(7\,A+\right.\right.}{210\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] -1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a^
3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a^3
*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] sage2
```

3.163 $\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx$

Optimal. Leaf size=142

$$\frac{c^2(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}}{30f\sqrt{c - c\sin(e + fx)}} + \frac{c(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}\sqrt{c - c\sin(e + fx)}}{15f} - \frac{B\cos(e +$$

```
[Out] ((3*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((3*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c -
 c*Sin[e + f*x])^(3/2))/(6*f)
```

Rubi [A] time = 0.359294, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2973, 2740, 2738}

$$\frac{c^2(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}}{30f\sqrt{c - c\sin(e + fx)}} + \frac{c(3A + B)\cos(e + fx)(a\sin(e + fx) + a)^{7/2}\sqrt{c - c\sin(e + fx)}}{15f} - \frac{B\cos(e +$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3
/2),x]
```


```
[Out] ((3*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(30*f*Sqrt[c - c*Si
n[e + f*x]]) + ((3*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(15*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c -
 c*Sin[e + f*x])^(3/2))/(6*f)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+}{6f}$$

$$= \frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c}}{15f}$$

$$= \frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{30f\sqrt{c-c\sin(e+fx)}} + \frac{(3}{}$$

Mathematica [A] time = 1.85639, size = 212, normalized size = 1.49

$$-\frac{a^3c(\sin(e+fx)-1)(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(-15(16A+11B)\cos(2(e+fx))-30(2A}{960f\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] -(a^3*c*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]
*Sqrt[c - c*Sin[e + f*x]]*(-15*(16*A + 11*B)*Cos[2*(e + f*x)] - 30*(2*A + B
)*Cos[4*(e + f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] + 240*B*Sin[
e + f*x] + 60*A*Sin[3*(e + f*x)] - 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e +
```

```
f*x)] - 24*B*Sin[5*(e + f*x)]))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A] time = 0.329, size = 185, normalized size = 1.3

$$\frac{\left(5 B \sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+6 A\left(\cos\left(fx+e\right)\right)^{4}+12 B\left(\cos\left(fx+e\right)\right)^{4}-15 A\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)-10 B\left(\right.}{30 f\left(\left(\cos\left(fx+e\right)\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4+12*B*cos(f*x+e)^4-15*A
*cos(f*x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^2*sin(f*x+e)-12*A*cos(f*x+e)^2-4*B
*cos(f*x+e)^2-15*A*sin(f*x+e)-10*B*sin(f*x+e)-24*A-8*B)*(-c*(-1+sin(f*x+e))
)^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)/c
os(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [A] time = 1.56558, size = 356, normalized size = 2.51

$$\frac{\left(5 Ba^{3}c\cos\left(fx+e\right)^{6}-15\left(A+B\right)a^{3}c\cos\left(fx+e\right)^{4}+5\left(3 A+2 B\right)a^{3}c-2\left(3\left(A+2 B\right)a^{3}c\cos\left(fx+e\right)^{4}-2\left(3 A+B\right)a^{3}c\right.\right.}{30 f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] 1/30*(5*B*a^3*c*cos(f*x + e)^6 - 15*(A + B)*a^3*c*cos(f*x + e)^4 + 5*(3*A +
 2*B)*a^3*c - 2*(3*(A + 2*B)*a^3*c*cos(f*x + e)^4 - 2*(3*A + B)*a^3*c*cos(f
*x + e)^2 - 4*(3*A + B)*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rubi [A] time = 0.324937, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e +
 f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A$$
$$= \frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(}{}$$

Mathematica [A] time = 1.02277, size = 121, normalized size = 1.26

$$\frac{a^3\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(4(60A+23B)\sin(e+fx)+\cos(4(e+fx))(5A+4B\sin(e+fx)+}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(6
0*A + 23*B)*Sin[e + f*x] + Cos[4*(e + f*x)]*(5*A + 15*B + 4*B*Sin[e + f*x])
 - 4*Cos[2*(e + f*x)]*(5*(7*A + 5*B) + 4*(5*A + 6*B)*Sin[e + f*x])))/(160*f
)
```

Maple [B] time = 0.359, size = 174, normalized size = 1.8

$$\frac{\left(-4B\left(\cos\left(fx+e\right)\right)^4+5A\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+15B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+20A\left(\cos\left(fx+e\right)\right)^2+28}{20f\left(\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+3\left(\cos\left(fx+e\right)\right)^2-4\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*sin
(f*x+e)+20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)-25*B*sin(f*x+e)
-40*A-24*B)*(-c*(-1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/
```

```
(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/cos(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}\sqrt{-c\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [A] time = 1.42321, size = 340, normalized size = 3.54

$$\frac{\left(5\,(A+3\,B)a^{3}\cos\left(fx+e\right)^{4}-40\,(A+B)a^{3}\cos\left(fx+e\right)^{2}+5\,(7\,A+5\,B)a^{3}+4\left(Ba^{3}\cos\left(fx+e\right)^{4}-(5\,A+7\,B)a^{3}\cos\left(f\right.\right.\right.}{20\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(f*x + e)^4 - 40*(A + B)*a^3*cos(f*x + e)^2 + 5*(7
*A + 5*B)*a^3 + 4*(B*a^3*cos(f*x + e)^4 - (5*A + 7*B)*a^3*cos(f*x + e)^2 +
2*(5*A + 3*B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=239

$$-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{8a^4(A+B)\cos(e+fx)\log(}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-}}$$

```
[Out] (-8*a^4*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (4*a^3*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + B)*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x
]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f
*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.568527, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}-\frac{a^2(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f\sqrt{c-c\sin(e+fx)}}-\frac{8a^4(A+B)\cos(e+fx)\log(}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*
x]],x]
```


```
[Out] (-8*a^4*(A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (4*a^3*(A + B)*Cos[e + f*x]*Sqrt[a + a*Sin
[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + B)*Cos[e + f*x]*(a + a
*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) - (a*(A + B)*Cos[e + f*x
]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f
*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}$$
$$= -\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(}{4f\sqrt{c-c\sin(e+}}$$
$$= -\frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a}{3f\sqrt{c-c\sin}}$$
$$= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos(e+fx)(a}{f\sqrt{c-c\sin(}}$$
$$= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos(e+fx)(a}{f\sqrt{c-c\sin(}}$$
$$= -\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+B)\cos(e+fx)(a}{f\sqrt{c-c\sin(}}$$
$$= -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^3(A+B)\cos(e+fx}{f\sqrt{c-c\sin}}$$

Mathematica [A] time = 2.80129, size = 183, normalized size = 0.77

$$-\frac{a^3(\sin(e+fx)+1)^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(29A+36B)\sin(e+fx)-8(A+4B)\sin\right.}{96f\sqrt{c-c\sin(e+fx)}\left(\sin\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] -(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[a*(1
+ Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)]
+ 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A + 36*B)*
Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(96*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.342, size = 671, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(-64*A-67*B+64*A*sin(f*x+e)+68*A*cos(f*x+e)^2+20*A*cos(f*x+e)^2*sin(
f*x+e)+96*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-3*B*sin(f*x+e)*cos(f*x+e)^4+4*A
*cos(f*x+e)^3*sin(f*x+e)+32*B*cos(f*x+e)^2*sin(f*x+e)-24*A*cos(f*x+e)+16*B*
cos(f*x+e)^3*sin(f*x+e)-192*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))+24*A*cos(f*x+e)^3+48*B*cos(f*x+e)^3-45*B*cos(f*x+e)+96*A*sin(f*x+e
)*ln(2/(cos(f*x+e)+1))-192*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))-112*B*sin(f*x+e)*cos(f*x+e)+96*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-19
2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+96*B*sin(f*x+e)*l
n(2/(cos(f*x+e)+1))-192*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))-88*A*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)^5-4*A*cos(f*x+e)^4-13*B*cos(
f*x+e)^4+80*B*cos(f*x+e)^2-96*A*ln(2/(cos(f*x+e)+1))+192*A*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-96*B*ln(2/(cos(f*x+e)+1))+192*B*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+67*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(7/2)/(sin(f
*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*
sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+s
in(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A+B)a^3\right)\right.}{c\sin\left(fx+e\right)-c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x +
 e) + c), x)
```

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^4(3A+5B)\cos(e+}{cf\sqrt{a\sin(e+fx)+}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^4*(3*A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^3*(3*A + 5*B)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a^2*(3*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*
x]]) + (a*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.593029, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{4a^4(3A+5B)\cos(e+}{cf\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(3/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x])
^(3/2)) + (4*a^4*(3*A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^3*(3*A + 5*B)*Cos[e +
f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a^2*(3*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*
x]]) + (a*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[
c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} - \frac{(3A+5B)\int \frac{(a+a\sin(e+fx))}{\sqrt{c-c\sin(e+fx}}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a(3A+5B)\cos(e+fx)(a}{6cf\sqrt{c-c\sin}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(3A+5B)\cos(e+fx)(}{2cf\sqrt{c-c\sin}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5B)\cos(e+fx}{cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5B)\cos(e+fx}{cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(3A+5B)\cos(e+fx}{cf\sqrt{c-c\sin(}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{4a^4(3A+5B)\cos(e+fx}{cf\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 3.56677, size = 292, normalized size = 1.08

$$\frac{a^3\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2(27A+59B)\cos(2(e+fx))-117A\sin(e+fx)-3A\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-132
*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 960*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] - 13*B*Sin[3*(e + f*x)
]))/(24*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[
```

```
c - c*Sin[e + f*x]])
```

Maple [B] time = 0.281, size = 927, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

[Out] -1/6/f*(-102*A-166*B+102*A*sin(f*x+e)+99*A*cos(f*x+e)^2-24*A*cos(f*x+e)^2*s
in(f*x+e)+72*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+120*B*cos(f*x+e)^2*ln(2/(cos
(f*x+e)+1))+2*B*sin(f*x+e)*cos(f*x+e)^4-3*A*cos(f*x+e)^3*sin(f*x+e)-48*B*co
s(f*x+e)^2*sin(f*x+e)+27*A*cos(f*x+e)+144*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-72*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+
e)+1))-13*B*cos(f*x+e)^3*sin(f*x+e)-144*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-240*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))-27*A*cos(f*x+e)^3-61*B*cos(f*x+e)^3+59*B*cos(f*x+e)-120*B*ln(2/(cos
(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+240*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))*sin(f*x+e)*cos(f*x+e)+144*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-288*A*s
in(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-107*B*sin(f*x+e)*cos(f
*x+e)+120*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-240*B*cos(f*x+e)*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))+240*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-480*B*si
n(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-75*A*sin(f*x+e)*cos(f*x
+e)-144*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+72*A*cos(
f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)^5+3*A*cos(f*x+e)^4+11*B*cos(f*
x+e)^4+155*B*cos(f*x+e)^2-144*A*ln(2/(cos(f*x+e)+1))+288*A*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-240*B*ln(2/(cos(f*x+e)+1))+480*B*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))+166*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(7/2)/(sin
(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-
4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1
+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A+B)a^3\right)\right.}{c^2\cos\left(fx+e\right)^2+2c^2\sin\left(fx+e\right)-2c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*si
n(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{3a^3(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{6a^4(A+3B)\cos(e+f}{c^2f\sqrt{a\sin(e+fx)+}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (6*a^4*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x
]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*(A +
 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]
]) - (3*a^2*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c^2*f*Sqr
t[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.595239, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$-\frac{3a^3(A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2(A+3B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}-\frac{6a^4(A+3B)\cos(e+f}{c^2f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(5/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x])
^(5/2)) - (a*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c -
 c*Sin[e + f*x])^(3/2)) - (6*a^4*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x
]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^3*(A +
 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*Sqrt[c - c*Sin[e + f*x]
]) - (3*a^2*(A + 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c^2*f*Sqr
t[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{(A+3B)\int\frac{(a+a\sin(e+fx))^{7/}}{(c-c\sin(e+fx))^{3/}}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a}{2cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a}{2cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a}{2cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a}{2cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a}{2cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a}{2cf(c-c\sin(e}$$

Mathematica [A] time = 2.59541, size = 251, normalized size = 0.95

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4(A+6B)\sin(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A
+ B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(e
+ f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(A
```

```
+ 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

Maple [B] time = 0.28, size = 1189, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(32*A+100*B-32*A*sin(f*x+e)-34*A*cos(f*x+e)^2+22*A*cos(f*x+e)^2*sin(
f*x+e)-24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-108*B*cos(f*x+e)^2*ln(2/(cos(f*
x+e)+1))+B*sin(f*x+e)*cos(f*x+e)^4-2*A*cos(f*x+e)^3*sin(f*x+e)+63*B*cos(f*x
+e)^2*sin(f*x+e)-20*A*cos(f*x+e)-48*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))
-10*B*cos(f*x+e)^3*sin(f*x+e)+48*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))+216*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
+20*A*cos(f*x+e)^3+53*B*cos(f*x+e)^3-54*B*cos(f*x+e)+72*B*ln(2/(cos(f*x+e)+
1))*sin(f*x+e)*cos(f*x+e)-144*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*
sin(f*x+e)*cos(f*x+e)-48*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+96*A*sin(f*x+e)*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+46*B*sin(f*x+e)*cos(f*x+e)-72*B*
cos(f*x+e)*ln(2/(cos(f*x+e)+1))+144*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-144*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+288*B*sin(f*x+e)*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*A*sin(f*x+e)*cos(f*x+e)+72*A*cos
(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-36*A*cos(f*x+e)^2*ln(2
/(cos(f*x+e)+1))+B*cos(f*x+e)^5-24*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))*cos(f*x+e)^3+12*A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-24*A*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+12*A*ln(2/(cos(f*x+e)
+1))*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^4+9*B*cos(f*x+e)^4-109*B*cos(f*
x+e)^2+48*A*ln(2/(cos(f*x+e)+1))-96*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+144*B*ln(2/(cos(f*x+e)+1))-288*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))-72*B*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+36*B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+36*B*cos(f*x+e)^3*ln(
2/(cos(f*x+e)+1))-72*B*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-100*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos
(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-
8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A+B)a^3\right)}{3c^3\cos\left(fx+e\right)^2-4c^3-\left(c^3\cos\left(fx+e\right)^2-4c^3\right)\sin\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3
- (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=264

$$\frac{a^3(A+7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^3f\sqrt{c-c\sin(e+fx)}}+\frac{a^2(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^4(A+7B)\cos(e+fx)\log}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*c*f*(c
- c*Sin[e + f*x])^(5/2)) + (a^2*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(4*c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^4*(A + 7*B)*Cos[e + f*x]*
Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (a^3*(A + 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^3*f*Sqrt
[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.608763, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{a^3(A+7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2c^3f\sqrt{c-c\sin(e+fx)}}+\frac{a^2(A+7B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}}+\frac{a^4(A+7B)\cos(e+fx)\log}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x])
^(7/2)) - (a*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*c*f*(c
- c*Sin[e + f*x])^(5/2)) + (a^2*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(4*c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^4*(A + 7*B)*Cos[e + f*x]*
Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (a^3*(A + 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^3*f*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{(A+7B)\int \frac{(a+a\sin(e+fx))^{7/}}{(c-c\sin(e+fx))^{5/}}}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a}{12cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a}{12cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a}{12cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a}{12cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a}{12cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{a(A+7B)\cos(e+fx)(a}{12cf(c-c\sin(e}$$

Mathematica [A] time = 2.98761, size = 244, normalized size = 0.92

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(18(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4-6(3A+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + 
f*x])^(7/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A + 
 B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B)*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[(e + 
 f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[
```

```
(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

Maple [B] time = 0.282, size = 1455, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] -1/3/f*(-20*A-116*B+20*A*sin(f*x+e)+28*A*cos(f*x+e)^2-14*A*cos(f*x+e)^2*sin
(f*x+e)+12*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+168*B*cos(f*x+e)^2*ln(2/(cos(f
*x+e)+1))+3*B*sin(f*x+e)*cos(f*x+e)^4+8*A*cos(f*x+e)^3*sin(f*x+e)-98*B*cos(
f*x+e)^2*sin(f*x+e)+6*A*cos(f*x+e)+24*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))-12*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1
))+41*B*cos(f*x+e)^3*sin(f*x+e)-24*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))-336*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-6*A*cos(f*x+e)^3-57*B*cos(f*x+e)^3+54*B*cos(f*x+e)-84*B*ln(2/(cos(f*x+e)
+1))*sin(f*x+e)*cos(f*x+e)+168*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
*sin(f*x+e)*cos(f*x+e)+24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*sin(f*x+e)
*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-62*B*sin(f*x+e)*cos(f*x+e)+84*B
*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-168*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+168*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-336*B*sin(f*x+e)*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-14*A*sin(f*x+e)*cos(f*x+e)-48*A*co
s(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)^2*ln(
2/(cos(f*x+e)+1))-21*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+42*B*cos(f*x+e)^4*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*A*cos(f*x+e)^4*ln(-(-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)^5-3*A*cos(f*x+e)^4*ln(2/(cos(f*x
+e)+1))+18*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3-9*A*ln
(2/(cos(f*x+e)+1))*cos(f*x+e)^3+24*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))*cos(f*x+e)^2*sin(f*x+e)-12*A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x
+e)+3*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))*sin(f*x+e)-6*A*cos(f*x+e)^3*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-42*B*cos(f*x+e)^3*sin(f*x+
e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+21*B*cos(f*x+e)^3*sin(f*x+e)*
ln(2/(cos(f*x+e)+1))-8*A*cos(f*x+e)^4-44*B*cos(f*x+e)^4+160*B*cos(f*x+e)^2-
24*A*ln(2/(cos(f*x+e)+1))+48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1
68*B*ln(2/(cos(f*x+e)+1))+336*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
168*B*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-84
*B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-63*B*cos(f*x+e)^3*ln(2/(cos
(f*x+e)+1))+126*B*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1
16*B*sin(f*x+e))*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+
e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos
```

```
(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(7/2)
```

Maxima [B] time = 1.63585, size = 1011, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="maxima")
```

```
[Out] -1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/2)
*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2)*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(7/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7/2)*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x + e)/(cos(f*x
 + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 22*a^(
7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2)*sqrt(c)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^4*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
5*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{a^3B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}-\frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e
 + f*x])^(7/2)) + (a^2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*
(c - c*Sin[e + f*x])^(5/2)) - (a^3*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(c^3*f*(c - c*Sin[e + f*x])^(3/2)) - (a^4*B*Cos[e + f*x]*Log[1 - Sin[e + f
*x]])/(c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.598736, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{a^3B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}+\frac{a^2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}-\frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])
^(9/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e
 + f*x])^(7/2)) + (a^2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*
(c - c*Sin[e + f*x])^(5/2)) - (a^3*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(c^3*f*(c - c*Sin[e + f*x])^(3/2)) - (a^4*B*Cos[e + f*x]*Log[1 - Sin[e + f
*x]])/(c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{B\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}dx}{c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(}{3cf(c-c\sin(e+f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(}{3cf(c-c\sin(e+f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(}{3cf(c-c\sin(e+f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(}{3cf(c-c\sin(e+f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(}{3cf(c-c\sin(e+f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(}{3cf(c-c\sin(e+f}$$

Mathematica [A] time = 2.74836, size = 238, normalized size = 0.96

$$\frac{(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-3(A+7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6+9(A+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] ((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A
+ 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(9/2))
```

Maple [B] time = 0.323, size = 1019, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/3/f*(6*A-34*B-6*A*sin(f*x+e)-9*A*cos(f*x+e)^2+3*A*cos(f*x+e)^2*sin(f*x+e)
+60*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+8*B*sin(f*x+e)*cos(f*x+e)^4-3*A*cos
(f*x+e)^3*sin(f*x+e)-39*B*cos(f*x+e)^2*sin(f*x+e)+3*B*cos(f*x+e)^4*sin(f*x+
e)*ln(2/(cos(f*x+e)+1))+11*B*cos(f*x+e)^3*sin(f*x+e)-120*B*cos(f*x+e)^2*ln(
-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-28*B*cos(f*x+e)^3+20*B*cos(f*x+e)-2
4*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+48*B*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-14*B*sin(f*x+e)*cos(f*x+e)+24*B*c
os(f*x+e)*ln(2/(cos(f*x+e)+1))-48*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+48*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-96*B*sin(f*x+e)*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*A*sin(f*x+e)*cos(f*x+e)-6*B*cos(f*x+e
)^4*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-15*B*cos(f*x+e)^4
*ln(2/(cos(f*x+e)+1))+30*B*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))+3*B*cos(f*x+e)^5*ln(2/(cos(f*x+e)+1))+8*B*cos(f*x+e)^5-24*B*cos(f*x
+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*B*cos(f*x+e)
^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-6*B*cos(f*x+e)^5*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)^4-19*B*cos(f*x+e)^4+53*B*cos(f*x+e)^2-48
*B*ln(2/(cos(f*x+e)+1))+96*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+72*
B*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-36*B*c
os(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*B*cos(f*x+e)^3*ln(2/(cos(f*x
+e)+1))+48*B*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+34*B*s
in(f*x+e))*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4
*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e
)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Ba^3\cos\left(fx+e\right)^4-(3A+5B)a^3\cos\left(fx+e\right)^2+4(A+B)a^3-\left((A+3B)a^3\cos\left(fx+e\right)^2-4(A+B)a^3\right)\text{s}\right.}{5c^5\cos\left(fx+e\right)^4-20c^5\cos\left(fx+e\right)^2+16c^5-\left(c^5\cos\left(fx+e\right)^4-12c^5\cos\left(fx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*
cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*
c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=96

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c
- c*Sin[e + f*x])^(9/2))
```

Rubi [A] time = 0.271934, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2972, 2742}

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x]
)^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c
- c*Sin[e + f*x])^(9/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
```

```
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}}{10c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-9B)\cos(e+fx)(a+a}{80cf(c-c\sin(e+}$$

Mathematica [B] time = 6.91147, size = 434, normalized size = 4.52

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{2f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{2(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}{f(c-c\sin(e+fx))^{11/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(11/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2
)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + S
in[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*
(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c
- c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(
1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(11/2))
```

Maple [B] time = 0.298, size = 389, normalized size = 4.1

$$\left(A\left(\cos\left(fx+e\right)\right)^5 + A\left(\cos\left(fx+e\right)\right)^4\sin\left(fx+e\right) + B\left(\cos\left(fx+e\right)\right)^5 + B\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4 - 6A\left(\cos\left(fx-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

```
[Out] 1/10/f*(A*cos(f*x+e)^5+A*cos(f*x+e)^4*sin(f*x+e)+B*cos(f*x+e)^5+B*sin(f*x+e
)*cos(f*x+e)^4-6*A*cos(f*x+e)^4+5*A*cos(f*x+e)^3*sin(f*x+e)+4*B*cos(f*x+e)^
4-5*B*cos(f*x+e)^3*sin(f*x+e)-17*A*cos(f*x+e)^3-22*A*cos(f*x+e)^2*sin(f*x+e
)-7*B*cos(f*x+e)^3-2*B*cos(f*x+e)^2*sin(f*x+e)+32*A*cos(f*x+e)^2-10*A*sin(f
*x+e)*cos(f*x+e)-8*B*cos(f*x+e)^2+10*B*sin(f*x+e)*cos(f*x+e)+26*A*cos(f*x+e
)+36*A*sin(f*x+e)+6*B*cos(f*x+e)-4*B*sin(f*x+e)-36*A+4*B)*sin(f*x+e)*(a*(1+
sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*cos(f*x+e)^2*sin
(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)-
4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(11/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.41408, size = 489, normalized size = 5.09

$$\frac{\left(10\,Ba^3\cos\left(fx+e\right)^4-5\,(A+7\,B)a^3\cos\left(fx+e\right)^2+2\,(3\,A+13\,B)a^3-5\left((A-B)a^3\cos\left(fx+e\right)^2-2\,(A-B)a^3\right)\sin\left(\right.}{10\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+e\right)^3+16\,c^6f\cos\left(fx+e\right)-\left(c^6f\cos\left(fx+e\right)^5-12\,c^6f\cos\left(fx+e\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="fricas")
```

```
[Out] 1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A + 1
3*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 2
```

```
0*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12
*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(11/2), x)
```

3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=146

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(480*c^2*f*(c - c*Sin[e + f*x])^(9/2))
```

Rubi [A] time = 0.378984, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}+\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(13/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x]
)^(13/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c
- c*Sin[e + f*x])^(11/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(480*c^2*f*(c - c*Sin[e + f*x])^(9/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}}{6c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+fx)(a+a}{60cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+fx)(a+a}{60cf(c-c\sin(e+}$$

Mathematica [B] time = 6.94883, size = 442, normalized size = 3.03

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{3f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{3(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}{2f(c-c\sin(e+fx))^{13/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(13/2),x]
```

```
[Out] (4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2
)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
```

```
 f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(13/2)) + (3*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(13/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(13/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(13/2))
```

Maple [B] time = 0.331, size = 393, normalized size = 2.7

$$\frac{\left(3\,A\left(\cos\left(fx+e\right)\right)^{6}-3\,A\left(\cos\left(fx+e\right)\right)^{5}\sin\left(fx+e\right)+18\,A\left(\cos\left(fx+e\right)\right)^{5}+21\,A\left(\cos\left(fx+e\right)\right)^{4}\sin\left(fx+e\right)-72}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)
```

```
[Out] 1/30/f*(3*A*cos(f*x+e)^6-3*A*cos(f*x+e)^5*sin(f*x+e)+18*A*cos(f*x+e)^5+21*A
*cos(f*x+e)^4*sin(f*x+e)-72*A*cos(f*x+e)^4+51*A*cos(f*x+e)^3*sin(f*x+e)+15*
B*cos(f*x+e)^4-15*B*cos(f*x+e)^3*sin(f*x+e)-106*A*cos(f*x+e)^3-157*A*cos(f*
x+e)^2*sin(f*x+e)-10*B*cos(f*x+e)^3+5*B*cos(f*x+e)^2*sin(f*x+e)+235*A*cos(f
*x+e)^2-78*A*sin(f*x+e)*cos(f*x+e)-35*B*cos(f*x+e)^2+30*B*sin(f*x+e)*cos(f*
x+e)+118*A*cos(f*x+e)+196*A*sin(f*x+e)+10*B*cos(f*x+e)-20*B*sin(f*x+e)-196*
A+20*B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*
x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*c
os(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(13/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.28966, size = 535, normalized size = 3.66

$$-\frac{\left(15\,Ba^{3}\cos\left(fx+e\right)^{4}-15\,(A+3\,B)a^{3}\cos\left(fx+e\right)^{2}+6\,(3\,A+5\,B)a^{3}-2\left(5\,(A+B)a^{3}\cos\left(fx+e\right)^{2}-(11\,A+5\,B)a^{3}\right)\cdots}{30\left(c^{7}f\cos\left(fx+e\right)^{7}-18\,c^{7}f\cos\left(fx+e\right)^{5}+48\,c^{7}f\cos\left(fx+e\right)^{3}-32\,c^{7}f\cos\left(fx+e\right)+2\left(3\,c^{7}f\cos\left(fx+e\right)^{5}-16\,\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="fricas")
```

```
[Out] -1/30*(15*B*a^3*cos(f*x + e)^4 - 15*(A + 3*B)*a^3*cos(f*x + e)^2 + 6*(3*A +
 5*B)*a^3 - 2*(5*(A + B)*a^3*cos(f*x + e)^2 - (11*A + 5*B)*a^3)*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7
 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e
) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x
+ e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(13/2), x)
```

3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=202

$$\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}+\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}}+\frac{(3A-11B)\cos(e+fx)(a}{168cf(c-c\sin(e}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x]
)^(15/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(168*c*f
*(c - c*Sin[e + f*x])^(13/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(7/2))/(840*c^2*f*(c - c*Sin[e + f*x])^(11/2)) + ((3*A - 11*B)*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2))/(6720*c^3*f*(c - c*Sin[e + f*x])^(9/2))
```

Rubi [A] time = 0.49066, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}+\frac{(3A-11B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}}+\frac{(3A-11B)\cos(e+fx)(a}{168cf(c-c\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x]
)^(15/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(168*c*f
*(c - c*Sin[e + f*x])^(13/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(7/2))/(840*c^2*f*(c - c*Sin[e + f*x])^(11/2)) + ((3*A - 11*B)*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(7/2))/(6720*c^3*f*(c - c*Sin[e + f*x])^(9/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\int \frac{(a+a\sin(e+fx}{(c-c\sin(e+fx)}}{14c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a}{168cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a}{168cf(c-c\sin(e}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a}{168cf(c-c\sin(e}$$

Mathematica [B] time = 7.14892, size = 442, normalized size = 2.19

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{4f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7} + \frac{6(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+f\right.\right.}{5f(c-c\sin(e+fx))^{15/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(15/2),x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/
2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2
)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
 f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7
*(c - c*Sin[e + f*x])^(15/2))
```

Maple [B] time = 0.366, size = 505, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -1/420/f*(5016*A-472*B+39*A*cos(f*x+e)^7-3*B*cos(f*x+e)^7-5016*A*sin(f*x+e)
-7404*A*cos(f*x+e)^2-1209*A*cos(f*x+e)^4*sin(f*x+e)+5136*A*cos(f*x+e)^2*sin
(f*x+e)+273*A*cos(f*x+e)^5*sin(f*x+e)+93*B*sin(f*x+e)*cos(f*x+e)^4+39*A*cos
(f*x+e)^6*sin(f*x+e)-1911*A*cos(f*x+e)^3*sin(f*x+e)-352*B*cos(f*x+e)^2*sin(
f*x+e)-2748*A*cos(f*x+e)+287*B*cos(f*x+e)^3*sin(f*x+e)-21*B*cos(f*x+e)^5*si
n(f*x+e)+3225*A*cos(f*x+e)^3-65*B*cos(f*x+e)^3-4*B*cos(f*x+e)-312*A*cos(f*x
+e)^6+24*B*cos(f*x+e)^6-476*B*sin(f*x+e)*cos(f*x+e)+2268*A*sin(f*x+e)*cos(f
*x+e)-3*B*cos(f*x+e)^6*sin(f*x+e)-936*A*cos(f*x+e)^5+72*B*cos(f*x+e)^5+3120
*A*cos(f*x+e)^4-380*B*cos(f*x+e)^4+828*B*cos(f*x+e)^2+472*B*sin(f*x+e))*sin
(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+cos(f*x+e)^4-4*co
s(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2
+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(15/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.13455, size = 585, normalized size = 2.9

$$-\frac{\left(140\,Ba^3\cos\left(fx+e\right)^4-7\,(27\,A+61\,B)a^3\cos\left(fx+e\right)^2+4\,(57\,A+71\,B)a^3-7\left(5\,(3\,A+5\,B)a^3\cos\left(fx+e\right)^2\right.\right.}{420\left(7\,c^8f\cos\left(fx+e\right)^7-56\,c^8f\cos\left(fx+e\right)^5+112\,c^8f\cos\left(fx+e\right)^3-64\,c^8f\cos\left(fx+e\right)-\left(c^8f\cos\left(fx+e\right)^7-24\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="fricas")
```

```
[Out] -1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*(
57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^3)
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*
cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^
8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8
*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(15/2), x)
```

3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e}{224c^2f(c-c\sin(e+f}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x]
)^(17/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c
- c*Sin[e + f*x])^(15/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(224*c^2*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^(7/2))/(1120*c^3*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*
B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8960*c^4*f*(c - c*Sin[e + f*x]
)^(9/2))
```

Rubi [A] time = 0.590887, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}}+\frac{(A-3B)\cos(e+fx)(a\sin(e}{224c^2f(c-c\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^
(17/2),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x]
)^(17/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c
- c*Sin[e + f*x])^(15/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2))/(224*c^2*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^(7/2))/(1120*c^3*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*
B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8960*c^4*f*(c - c*Sin[e + f*x]
)^(9/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
```

```
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}}{4c}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a}{56cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a}{56cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a}{56cf(c-c\sin(e+}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a}{56cf(c-c\sin(e+}$$

Mathematica [A] time = 7.1124, size = 436, normalized size = 1.77

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}{5f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{(A+3B)(a(\sin(e+fx)+1))^{7/2}\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}{f(c-c\sin(e+fx))^{17/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(17/2),x]
```

```
[Out] ((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)
)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
 (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x
]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x]
)^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[
e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1
+ Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c
*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 +
 Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*
Sin[e + f*x])^(17/2))
```

Maple [B] time = 0.405, size = 560, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] -1/140/f*(3076*A-268*B+96*A*cos(f*x+e)^7-8*B*cos(f*x+e)^7-3076*A*sin(f*x+e)
-5348*A*cos(f*x+e)^2-1332*A*cos(f*x+e)^4*sin(f*x+e)+3880*A*cos(f*x+e)^2*sin
(f*x+e)+372*A*cos(f*x+e)^5*sin(f*x+e)+111*B*sin(f*x+e)*cos(f*x+e)^4+108*A*c
os(f*x+e)^6*sin(f*x+e)-1548*A*cos(f*x+e)^3*sin(f*x+e)-300*B*cos(f*x+e)^2*si
n(f*x+e)-1608*A*cos(f*x+e)+164*B*cos(f*x+e)^3*sin(f*x+e)-B*cos(f*x+e)^8-31*
B*cos(f*x+e)^5*sin(f*x+e)+2332*A*cos(f*x+e)^3-136*B*cos(f*x+e)^3+64*B*cos(f
*x+e)-480*A*cos(f*x+e)^6+40*B*cos(f*x+e)^6-204*B*sin(f*x+e)*cos(f*x+e)+1468
*A*sin(f*x+e)*cos(f*x+e)-9*B*cos(f*x+e)^6*sin(f*x+e)-960*A*cos(f*x+e)^5+80*
B*cos(f*x+e)^5+B*cos(f*x+e)^7*sin(f*x+e)-12*A*cos(f*x+e)^7*sin(f*x+e)+2880*
A*cos(f*x+e)^4-275*B*cos(f*x+e)^4+504*B*cos(f*x+e)^2+268*B*sin(f*x+e)+12*A*
```

```
cos(f*x+e)^8)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3+
cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+
e)-8*cos(f*x+e)^2+8*sin(f*x+e)-4*cos(f*x+e)+8)/(-c*(-1+sin(f*x+e)))^(17/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.19861, size = 613, normalized size = 2.49

$$\frac{\left(35\,Ba^{3}\cos\left(fx+e\right)^{4}-56\,(A+2\,B)a^{3}\cos\left(fx+e\right)^{2}+4\,(17\,A+19\,B)a^{3}-4\left(7\,(A+2\,B)a^{3}\cos\left(fx\right.\right.}{140\left(c^{9}f\cos\left(fx+e\right)^{9}-32\,c^{9}f\cos\left(fx+e\right)^{7}+160\,c^{9}f\cos\left(fx+e\right)^{5}-256\,c^{9}f\cos\left(fx+e\right)^{3}+128\,c^{9}f\cos\left(fx+e\right)+8\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="fricas")
```

```
[Out] 1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A
+ 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*x +
 e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*cos(
f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f*co
s(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{7}{2}}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(17/2), x)
```

3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=197

$$\frac{2c^2(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{4c^3(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)(c-c\sin}{2f\sqrt{a\sin(e+fx)+}}$$

```
[Out] (4*(A - B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*(A - B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[
e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*(c - c*Si
n[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.462934, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}+\frac{4c^3(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)(c-c\sin}{2f\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (4*(A - B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*(A - B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[
e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*(c - c*Si
n[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*
Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

<u>Rule 2973</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+}{3f\sqrt{a+a\sin(e+fx}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c}{2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c}{2f\sqrt{a+a\sin(e+}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)(c-c}{2f\sqrt{a+a\sin(e+}}$$

$$= \frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2(A-B)c^2\cos(e+fx)\surd}{f\sqrt{a+a\sin(e}}$$

Mathematica [A] time = 1.31412, size = 185, normalized size = 0.94

$$-\frac{c^2(\sin(e+fx)-1)^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((36A-51B)\sin(e+fx)+3(A-3B)\cos(2\right.}{12f\sqrt{a(\sin(e+fx)+1)}\left(\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c -
c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (36*A
- 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 0.378, size = 595, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/6/f*(15*A-17*B+15*A*sin(f*x+e)-48*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))+48*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-15*A*cos(f*x+e)^2+3*A*cos
(f*x+e)^2*sin(f*x+e)-24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-7*B*cos(f*x+e)^2*
sin(f*x+e)+3*A*cos(f*x+e)-2*B*cos(f*x+e)^3*sin(f*x+e)-3*A*cos(f*x+e)^3+9*B*
cos(f*x+e)^3-9*B*cos(f*x+e)+24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+26*B*sin(f
*x+e)*cos(f*x+e)+24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-24*B*sin(f*x+e)*ln(2/
(cos(f*x+e)+1))-18*A*sin(f*x+e)*cos(f*x+e)+48*A*cos(f*x+e)*ln((1-cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))-48*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))-48*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*B*sin(f*
x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^4+19*B*cos(f*x
+e)^2+24*A*ln(2/(cos(f*x+e)+1))-24*B*ln(2/(cos(f*x+e)+1))-17*B*sin(f*x+e))*
(-c*(-1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+
e)*cos(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e))
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left((A-2\,B)c^2\cos\left(fx+e\right)^2-2\,(A-B)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2\,(A-B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*
x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=146

$$\frac{2c^2(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (2*(A - B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e +
f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.365348, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*(A - B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*
x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e +
f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}dx = -\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+}{2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+}{2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{B\cos(e+fx)(c-c\sin(e+}{2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)c\cos(e+fx)\sqrt{}}{f\sqrt{a+a\sin(}}$$

Mathematica [A] time = 0.673589, size = 146, normalized size = 1.

$$-\frac{c(\sin(e+fx)-1)\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(B\cos(2(e+fx))-4\left((A-2B)\sin(e+fx)+\right.\right.}{4f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] -(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Si
n[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x])))/(4*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 0.345, size = 504, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e)^3+2*A*sin(f*x+e)*cos(f*x+e)-
4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+2*A*cos(f*x+e)^2+4*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*A*c
os(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*sin(f*x+e)*cos(f*x+e
)+4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-8*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^2-4*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+8*B
*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)-4*A*ln(
2/(cos(f*x+e)+1))+8*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*sin(f*x+
e)+B*cos(f*x+e)+4*B*ln(2/(cos(f*x+e)+1))-8*B*ln((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-2*A+3*B)*(-c*(-1+sin(f*x+e)))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f
*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c*
sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x +
 e) + a), x)
```

3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=96

$$\frac{c(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] ((A - B)*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.322328, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2738, 2737, 2667, 31}

$$\frac{c(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] ((A - B)*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
```

```
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{B\int\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A+B)\int\frac{\sqrt{c-c\sin(e}}{\sqrt{a+a\sin(e}}$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{(a(-A+B)c\cos(e+fx))\int\frac{\cos(e+}{a+a\sin(e}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}$$

$$= -\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}} - \frac{((-A+B)c\cos(e+fx))\,\text{Subst}\left(\int\right.}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+}}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 1.13768, size = 119, normalized size = 1.24

$$\frac{\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(B\sin(e+fx)+(A-B)\left(2\log\left(e^{i(e+fx)}+i\right)-ifx\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e
+ f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*((-I)*f*x + 2*Log[I + E^(I*
(e + f*x))]) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 0.34, size = 399, normalized size = 4.2

$$-\frac{1}{f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\left(A\sin\left(fx+e\right)\ln\left(2\left(\cos\left(fx+e\right)+1\right)^{-1}\right)-2\,A\sin\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)}{\sin\left(fx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*ln((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*cos(f*x+e)-B*sin(f*x+e)*ln
(2/(cos(f*x+e)+1))+2*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+
B*cos(f*x+e)^2+B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)*ln((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln(2/(cos(f*x+e)+1))-2*A*ln((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))-B*sin(f*x+e)-B*ln(2/(cos(f*x+e)+1))+2*B*ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-B)*(-c*(-1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)+s
in(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [A] time = 1.56585, size = 238, normalized size = 2.48

$$\frac{B\left(\frac{2\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)}{\sqrt{a}}-\frac{2\sqrt{a}\sqrt{c}\sin\left(fx+e\right)}{\left(a+\frac{a\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\right)}\right)-A\left(\frac{2\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{\sqrt{a}}-\frac{\sqrt{c}\log\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right.}{\sqrt{a}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] (B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*sin(
f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))
- A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e)
 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f
*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e
) + a), x)
```

3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=113

$$\frac{(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(
2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.363592, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2969, 2737, 2667, 31}

$$\frac{(A-B)\cos(e+fx)\log(\sin(e+fx)+1)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] -((A + B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) + ((A - B)*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(
2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2969

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist
[(A*b + a*B)/(2*a*b), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]]
, x], x] + Dist[(B*c + A*d)/(2*c*d), Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a +
b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{(A+B)\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{2a}+\frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{2c}\\
&= \frac{(a(A-B)\cos(e+fx))\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{((A+B)c\cos(e+fx))\int \frac{\cos(e+}{c-c\sin(}}{2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(}}\\
&= \frac{((A-B)\cos(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{((A+B)\cos(e}{2f\sqrt{a}}\\
&= -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)\log(1+s}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c}}
\end{aligned}$$

Mathematica [A] time = 0.328094, size = 97, normalized size = 0.86

$$-\frac{\cos(e+fx)\left((A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+(B-A)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*((A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (-A + B
)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*
```

```
Sqrt[c - c*Sin[e + f*x]]))
```

Maple [A] time = 0.323, size = 165, normalized size = 1.5

$$-\frac{\cos\left(fx+e\right)}{f}\left(A\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+B\ln\left(-\frac{-1+\cos\left(\right.}{\mathrm{s}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos
(f*x+e)+1))+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+si
n(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(a*c*cos(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sin\left(e + fx\right)}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}\sqrt{-c\left(\sin\left(e + fx\right) - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e +
f*x) - 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx + e\right) + A}{\sqrt{a\sin\left(fx + e\right) + a}\sqrt{-c\sin\left(fx + e\right) + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)), x)
```

3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] ((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.251913, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2741, 3770}

$$\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] ((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c}}}{2c}$$
$$= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{((A-B)\cos(e+fx))\int \mathrm{se}}{2c\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$
$$= \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\tanh^{-1}(\sin(e+fx}{2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

Mathematica [A] time = 0.522785, size = 191, normalized size = 1.85

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2 \mathrm{lo}\right.}{2f\sqrt{a(\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)),x]
```

```
[Out] ((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*x])]
*(c - c*Sin[e + f*x])^(3/2))
```

Maple [B] time = 0.325, size = 302, normalized size = 2.9

$$\frac{\cos\left(fx+e\right)}{2f}\left(A\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\sin\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)
*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+A*sin(f*x+e)-A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+sin
(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2)), x)
```

Fricas [A] time = 2.04792, size = 869, normalized size = 8.44

$$\left[-\frac{\left((A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-(A-B)\cos\left(fx+e\right)\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)+2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(\right.}}{\cos\left(fx+e\right)^{3}}\right.}{4\left(ac^{2}f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^{2}f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c)
*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*sin(f*
x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x +
e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sin\left(e + fx\right)}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}\left(-c\left(\sin\left(e + fx\right) - 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
 - 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx + e\right) + A}{\sqrt{a\sin\left(fx + e\right) + a}\left(-c\sin\left(fx + e\right) + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2)), x)
```

3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

```
[Out] ((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)) + ((A - B)*Cos[e + f*x])/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.355057, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```


```
[Out] ((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)) + ((A - B)*Cos[e + f*x])/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-}}{2c}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e}{4cf\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e}{4cf\sqrt{a+a\sin(e+fx)}(c}$$

$$= \frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} + \frac{(A-B)\cos(e}{4cf\sqrt{a+a\sin(e+fx)}(c}$$

Mathematica [A] time = 0.618092, size = 222, normalized size = 1.45

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(5/2)),x]
```

```
[Out] ((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[Co
s[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^4)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/
2))
```

Maple [B] time = 0.344, size = 465, normalized size = 3.

$$\frac{\cos\left(fx+e\right)}{4f}\left(A\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(f\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4/f*(A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*cos(f*x+e)^2*ln(-(-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos
(f*x+e)^2+2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin
(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)^2-2*B*sin(f
*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)*ln((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*sin(f*x+e)-2*A*ln(-(-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)+
2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-2*A)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(-1+sin(f*x+e
)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2)), x)
```

Fricas [A] time = 2.1617, size = 1083, normalized size = 7.08

$$\left[-\frac{\left((A-B)\cos\left(fx+e\right)^3+2\,(A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A-B)\cos\left(fx+e\right)\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(f\cdots\right.}{\cdots}\right.}{8\left(ac^3f\cos\left(fx+e\right)^3+2\,ac^3f\cos\left(fx\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A
 - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
 + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
 e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x
+ e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x + e)^3
+ 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt(-a*c)*
arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*c
os(f*x + e)*sin(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(
f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2)), x)
```

3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=271

$$-\frac{2c^3(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c^2(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{4c^4(3A-5B)\cos(e+f}{af\sqrt{a\sin(e+fx)+}}$$

```
[Out] (-4*(3*A - 5*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(3*A - 5*B)*c^3*Cos[e + f*x]*Sqrt
[c - c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((3*A - 5*B)*c^2*Cos
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((
3*A - 5*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[a + a*Sin
[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(2*f*(a + a
*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.577265, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$-\frac{2c^3(3A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{c^2(3A-5B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{4c^4(3A-5B)\cos(e+f}{af\sqrt{a\sin(e+fx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```


```
[Out] (-4*(3*A - 5*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(3*A - 5*B)*c^3*Cos[e + f*x]*Sqrt
[c - c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((3*A - 5*B)*c^2*Cos
[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((
3*A - 5*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[a + a*Sin
[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(2*f*(a + a
*Sin[e + f*x])^(3/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(3A-5B)\int \frac{(c-c\sin(e+fx))}{\sqrt{a+a\sin(e+f}}}{2a}$$

$$= -\frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c}{2f(a+a\sin(}$$

$$= -\frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+}{6af\sqrt{a+}}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+}{2af\sqrt{a+}}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+}{2af\sqrt{a+}}$$

$$= -\frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+}{2af\sqrt{a+}}$$

$$= -\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(3A-5B)c^3\cos(e}{af\sqrt{a+}}$$

Mathematica [A] time = 3.51203, size = 271, normalized size = 1.

$$-\frac{c^3\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(27A-59B)\cos(2(e+fx))-117A\sin(e+fx)-3A\sin(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(132*A
 - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 576*A*Log
[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*(e + f*x)])
)/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.288, size = 938, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(-102*A+166*B-102*A*sin(f*x+e)+288*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))-480*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+99*A*cos(f*x+e)^2+2
4*A*cos(f*x+e)^2*sin(f*x+e)+72*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-120*B*cos(
f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*sin(f*x+e)*cos(f*x+e)^4+3*A*cos(f*x+e)^3*
sin(f*x+e)-48*B*cos(f*x+e)^2*sin(f*x+e)+27*A*cos(f*x+e)+72*A*cos(f*x+e)*sin
(f*x+e)*ln(2/(cos(f*x+e)+1))-13*B*cos(f*x+e)^3*sin(f*x+e)-27*A*cos(f*x+e)^3
+61*B*cos(f*x+e)^3-59*B*cos(f*x+e)-120*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*co
s(f*x+e)-144*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-107*B*sin(f*x+e)*cos(f*x+e)-
120*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+240*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))
+75*A*sin(f*x+e)*cos(f*x+e)+72*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*B*cos(
f*x+e)^5+240*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-144*A*
ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-144*A*cos(f*x+e)*ln((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+240*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))+288*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))-480*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)
^4+240*B*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144
*A*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-11*B*cos(
f*x+e)^4-155*B*cos(f*x+e)^2-144*A*ln(2/(cos(f*x+e)+1))+240*B*ln(2/(cos(f*x+
e)+1))+166*B*sin(f*x+e))*(-c*(-1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4-sin(f*x+e
)*cos(f*x+e)^3+3*cos(f*x+e)^3+4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+4*si
n(f*x+e)*cos(f*x+e)-4*cos(f*x+e)-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bc^3\cos\left(fx+e\right)^4+(3\,A-5\,B)c^3\cos\left(fx+e\right)^2-4\,(A-B)c^3-\left((A-3\,B)c^3\cos\left(fx+e\right)^2-4\,(A-B)c^3\right)\sin}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=210

$$-\frac{2c^2(A-2B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{4c^3(A-2B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-2B)\cos(e+fx)(c}{2af\sqrt{a\sin(e+}}$$

```
[Out] (-4*(A - 2*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(A - 2*B)*c^2*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - 2*B)*c*Cos[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*C
os[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.478091, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$-\frac{2c^2(A-2B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}-\frac{4c^3(A-2B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-2B)\cos(e+fx)(c}{2af\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*(A - 2*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (2*(A - 2*B)*c^2*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - 2*B)*c*Cos[e + f*x
]*(c - c*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*C
os[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-2B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}}{a}$$

$$= -\frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-}{2f(a+a\sin(e}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e+fx)(}{2af\sqrt{a+a\sin}}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e+fx)(}{2af\sqrt{a+a\sin}}$$

$$= -\frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e+fx)(}{2af\sqrt{a+a\sin}}$$

$$= -\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(A-2B)c^2\cos(e+}{af\sqrt{a+a}}$$

Mathematica [A] time = 1.58575, size = 212, normalized size = 1.01

$$-\frac{c^2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(2A-7B)\cos(2(e+fx))+\sin(e+fx)\left(64(A-2B)\log\left(\mathrm{s}\right.\right.\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(28*A
- 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*A + 31*
B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + B
*Sin[3*(e + f*x)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(3/2))
```

Maple [B] time = 0.273, size = 853, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(12*A-22*B+12*A*sin(f*x+e)-32*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+64*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*A*cos(f*x+e)^2-2*A*co
s(f*x+e)^2*sin(f*x+e)-8*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+16*B*cos(f*x+e)^2
*ln(2/(cos(f*x+e)+1))+6*B*cos(f*x+e)^2*sin(f*x+e)-2*A*cos(f*x+e)-8*A*cos(f*
x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+B*cos(f*x+e)^3*sin(f*x+e)+2*A*cos(f*x+
e)^3-7*B*cos(f*x+e)^3+7*B*cos(f*x+e)+16*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*c
os(f*x+e)+16*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+15*B*sin(f*x+e)*cos(f*x+e)+1
6*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-32*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-10
*A*sin(f*x+e)*cos(f*x+e)-8*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-32*B*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+16*A*ln((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))*cos(f*x+e)^2+16*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))-32*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*A*
sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+64*B*sin(f*x+e)*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+16*A*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))+B*cos(f*x+e)^4+21*B*cos(f*x+e)^2+16*A*ln(2/(cos(f*x+e)+
1))-32*B*ln(2/(cos(f*x+e)+1))-22*B*sin(f*x+e))*(-c*(-1+sin(f*x+e)))^(5/2)/(
cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(f*x+e)^2
-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left((A-2B)c^2\cos\left(fx+e\right)^2-2(A-B)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2(A-B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+}}{a^2\cos\left(fx+e\right)^2-2a^2\sin\left(fx+e\right)-2a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{c^2(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c-c\sin}{2f(a\sin(e+fx)+a)}$$

```
[Out] -(((A - 3*B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])) - ((A - 3*B)*c*Cos[e + f*x]*Sqrt[c - c*S
in[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c -
 c*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.385612, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2740, 2737, 2667, 31}

$$-\frac{c^2(A-3B)\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e+fx)(c-c\sin}{2f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -(((A - 3*B)*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])) - ((A - 3*B)*c*Cos[e + f*x]*Sqrt[c - c*S
in[e + f*x]])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c -
 c*Sin[e + f*x])^(3/2))/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}} - \frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}}{2a}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\,\mathrm{s}}{2f(a+a\sin(e+f}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\,\mathrm{s}}{2f(a+a\sin(e+f}$$

$$= -\frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\,\mathrm{s}}{2f(a+a\sin(e+f}$$

$$= -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{(A-3B)c\cos(e+fx)\sqrt{}}{2af\sqrt{a+a\,\mathrm{sin}}}$$

Mathematica [A] time = 0.865434, size = 190, normalized size = 1.19

$$-\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\sin(e+fx)\left(2(A-3B)\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right)\right.\right.\right.}{2f(a(\sin(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*A - 3
*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 12
*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.29, size = 760, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(2*A-4*B+2*A*sin(f*x+e)-4*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12
*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2-A*cos(f*x+e)*l
n(2/(cos(f*x+e)+1))+3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+B*cos(f*x+e)^2*si
n(f*x+e)-A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^3+B*cos(
f*x+e)+3*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*A*sin(f*x+e)*ln(2/(
cos(f*x+e)+1))+3*B*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1)
)-6*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*cos(f*x+e)-A*cos(f*x+e
)^2*ln(2/(cos(f*x+e)+1))-6*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f
*x+e)^2+2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*A*cos(f
*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*B*cos(f*x+e)*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-4*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))+12*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*B*cos(f
*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)*si
n(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*cos(f*x+e)^2+2*A*ln(2
/(cos(f*x+e)+1))-6*B*ln(2/(cos(f*x+e)+1))-4*B*sin(f*x+e))*(-c*(-1+sin(f*x+e
)))^(3/2)/(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a
*(1+sin(f*x+e)))^(3/2)
```

Maxima [B] time = 1.59481, size = 495, normalized size = 3.11

$$-\frac{B\left(\frac{6c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{3c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{2\left(\frac{3c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)-A\left(\frac{2c^{\frac{3}{2}}\log\left(\frac{\sin}{\cos}\right.}{a^{\frac{3}{2}}}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] -(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^(3/2)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4)) - A*(2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^(3/2) - c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 4
```

```
*sqrt(a)*c^(3/2)*sin(f*x + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
 + a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{Bc\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])) + (B*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.346303, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2971, 2737, 2667, 31, 2738}

$$\frac{Bc\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])) + (B*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(e+fx))\int \frac{\cos(e}{a+a\sin}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\,s}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(Bc\cos(e+fx))\,\mathrm{Subst}\left(\int\right.}{af\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)\log(1+s}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

Mathematica [C] time = 1.15604, size = 143, normalized size = 1.43

$$\frac{\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+2B\log\left(e^{i(e+fx)}+i\right)+B\left(2\log\left(e^{i(e+fx)}+i\right)-ifx\right)\sin(e+\right.}{f(a(\sin(e+fx)+1))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-A + B - I
*B*f*x + 2*B*Log[I + E^(I*(e + f*x))] + B*((-I)*f*x + 2*Log[I + E^(I*(e + f
*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(3/2))
```

Maple [B] time = 0.328, size = 408, normalized size = 4.1

$$-\frac{1}{f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}\left(2\,B\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^{2}-B\left(\cos\left(fx+e\right)\right)^{2}\ln\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*cos(f*x+e
)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)^
2+A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*
x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)*ln(2/(cos(f*x+e)
+1))-4*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)
*ln(2/(cos(f*x+e)+1))-A*sin(f*x+e)+B*sin(f*x+e)-4*B*ln((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+2*B*ln(2/(cos(f*x+e)+1))-A+B)*(-c*(-1+sin(f*x+e)))^(1/2)/
(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{a^2\cos\left(fx+e\right)^2-2\,a^2\sin\left(fx+e\right)-2\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
+ 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(3/2), x)
```

3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=103

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.255653, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2741, 3770}

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```


```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c\cdots}}}{2a}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int \mathrm{s}\cdots}{2a\sqrt{a+a\sin(e+fx)}\sqrt{c-\cdots}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\tanh^{-1}(\sin(e+f\cdots}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c\cdots}}$$

Mathematica [A] time = 0.552112, size = 186, normalized size = 1.81

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\cdots\right.}{2f(a(\sin(e+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/
2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] time = 0.331, size = 303, normalized size = 2.9

$$\frac{\cos\left(fx+e\right)}{2f}\left(A\sin\left(fx+e\right)\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\sin\left(fx+e\right)\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln(-(-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+A*sin(f*x+e)+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e))*cos(f*x+e)/(a*(1+sin
(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Fricas [A] time = 2.33261, size = 868, normalized size = 8.43

$$\left[\frac{\left(\left(A+B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(A+B\right)\cos\left(fx+e\right)\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(f\right.}}{\cos\left(fx+e\right)^{3}}\right.}{4\left(a^{2}cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^{2}cf\cos\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)*
log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*sin(f*x
 + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e
)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sin\left(e + fx\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e + fx\right) - 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx + e\right) + A}{\left(a\sin\left(fx + e\right) + a\right)^{\frac{3}{2}}\sqrt{-c\sin\left(fx + e\right) + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)\tanh^{-1}}{2acf\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(3/2)) + (A*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) + (A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.373341, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2972, 2743, 2741, 3770}

$$-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)}{2af\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{A\cos(e+fx)\tanh^{-1}}{2acf\sqrt{a\sin(e+fx)+a}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(3/2)) + (A*Cos[e + f*x])/(2*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) + (A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\int \frac{1}{\sqrt{a+a\sin(e+fx)}(c-}}{a}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\,\mathrm{co}}{2af\sqrt{a+a\sin(e+}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\,\mathrm{co}}{2af\sqrt{a+a\sin(e+}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\,\mathrm{co}}{2af\sqrt{a+a\sin(e+}}$$

Mathematica [A] time = 0.672257, size = 178, normalized size = 1.19

$$-\frac{\cos(e+fx)\left(2A\sin(e+fx)-A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+A\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{4cf(\sin(e+fx)-1)(a(\sin(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] -(Cos[e + f*x]*(2*B - A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + A*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*A*S
in[e + f*x]))/(4*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[
c - c*Sin[e + f*x]])
```

Maple [A] time = 0.269, size = 132, normalized size = 0.9

$$-\frac{\cos\left(fx+e\right)}{2f}\left(A\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+B*cos(f*x+e)^2-A*sin(f*x+e)-
B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Fricas [A] time = 2.43705, size = 703, normalized size = 4.69

$$\left[\frac{\sqrt{ac}A\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\left(A\sin\left(fx+e\right)+B\right)\sqrt{\cdots}}{4\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x +
 e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*
x + e))/cos(f*x + e)^3) + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a)*s
qrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*A*ar
ctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos
(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(3A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(3A-B)\cos(e}{8af\sqrt{a\sin(e+fx)+a}(c-}$$

```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*
Sin[e + f*x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A - B)*ArcTanh[Sin[e + f*x]]*Cos[e
 + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.479328, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(3A-B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8ac^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(3A-B)\cos(e}{8af\sqrt{a\sin(e+fx)+a}(c-}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```


```
[Out] -((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x]
)^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*
Sin[e + f*x])^(5/2)) + ((3*A - B)*Cos[e + f*x])/(8*a*c*f*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A - B)*ArcTanh[Sin[e + f*x]]*Cos[e
 + f*x])/(8*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A-B)\int \frac{}{\sqrt{a+a\sin(e+}}}{2a}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A-B)\,c}{8af\sqrt{a+a\sin(e+f}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A-B)\,c}{8af\sqrt{a+a\sin(e+f}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A-B)\,c}{8af\sqrt{a+a\sin(e+f}}$$

$$= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{(3A-B)\,c}{8af\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 0.948476, size = 306, normalized size = 1.41

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((B-A)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 +
(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/
2))
```

Maple [B] time = 0.272, size = 431, normalized size = 2.

$$\frac{\cos\left(fx+e\right)}{8f}\left(3A\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-3A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/8/f*(3*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+
e)-3*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-B*c
os(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2*
sin(f*x+e)-3*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*
ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*B*cos(f*x+e)^2*sin(
f*x+e)+B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+A*cos(f*x+e)^2-3*B*cos(f*x+e)
^2+3*A*sin(f*x+e)-B*sin(f*x+e)-A+3*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(
-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Fricas [A] time = 2.62385, size = 1060, normalized size = 4.88

$$\Bigg[-\frac{\left((3A-B)\cos(fx+e)^3\sin(fx+e)-(3A-B)\cos(fx+e)^3\right)\sqrt{ac}\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)+2\sqrt{ac}\sqrt{a\sin(fx+e)+a}\sqrt{-}}{\cos(fx+e)^3}\right.}{16\left(a^2c^3f\cos(fx+e)^3\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*
sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
 + 2*((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f
*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8*(((3*A - B)*cos(f*x + e)^3*sin(f*
x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) +
((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(a*sin(f*
x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x +
e) - a^2*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=323

$$\frac{4c^4(3A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^2(3A-7B)\cos(e+fx)(c}{3a^2f\sqrt{a\sin(e+}}$$

```
[Out] (8*(3*A - 7*B)*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*(3*A - 7*B)*c^4*Cos[e + f*x]*Sqr
t[c - c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^3*
Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 ((3*A - 7*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a
+ a*Sin[e + f*x]]) + ((3*A - 7*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2)
)/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e +
 f*x])^(9/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.712862, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{4c^4(3A-7B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(3A-7B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^2(3A-7B)\cos(e+fx)(c}{3a^2f\sqrt{a\sin(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```


```
[Out] (8*(3*A - 7*B)*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*(3*A - 7*B)*c^4*Cos[e + f*x]*Sqr
t[c - c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^3*
Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 ((3*A - 7*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a
+ a*Sin[e + f*x]]) + ((3*A - 7*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2)
)/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e +
 f*x])^(9/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
```

```
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A-7B)\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{3/2}}}{4a}$$

$$= \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-}{4f(a+a\sin(e+}$$

$$= \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c\cos(e+fx)}{4af(a+a\sin}$$

$$= \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^2\cos(e+fx}{3a^2f\sqrt{a+a\sin}}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)}{a^2f\sqrt{a+a\sin}}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)}{a^2f\sqrt{a+a\sin}}$$

$$= \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)c^3\cos(e+fx)}{a^2f\sqrt{a+a\sin}}$$

$$= \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4(3A-7B)c^4\cos(e+}{a^2f\sqrt{a+a}}$$

Mathematica [A] time = 7.04568, size = 573, normalized size = 1.77

$$-\frac{(28A-97B)\sin(e+fx)(c-c\sin(e+fx))^{9/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{4f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9} - \frac{(A-7B)\cos(2(e+fx))(c-c\sin(}{4f(a(\sin(e+fx)+1))^{5/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (-8*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2
))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
 + (16*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f
*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]
))^(5/2)) - ((A - 7*B)*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^5*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (16*(3*A - 7*B)*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x
])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))
^(5/2)) - ((28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*
x]*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
*(a*(1 + Sin[e + f*x]))^(5/2)) - (B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.3, size = 1287, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(396*A-932*B+396*A*sin(f*x+e)-1152*A*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+2688*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-429*A*cos(f*x+e)^
2-3*A*cos(f*x+e)^4*sin(f*x+e)-255*A*cos(f*x+e)^2*sin(f*x+e)-288*A*cos(f*x+e
)*ln(2/(cos(f*x+e)+1))+1008*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+15*B*sin(f*
x+e)*cos(f*x+e)^4+36*A*cos(f*x+e)^3*sin(f*x+e)+581*B*cos(f*x+e)^2*sin(f*x+e
)-222*A*cos(f*x+e)-288*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-108*B*c
os(f*x+e)^3*sin(f*x+e)+2*B*cos(f*x+e)^5*sin(f*x+e)+219*A*cos(f*x+e)^3-473*B
*cos(f*x+e)^3+490*B*cos(f*x+e)+672*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*
x+e)+2*B*cos(f*x+e)^6+576*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+442*B*sin(f*x+e
)*cos(f*x+e)+672*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-1344*B*sin(f*x+e)*ln(2/(
cos(f*x+e)+1))-174*A*sin(f*x+e)*cos(f*x+e)-432*A*cos(f*x+e)^2*ln(2/(cos(f*x
+e)+1))+3*A*cos(f*x+e)^5-17*B*cos(f*x+e)^5+144*A*ln(2/(cos(f*x+e)+1))*cos(f
*x+e)^3+672*B*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-288*A*c
os(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2016*B*ln((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+864*A*ln((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))*cos(f*x+e)^2-144*A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+
288*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-672*
B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+576*A*co
s(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1344*B*cos(f*x+e)*ln((1-c
```

```
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-1152*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+2688*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+33*A*cos(f*x+e)^4-1344*B*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+576*A*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))-93*B*cos(f*x+e)^4+1023*B*cos(f*x+e)^2+576*A*ln(2/(cos(f*x+e)+1))-1
344*B*ln(2/(cos(f*x+e)+1))+336*B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1
))-336*B*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-932*B*sin(f*x+e))*(-c*(-1+sin(f*
x+e)))^(9/2)/(cos(f*x+e)^5+sin(f*x+e)*cos(f*x+e)^4-5*cos(f*x+e)^4+4*sin(f*x
+e)*cos(f*x+e)^3-8*cos(f*x+e)^3-12*cos(f*x+e)^2*sin(f*x+e)+20*cos(f*x+e)^2-
8*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)+16*sin(f*x+e)-16)/(a*(1+sin(f*x+e)))^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{9}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$\frac{3c^3(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a\sin(e+fx)+a}}+\frac{6c^4(A-3B)\cos(e+fx)\,\mathrm{lo}}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c}}$$

```
[Out] (6*(A - 3*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (3*(A - 3*B)*c^3*Cos[e + f*x]*Sqrt[c
- c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*(A - 3*B)*c^2*Cos[
e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (
(A - 3*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*a*f*(a + a*Sin[e +
f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*(a +
a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.607353, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{3c^3(A-3B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}+\frac{3c^2(A-3B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a\sin(e+fx)+a}}+\frac{6c^4(A-3B)\cos(e+fx)\,\mathrm{lo}}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (6*(A - 3*B)*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[
e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (3*(A - 3*B)*c^3*Cos[e + f*x]*Sqrt[c
- c*Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*(A - 3*B)*c^2*Cos[
e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (
(A - 3*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(2*a*f*(a + a*Sin[e +
f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*f*(a +
a*Sin[e + f*x])^(5/2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}}{2a}$$

$$= \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c}{4f(a+a\sin(e+}$$

$$= \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(}{2af(a+a\sin(}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(}{4a^2f\sqrt{a+a\sin}}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(}{4a^2f\sqrt{a+a\sin}}$$

$$= \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(}{4a^2f\sqrt{a+a\sin}}$$

$$= \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3(A-3B)c^3\cos(e+f}{a^2f\sqrt{a+a\sin}}$$

Mathematica [A] time = 2.55644, size = 243, normalized size = 0.92

$$\frac{(c-c\sin(e+fx))^{7/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-4(A-6B)\sin(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A -
 B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e +
 f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4*(A
```

```
- 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.285, size = 1205, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/2/f*(32*A-100*B+32*A*sin(f*x+e)-96*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+288*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-34*A*cos(f*x+e)^2-22*A*
cos(f*x+e)^2*sin(f*x+e)-24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+108*B*cos(f*x+
e)^2*ln(2/(cos(f*x+e)+1))+B*sin(f*x+e)*cos(f*x+e)^4+2*A*cos(f*x+e)^3*sin(f*
x+e)+63*B*cos(f*x+e)^2*sin(f*x+e)-20*A*cos(f*x+e)-24*A*cos(f*x+e)*sin(f*x+e
)*ln(2/(cos(f*x+e)+1))-10*B*cos(f*x+e)^3*sin(f*x+e)+20*A*cos(f*x+e)^3-53*B*
cos(f*x+e)^3+54*B*cos(f*x+e)+72*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e
)+48*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+46*B*sin(f*x+e)*cos(f*x+e)+72*B*cos(
f*x+e)*ln(2/(cos(f*x+e)+1))-144*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-12*A*sin(
f*x+e)*cos(f*x+e)-36*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^5+12*
A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+72*B*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-24*A*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-216*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+72*A*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-12*A*ln(2/(cos(f*x+e)+1))*c
os(f*x+e)^2*sin(f*x+e)+24*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*
x+e)^2*sin(f*x+e)-72*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^
2*sin(f*x+e)+48*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B
*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*A*sin(f*x+e)*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+288*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+2*A*cos(f*x+e)^4-144*B*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))+48*A*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-9*B*cos(f*x+e)^4+109*B*cos(f*x+e)^2+48*A*ln(2/(cos(f*
x+e)+1))-144*B*ln(2/(cos(f*x+e)+1))+36*B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(
f*x+e)+1))-36*B*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-100*B*sin(f*x+e))*(-c*(-1
+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4-sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3+4*
cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)
-8*sin(f*x+e)+8)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(Bc^3\cos\left(fx+e\right)^4+(3A-5B)c^3\cos\left(fx+e\right)^2-4(A-B)c^3-\left((A-3B)c^3\cos\left(fx+e\right)^2-4(A-B)c^3\right)\sin\left(\right.}{3a^3\cos\left(fx+e\right)^2-4a^3+\left(a^3\cos\left(fx+e\right)^2-4a^3\right)\sin\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{7}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=211

$$\frac{c^2(A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-5B)\cos(e+fx)(c-c\sin}{4af(a\sin(e+fx)+}$$

```
[Out] ((A - 5*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - 5*B)*c^2*Cos[e + f*x]*Sqrt[c - c*
Sin[e + f*x]])/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((A - 5*B)*c*Cos[e + f*
x]*(c - c*Sin[e + f*x])^(3/2))/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B
)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.49444, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{c^2(A-5B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a\sin(e+fx)+a}}+\frac{c^3(A-5B)\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A-5B)\cos(e+fx)(c-c\sin}{4af(a\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```



```
[Out] ((A - 5*B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]]) + ((A - 5*B)*c^2*Cos[e + f*x]*Sqrt[c - c*
Sin[e + f*x]])/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((A - 5*B)*c*Cos[e + f*
x]*(c - c*Sin[e + f*x])^(3/2))/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B
)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(A-5B)\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}}{4a}$$

$$= \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c}{4f(a+a\sin(e+}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+fx)(c-}{4af(a+a\sin(e+}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+fx)(c-}{4af(a+a\sin(e+}$$

$$= \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+fx)(c-}{4af(a+a\sin(e+}$$

$$= \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-5B)c^2\cos(e+fx)\sqrt{}}{2a^2f\sqrt{a+a\sin}}$$

Mathematica [A] time = 1.1438, size = 199, normalized size = 0.94

$$\frac{(c-c\sin(e+fx))^{5/2}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-2B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+2(A-5B)\left(\right.\right.}{f(a(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A + 2
*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.269, size = 1106, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(2*A-14*B+2*A*sin(f*x+e)-8*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4
0*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2-2*A*cos(f*x+e
)^2*sin(f*x+e)-2*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+15*B*cos(f*x+e)^2*ln(2/(
cos(f*x+e)+1))+9*B*cos(f*x+e)^2*sin(f*x+e)-2*A*cos(f*x+e)-2*A*cos(f*x+e)*si
n(f*x+e)*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^3*sin(f*x+e)+2*A*cos(f*x+e)^3-8*
B*cos(f*x+e)^3+8*B*cos(f*x+e)+10*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+
e)+4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+6*B*sin(f*x+e)*cos(f*x+e)+10*B*cos(f
*x+e)*ln(2/(cos(f*x+e)+1))-20*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-3*A*cos(f*x
+e)^2*ln(2/(cos(f*x+e)+1))+A*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+10*B*cos(f*x
+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^3*ln((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))-30*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
*cos(f*x+e)^2+6*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-A*l
n(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+2*A*ln((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-10*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))-20*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*A*
sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+40*B*sin(f*x+e)*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*B*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+4*A*cos(f*x+e)*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-B*cos(f*x+e)^4+15*B*cos(f*x+e)^2+4*A*ln(2/(cos(f*x+e)+1)
)-20*B*ln(2/(cos(f*x+e)+1))+5*B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1)
)-5*B*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))-14*B*sin(f*x+e))*(-c*(-1+sin(f*x+e)
))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*co
s(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [B] time = 1.59875, size = 680, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] ((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e)
 + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(
```

```
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c^(5/2
)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)/(cos(f
*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*c^(5/2)*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/2) + 4*a^(
5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^(5/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left((A-2B)c^2\cos\left(fx+e\right)^2-2(A-B)c^2+\left(Bc^2\cos\left(fx+e\right)^2+2(A-B)c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{3a^3\cos\left(fx+e\right)^2-4a^3+\left(a^3\cos\left(fx+e\right)^2-4a^3\right)\sin\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin
(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{Bc^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((B*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) - (B*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 0.39236, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{Bc^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}-\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```



```
[Out] -((B*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]])) - (B*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
(a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2737

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}} + \frac{B\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}dx}{a}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+}{4f(a+a\sin(e+fx))^{5/2}}$$

$$= -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)\sqrt{c-c\sin(}{af(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.975258, size = 179, normalized size = 1.2

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin(e+fx)\left(A-4B\log\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)-3\right.\right.}{f(a(\sin(e+fx)+1))^{5/2}\left(\cos\left(\frac{1}{2}(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2*
(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 0.28, size = 604, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(-A-3*B-A*sin(f*x+e)+8*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*co
s(f*x+e)^2+3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)^2*sin(f*x+e
)-2*B*cos(f*x+e)^3+2*B*cos(f*x+e)+2*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f
*x+e)+B*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-4*B*sin(f
*x+e)*ln(2/(cos(f*x+e)+1))+A*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^3*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))*cos(f*x+e)^2-2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*
sin(f*x+e)-4*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*B*sin(
f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*cos(f*x+e)*sin(f*x+e)*l
n((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)^2-4*B*ln(2/(cos(f*x+
e)+1))+B*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^3*ln(2/(
cos(f*x+e)+1))-3*B*sin(f*x+e))*(-c*(-1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2-sin
(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 +
 (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((A - B)*c*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
+ f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]])
```

Rubi [A] time = 0.333082, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2971, 2738}

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

```
[Out] -((A - B)*c*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
+ f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
```

```
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{a} - (-A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$$
$$= -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx}{af(a+a\sin(e+fx))^{3/2}\sqrt{c}}$$

Mathematica [A] time = 0.505045, size = 99, normalized size = 1.05

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{c-c\sin(e+fx)}(A+2B\sin(e+fx)+B)}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]])/(2*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5)
```

Maple [A] time = 0.32, size = 135, normalized size = 1.4

$$-\frac{\left(A\left(\cos\left(fx+e\right)\right)^2+A\sin\left(fx+e\right)\cos\left(fx+e\right)+B\left(\cos\left(fx+e\right)\right)^2+B\sin\left(fx+e\right)\cos\left(fx+e\right)+2A\cos\left(fx+e\right)-}{2f\left(-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/f*(A*cos(f*x+e)^2+A*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2+B*sin(f*x+e)*
cos(f*x+e)+2*A*cos(f*x+e)-3*A*sin(f*x+e)-B*sin(f*x+e)-3*A-B)*sin(f*x+e)*(-c
*(-1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(5/2), x)
```

Fricas [A] time = 1.9835, size = 223, normalized size = 2.37

$$\frac{\left(2\,B\sin\left(fx+e\right)+A+B\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] 1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*
cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
a)^(5/2), x)
```

3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=151

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e +
f*x]]) - ((A + B)*Cos[e + f*x])/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c -
c*Sin[e + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.353968, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{4a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)}{4af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```


```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e +
f*x]]) - ((A + B)*Cos[e + f*x])/(4*a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c -
c*Sin[e + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*a^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\int \frac{1}{(a+a\sin(e+fx))^{3/2}\ldots}}{2a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e\ldots}{4af(a+a\sin(e+fx))^{3/2}\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e\ldots}{4af(a+a\sin(e+fx))^{3/2}\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e\ldots}{4af(a+a\sin(e+fx))^{3/2}\ldots}$$

Mathematica [A] time = 0.660088, size = 214, normalized size = 1.42

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-(A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^4))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [B] time = 0.336, size = 465, normalized size = 3.1

$$-\frac{\cos\left(fx+e\right)}{4f}\left(-A\left(\cos\left(fx+e\right)\right)^{2}\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)+A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)(\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/4/f*(-A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-B*cos(f*x+e)^2*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*c
os(f*x+e)^2+2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*s
in(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)^2+2*B*sin
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f*x+e)*ln((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A*sin(f*x+e)+2*A*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e
)+2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))-2*A)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(-1+sin(f*x
+e)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Fricas [A] time = 2.55936, size = 1081, normalized size = 7.16

$$\left[\frac{\left((A+B)\cos\left(fx+e\right)^3-2\,(A+B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A+B)\cos\left(fx+e\right)\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^3-2\,ac\cos\left(fx+e\right)}{}\right.}{8\left(a^3cf\cos\left(fx+e\right)^3-2\,a^3cf\cos\left(fx+e\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] [1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A
+ B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e)
- 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x +
e))/cos(f*x + e)^3) + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x +
 e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x + e)^3 -
 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(-a*c)*a
rctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*co
s(f*x + e)*sin(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f
*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(3A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(3A+B)\cos(e}{8af(a\sin(e+fx)+a)^{3/2}(c}$$

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x]
)^(3/2)) - ((3*A + B)*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c -
c*Sin[e + f*x])^(3/2)) + ((3*A + B)*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A + B)*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.480443, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{(3A+B)\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(3A+B)\cos(e}{8af(a\sin(e+fx)+a)^{3/2}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x]
)^(3/2)) - ((3*A + B)*Cos[e + f*x])/(8*a*f*(a + a*Sin[e + f*x])^(3/2)*(c -
c*Sin[e + f*x])^(3/2)) + ((3*A + B)*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((3*A + B)*ArcTanh[Sin[e + f*x]]*Cos
[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{(3A+B)\int \frac{}{(a+a\sin(e}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{(3A+}{8af(a+a\sin(e+f}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{(3A+}{8af(a+a\sin(e+f}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{(3A+}{8af(a+a\sin(e+f}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{(3A+}{8af(a+a\sin(e+f}$$

Mathematica [A] time = 0.947007, size = 305, normalized size = 1.47

$$\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((A+B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 +
 (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/
2))
```

Maple [B] time = 0.276, size = 431, normalized size = 2.1

$$-\frac{\cos\left(fx+e\right)}{8f}\left(3A\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)-3A\ln\left(\frac{1-\cos\left(fx+e\right)+\sin\left(\right.}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/f*(3*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x
+e)-3*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+B*
cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-2*A*cos(f*x+e)^2
*sin(f*x+e)+3*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A
*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+2*B*cos(f*x+e)^2*sin
(f*x+e)+B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+A*cos(f*x+e)^2+3*B*cos(f*x+e
)^2-3*A*sin(f*x+e)-B*sin(f*x+e)-A-3*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/
(-c*(-1+sin(f*x+e)))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Fricas [A] time = 2.53491, size = 1058, normalized size = 5.09

$$\left[\frac{\left((3A+B)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+(3A+B)\cos\left(fx+e\right)^{3}\right)\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-}}{\cos\left(fx+e\right)^{3}}\right.}{16\left(a^{3}c^{2}f\cos\left(fx+e\right)^{3}\sin\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*s
qrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)
- 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*
x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*(((3*A + B)*cos(f*x + e)^3*sin(f*x
 + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + (
(3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e
) + a^3*c^2*f*cos(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=245

$$\frac{3A\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3A\cos(e}{8a^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x]
)^(5/2)) - (A*Cos[e + f*x])/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(3/2)) + (3*A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])
/(8*a^2*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.569916, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2972, 2743, 2741, 3770}

$$\frac{3A\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{3A\cos(e}{8a^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```


```
[Out] -((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x]
)^(5/2)) - (A*Cos[e + f*x])/(2*a*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (3*A*Cos[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(3/2)) + (3*A*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])
/(8*a^2*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2741

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{A\int \frac{1}{(a+a\sin(e+fx))^{3/2}(\ldots}}{a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\,c\ldots}{2af(a+a\sin(e+f\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\,c\ldots}{2af(a+a\sin(e+f\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\,c\ldots}{2af(a+a\sin(e+f\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\,c\ldots}{2af(a+a\sin(e+f\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{A\,c\ldots}{2af(a+a\sin(e+f\ldots}$$

Mathematica [A] time = 0.939692, size = 246, normalized size = 1.

$$\frac{\sec^3(e+fx)\left(22A\sin(e+fx)+6A\sin(3(e+fx))-9A\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12A\cos(2(e+fx))\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sec[e + f*x]^3*(16*B - 9*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*A
*Cos[2*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]]) - 3*A*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*A*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + 22*A*Sin[e + f*x] + 6*A*Sin[3*(e + f*x
)]))/(64*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A] time = 0.297, size = 151, normalized size = 0.6

$$-\frac{\cos\left(fx+e\right)}{8f}\left(3A\left(\cos\left(fx+e\right)\right)^4\ln\left(-\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)-3A\left(\cos\left(fx+e\right)\right)^4\ln\left(\frac{1-\cos\left(fx+e\right)+\ldots}{\sin\left(fx+\ldots\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(3*A*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A*cos
(f*x+e)^4*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*cos(f*x+e)^4-3*A*cos
(f*x+e)^2*sin(f*x+e)-2*A*sin(f*x+e)-2*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2
)/(-c*(-1+sin(f*x+e)))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Fricas [A] time = 2.59602, size = 783, normalized size = 3.2

$$\left[\frac{3\sqrt{ac}A\cos\left(fx+e\right)^{5}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)+2\left(\left(3A\cos\left(fx+e\right)^{2}+2\right.\right.}{16\,a^{3}c^{3}f\cos\left(fx+e\right)^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*
x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin
(f*x + e))/cos(f*x + e)^3) + 2*((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2
```

```
*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x
+ e)^5), -1/8*(3*sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^5 - (
(3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2*B)*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=174

$$\frac{c2^{n+\frac{1}{2}}(A(m+n+1)+B(m-n))\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1\right.}{f(2m+1)(m+n+1)}$$

```
[Out] (2^(1/2 + n)*c*(B*(m - n) + A*(1 + m + n))*Cos[e + f*x]*Hypergeometric2F1[(
1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*
x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 +
 2*m)*(1 + m + n)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n)/(f*(1 + m + n))
```

Rubi [A] time = 0.314561, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c2^{n+\frac{1}{2}}(A(m+n+1)+B(m-n))\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2}(2m+1\right.}{f(2m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n)*c*(B*(m - n) + A*(1 + m + n))*Cos[e + f*x]*Hypergeometric2F1[(
1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*
x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*(1 +
 2*m)*(1 + m + n)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n)/(f*(1 + m + n))
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
```

```
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{n}}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{n}}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{n}}{f(1+m+n)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{n}}{f(1+m+n)}$$

$$= \frac{2^{\frac{1}{2}+n}c\left(A+\frac{B(m-n)}{1+m+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-\right.}{}$$

Mathematica [C] time = 14.1725, size = 2903, normalized size = 16.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (4*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 +
 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m
+ n))*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(A*Cos[(-e + Pi/2 - f*x
)/2]^(2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*n) + B*Cos[(-e + Pi/2 - f*x)/2]^(2*m
)*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Sin[e + f*x])*Tan[(-e + Pi/2 - f*x)/4])/(f
*(1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*((-4*m*(8*B*AppellF1[1/2
+ n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2
 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]
^2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]
^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/(1 + 2*n) - ((8*B*AppellF1[
1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
```

```
 + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 +
 n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1
[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x
)/4]^2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n))/((1 + 2*n)*(1 - Tan
[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (4*n*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 +
m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] -
 (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*
(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])
*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n)
)*Sin[(-e + Pi/2 - f*x)/2]^(-1 + 2*n)*Tan[(-e + Pi/2 - f*x)/4])/((1 + 2*n)*
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (4*m*(8*B*AppellF1[1/2 + n, -2*m,
 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)
/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m
, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x
)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(
2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(1 + 2*n)*Tan[(-e + Pi/2 - f*x)/4])/((1
 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (4*(m + n)*(8*B*AppellF1[
1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 +
 n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1
[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x
)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]
^2)/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (4*Cos[(-e + Pi/2
- f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m + n))*Sin[(-e + Pi/2 - f
*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]*(-((A + B)*(-((m*(1/2 + n)*AppellF1[3
/2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])
/(3/2 + n)) - ((1/2 + n)*(1 + 2*(m + n))*AppellF1[3/2 + n, -2*m, 2 + 2*(m +
 n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[
(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 + n)))) - 8*B*(-((
m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 3 + 2*(m + n), 5/2 + n, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan
[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(3 + 2*(m + n))*AppellF1[3/2
 + n, -2*m, 4 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(
3/2 + n))) + 8*B*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(1 + m + n),
5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e +
 Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(1 + m
+ n)*AppellF1[3/2 + n, -2*m, 1 + 2*(1 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e +
 Pi/2 - f*x)/4])/(3/2 + n))))/((1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(
```

```
2*m))))
```

Maple [F] time = 2.369, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.196 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=145

$$\frac{a^4c^32^{m+\frac{1}{2}}(B(3-m)-A(m+4))\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+f\right.}{7f(m+4)}$$

```
[Out] (2^(1/2 + m)*a^4*c^3*(B*(3 - m) - A*(4 + m))*Cos[e + f*x]^7*Hypergeometric2
F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-4 + m))/(7*f*(4 + m)) - (a^3*B*c^3*Cos[e + f*x]^7*(a +
 a*Sin[e + f*x])^(-3 + m))/(f*(4 + m))
```

Rubi [A] time = 0.341807, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^4c^32^{m+\frac{1}{2}}(B(3-m)-A(m+4))\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+f\right.}{7f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```


```
[Out] (2^(1/2 + m)*a^4*c^3*(B*(3 - m) - A*(4 + m))*Cos[e + f*x]^7*Hypergeometric2
F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-4 + m))/(7*f*(4 + m)) - (a^3*B*c^3*Cos[e + f*x]^7*(a +
 a*Sin[e + f*x])^(-3 + m))/(f*(4 + m))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-3+m}(A+B\,\mathrm{s}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)} + \left(a^3c^3\left(\right.\right.$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)} + \frac{\left(a^5c^3\left(\right.\right.}{}$$

$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)} + \frac{\left(2^{-\frac{1}{2}+m}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a^4c^3\left(A-\frac{B(3-m)}{4+m}\right)\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},\frac{1}{2}-m;\frac{9}{2}\right.}{}$$

Mathematica [F] time = 180.084, size = 0, normalized size = 0.

$$\$\text{Aborted}$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3,x]
```

```
[Out] $Aborted
```

Maple [F] time = 3.02, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(Bc^3\cos\left(fx+e\right)^4+(3\,A-5\,B)c^3\cos\left(fx+e\right)^2-4\,(A-B)c^3-\left((A-3\,B)c^3\cos\left(fx+e\right)^2-4\,(A-B)c^3\right)\text{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*(a*sin
(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)^{3}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)
^m, x)
```

3.197 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=145

$$\frac{a^3c^22^{m+\frac{1}{2}}(B(2-m)-A(m+3))\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+\right.}{5f(m+3)}$$

```
[Out] (2^(1/2 + m)*a^3*c^2*(B*(2 - m) - A*(3 + m))*Cos[e + f*x]^5*Hypergeometric2
F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-3 + m))/(5*f*(3 + m)) - (a^2*B*c^2*Cos[e + f*x]^5*(a +
 a*Sin[e + f*x])^(-2 + m))/(f*(3 + m))
```

Rubi [A] time = 0.334866, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^3c^22^{m+\frac{1}{2}}(B(2-m)-A(m+3))\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+\right.}{5f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```


```
[Out] (2^(1/2 + m)*a^3*c^2*(B*(2 - m) - A*(3 + m))*Cos[e + f*x]^5*Hypergeometric2
F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a
 + a*Sin[e + f*x])^(-3 + m))/(5*f*(3 + m)) - (a^2*B*c^2*Cos[e + f*x]^5*(a +
 a*Sin[e + f*x])^(-2 + m))/(f*(3 + m))
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^2\, dx = \left(a^2c^2\right) \int \cos^4(e + fx)(a + a\sin(e + fx))^{-2+m}(A + B$$

$$= -\frac{a^2Bc^2\cos^5(e + fx)(a + a\sin(e + fx))^{-2+m}}{f(3 + m)} + \Big(a^2c^2$$

$$= -\frac{a^2Bc^2\cos^5(e + fx)(a + a\sin(e + fx))^{-2+m}}{f(3 + m)} + \frac{\Big(a^4c^2}{}$$

$$= -\frac{a^2Bc^2\cos^5(e + fx)(a + a\sin(e + fx))^{-2+m}}{f(3 + m)} + \frac{\Big(2^{-\frac{1}{2}}}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a^3c^2\left(A - \frac{B(2-m)}{3+m}\right)\cos^5(e + fx)\, {}_2F_1\left(\frac{5}{2}, \frac{1}{2} - m;\right.}{}$$

Mathematica [F] time = 180.043, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2,x]
```

```
[Out] $Aborted
```

Maple [F] time = 2.553, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^m \left(A + B\sin\left(fx + e\right)\right)\left(c - c\sin\left(fx + e\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left((A-2B)c^2\cos(fx+e)^2-2(A-B)c^2+\left(Bc^2\cos(fx+e)^2+2(A-B)c^2\right)\sin(fx+e)\right)(a\sin(fx+e)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^
m, x)
```

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=139

$$\frac{a^2c2^{m+\frac{1}{2}}(B(1-m)-A(m+2))\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx\right.}{3f(m+2)}$$

```
[Out] (2^(1/2 + m)*a^2*c*(B*(1 - m) - A*(2 + m))*Cos[e + f*x]^3*Hypergeometric2F1
[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a +
 a*Sin[e + f*x])^(-2 + m))/(3*f*(2 + m)) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin
[e + f*x])^(-1 + m))/(f*(2 + m))
```

Rubi [A] time = 0.289108, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^2c2^{m+\frac{1}{2}}(B(1-m)-A(m+2))\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx\right.}{3f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a^2*c*(B*(1 - m) - A*(2 + m))*Cos[e + f*x]^3*Hypergeometric2F1
[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a +
 a*Sin[e + f*x])^(-2 + m))/(3*f*(2 + m)) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin
[e + f*x])^(-1 + m))/(f*(2 + m))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx = (ac)\int \cos^2(e + fx)(a + a\sin(e + fx))^{-1+m}(A + B\sin($$

$$= -\frac{aBc\cos^3(e + fx)(a + a\sin(e + fx))^{-1+m}}{f(2 + m)} + \left(ac\left(A - \right.\right.$$

$$= -\frac{aBc\cos^3(e + fx)(a + a\sin(e + fx))^{-1+m}}{f(2 + m)} + \frac{\left(a^3c\left(A - \right.\right.}{}$$

$$= -\frac{aBc\cos^3(e + fx)(a + a\sin(e + fx))^{-1+m}}{f(2 + m)} + \frac{\left(2^{-\frac{1}{2}+m}a^3\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a^2c\left(A - \frac{B(1-m)}{2+m}\right)\cos^3(e + fx)\,{}_2F_1\left(\frac{3}{2}, \frac{1}{2} - m; \frac{5}{2}; \frac{1}{2}\right.}{}$$

Mathematica [C] time = 4.20376, size = 462, normalized size = 3.32

$$\frac{ic4^{-m-1}e^{ifmx}\left(1 + ie^{-i(e+fx)}\right)^{-2m}\left(-(-1)^{3/4}e^{-\frac{1}{2}i(e+fx)}\left(e^{i(e+fx)} + i\right)\right)^{2m}(\sin(e + fx) - 1)\sin^{-2m}\left(\frac{1}{4}(2e + 2fx + \pi)\right)(a(\sin(e + f}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),
x]
```

```
[Out] (I*4^(-1 - m)*c*E^(I*f*m*x)*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)*
(e + f*x))))^(2*m)*(((-I)*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, (-I)/E^
(I*(e + f*x))])/(E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*((-I)*A + B)*Hyper
geometric2F1[-1 - m, -2*m, -m, (-I)/E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*
x))*(1 + m)) + ((2*I)*A*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -
2*m, 2 - m, (-I)/E^(I*(e + f*x))])/(-1 + m) + (2*B*E^(I*(e - f*(-1 + m)*x))
*Hypergeometric2F1[1 - m, -2*m, 2 - m, (-I)/E^(I*(e + f*x))])/(-1 + m) + (I
*B*E^((2*I)*e - I*f*(-2 + m)*x)*Hypergeometric2F1[2 - m, -2*m, 3 - m, (-I)/
E^(I*(e + f*x))])/(-2 + m) + (4*A*Hypergeometric2F1[-2*m, -m, 1 - m, (-I)/E
^(I*(e + f*x))])/(E^(I*f*m*x)*m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x])
)^m)/((1 + I/E^(I*(e + f*x)))^(2*m)*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2*Sin[(2*e + Pi + 2*f*x)/4]^(2*m))
```

Maple [F] time = 1.549, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bc\cos\left(fx+e\right)^{2}-(A-B)c\sin\left(fx+e\right)+(A-B)c\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(f
*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-c\left(\int -A\left(a\sin\left(e+fx\right)+a\right)^{m}\,dx+\int A\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)dx+\int -B\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) +
a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x),
x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)-c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m
, x)
```

3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{B\,c}{}$$

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rubi [A] time = 0.0817716, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{B\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\,dx = -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m}{f(1 + m)} + \frac{(A + Am + Bm)\int (a + a\sin(e + f}{1 + m}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m}{f(1 + m)} + \frac{\left((A + Am + Bm)(1 + \sin(e + fx))\right.}{}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m}{f(1 + m)} - \frac{2^{\frac{1}{2}+m}(A + Am + Bm)\cos(e + fx)}{}$$

Mathematica [C] time = 1.82332, size = 275, normalized size = 2.35

$$-\frac{\sin^{-2m}\left(\frac{1}{4}(2e + 2fx + \pi)\right)(a(\sin(e + fx) + 1))^m\left(\frac{2\sqrt{2}A\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{(2m+1)\sqrt{1-\sin(e+fx)}}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*A*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

Maple [F] time = 1.133, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=123

$$\frac{2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cfm}-\frac{B\,\mathrm{se}}{}$$

```
[Out] (2^(1/2 + m)*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin
[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x]
)^m)/(c*f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*m)
```

Rubi [A] time = 0.303142, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cfm}-\frac{B\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```



```
[Out] (2^(1/2 + m)*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sin
[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x]
)^m)/(c*f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*m)
```


Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
```

```
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{1+m}(A+B\sin(e+fx))\,dx}{ac}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{(B+Am+Bm)\int \sec^2(e+fx}{acm}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(a(B+Am+Bm)\sec(e+fx\right.}{}$$

$$= -\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm} + \frac{\left(2^{-\frac{1}{2}+m}a(B+Am+Bm)\sec(\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx}{cfm}$$

Mathematica [C] time = 25.6499, size = 7409, normalized size = 60.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)}{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{A\left(a\sin\left(e+fx\right)+a\right)^{m}}{\sin\left(e+fx\right)-1}\,dx+\int \frac{B\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{\sin\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] -(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x) + Integral(B*(a
*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2c^2f(1-m)}+\frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2))\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m}}{3ac^2f(1-m)}$$

```
[Out] (2^(1/2 + m)*(A*(1 - m) - B*(2 + m))*Hypergeometric2F1[-3/2, 1/2 - m, -1/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(1 + m))/(3*a*c^2*f*(1 - m)) + (B*Sec[e + f*x]^3*(a + a*Sin[e
+ f*x])^(2 + m))/(a^2*c^2*f*(1 - m))
```

Rubi [A] time = 0.330138, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2c^2f(1-m)}+\frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2))\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m}}{3ac^2f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*(A*(1 - m) - B*(2 + m))*Hypergeometric2F1[-3/2, 1/2 - m, -1/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(1 + m))/(3*a*c^2*f*(1 - m)) + (B*Sec[e + f*x]^3*(a + a*Sin[e
+ f*x])^(2 + m))/(a^2*c^2*f*(1 - m))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{2+m}(A+B\sin(e+fx))\,dx}{a^2c^2}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(A-\frac{B(2+m)}{1-m}\right)\int \sec^4(e+fx)(a+}{a^2c^2}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(\left(A-\frac{B(2+m)}{1-m}\right)\sec^3(e+fx)(a-\right.}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right)\sec^3(e+f\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(2+m)}{1-m}\right){}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)}{3ac^2f}$$

Mathematica [C] time = 23.3283, size = 8371, normalized size = 56.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.754, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 +
 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^
2, x)
```

3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=148

$$\frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3))\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+f\right.}{5a^2c^3f(2-m)}$$

```
[Out] (2^(1/2 + m)*(A*(2 - m) - B*(3 + m))*Hypergeometric2F1[-5/2, 1/2 - m, -3/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(2 + m))/(5*a^2*c^3*f*(2 - m)) + (B*Sec[e + f*x]^5*(a + a*Sin[
e + f*x])^(3 + m))/(a^3*c^3*f*(2 - m))
```

Rubi [A] time = 0.331781, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3))\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+f\right.}{5a^2c^3f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1/2 + m)*(A*(2 - m) - B*(3 + m))*Hypergeometric2F1[-5/2, 1/2 - m, -3/2,
 (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*S
in[e + f*x])^(2 + m))/(5*a^2*c^3*f*(2 - m)) + (B*Sec[e + f*x]^5*(a + a*Sin[
e + f*x])^(3 + m))/(a^3*c^3*f*(2 - m))
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^6(e+fx)(a+a\sin(e+fx))^{3+m}(A+B\sin(e+fx))\,dx}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(A-\frac{B(3+m)}{2-m}\right)\int \sec^6(e+fx)(a}{a^3c^3}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(\left(A-\frac{B(3+m)}{2-m}\right)\sec^5(e+fx)(a}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{\left(2^{-\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right)\sec^5(e+\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}\left(A-\frac{B(3+m)}{2-m}\right){}_2F_1\left(-\frac{5}{2},\frac{1}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx}{5a^2c^3f}$$

Mathematica [C] time = 25.9016, size = 9702, normalized size = 65.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.931, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)}{\left(c-c\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2
 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(c\sin\left(fx+e\right)-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^3, x)
```

3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rubi [A] time = 0.294433, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```


```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^{m}}{\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int \sec(e}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e+fx))\,\mathrm{Subst}\Big(}{f\sqrt{a+a\sin(e+f}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+\right.}{f(1}$$

Mathematica [A] time = 4.86842, size = 200, normalized size = 1.69

$$\frac{2^{-2m-\frac{3}{2}} \sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m \left(2^{2m+1}(A+B)\,_2F_1\left(1,2m+1;2(\right.\right.}{(2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Si
n[e + f*x]])
```

Maple [F] time = 0.321, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] (-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*
x]])
```

Rubi [A] time = 0.288117, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e + f*
x]])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - (-A-B)\int \frac{(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)\cos(e+fx))\int \sec(e+}{\sqrt{a-a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} - \frac{((-A-B)c\cos(e+fx))\,\mathrm{Subst}\Big(}{f\sqrt{a-a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+m\right.}{f(1+}$$

Mathematica [A] time = 2.35267, size = 200, normalized size = 1.69

$$\frac{2^{-2m-\frac{3}{2}}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(\sin(e+fx)+1))^m\left(2^{2m+1}(A+B)\,{}_2F_1\left(1,2m+1;2\right.\right.}{(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[a - a*Si
n[e + f*x]])
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int \left(A+B\sin\left(fx+e\right)\right)\left(c+c\sin\left(fx+e\right)\right)^{m}\frac{1}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
+ a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{m}}{a\sin\left(fx+e\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)^m/(a*sin(f*x + e) - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(c\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f*
x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(c\sin\left(fx+e\right)+c\right)^{m}}{\sqrt{-a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
+ a), x)
```

3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=275

$$-\frac{16c^2(B(5-2m)-A(2m+7))\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+7)\left(4m^2+16m+15\right)}-\frac{64c^3(B(5-2m)-A(2m+7))\cos(e}{f(2m+5)(2m+7)\left(4m^2+8m+\right.}$$

```
[Out] (-64*c^3*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(
f*(5 + 2*m)*(7 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (16*c^2
*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c
*Sin[e + f*x]])/(f*(7 + 2*m)*(15 + 16*m + 4*m^2)) - (2*c*(B*(5 - 2*m) - A*(
7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(
f*(5 + 2*m)*(7 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Si
n[e + f*x])^(5/2))/(f*(7 + 2*m))
```

Rubi [A] time = 0.503458, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2973, 2740, 2738}

$$-\frac{16c^2(B(5-2m)-A(2m+7))\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+7)\left(4m^2+16m+15\right)}-\frac{64c^3(B(5-2m)-A(2m+7))\cos(e}{f(2m+5)(2m+7)\left(4m^2+8m+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-64*c^3*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(
f*(5 + 2*m)*(7 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (16*c^2
*(B*(5 - 2*m) - A*(7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c
*Sin[e + f*x]])/(f*(7 + 2*m)*(15 + 16*m + 4*m^2)) - (2*c*(B*(5 - 2*m) - A*(
7 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(
f*(5 + 2*m)*(7 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Si
n[e + f*x])^(5/2))/(f*(7 + 2*m))
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+}{f(7+2m)}$$

$$= -\frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin}{f(5+2m)(7+2m}$$

$$= -\frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\,\mathrm{s}}{f(3+2m)(5+2m)(7+}$$

$$= -\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\,\mathrm{s}}{f(1+2m)(3+2m)(5+2m)(7+2m)\sqrt{c-c\,\mathrm{si}}}$$

Mathematica [C] time = 6.82886, size = 667, normalized size = 2.43

$$(c-c\sin(e+fx))^{5/2}(a(\sin(e+fx)+1))^m\left(\frac{\left(32Am^3+304Am^2+1272Am+2100A-8Bm^3-68Bm^2-110Bm-1575B\right)\left(\left(\frac{1}{8}-\frac{i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)+\left(\frac{1}{8}+\frac{i}{8}\right)\right.}{(2m+1)(2m+3)(2m+5)(2m+7)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2100*A - 1575*B + 1
272*A*m - 110*B*m + 304*A*m^2 - 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 + I/8)
*Cos[(e + f*x)/2] + (1/8 - I/8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5
+ 2*m)*(7 + 2*m)) + ((2100*A - 1575*B + 1272*A*m - 110*B*m + 304*A*m^2 - 68
*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 - I/8)*Cos[(e + f*x)/2] + (1/8 + I/8)*Si
n[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((350*A - 385*
B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 - I/8)*Cos[(3*(e + f*x))
/2] - (1/8 + I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)) +
((350*A - 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 + I/8)*Cos
[(3*(e + f*x))/2] - (1/8 - I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)
*(7 + 2*m)) + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 + I/8)*Cos[(5*(e + f*x)
)/2] - (1/8 + I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((14*A -
35*B + 4*A*m - 6*B*m)*((-1/8 - I/8)*Cos[(5*(e + f*x))/2] - (1/8 - I/8)*Sin[
(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((1/8 - I/8)*B*Cos[(7*(e + f*x))
/2] - (1/8 + I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m) + ((1/8 + I/8)*B*Cos[(7
*(e + f*x))/2] - (1/8 - I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m)))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [F] time = 0.327, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^{m} \left(A + B\sin\left(fx + e\right)\right) \left(c - c\sin\left(fx + e\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [B] time = 1.73941, size = 979, normalized size = 3.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)*a^m*c^(
5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^m*c^(5/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 128*m^3
 + 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m + 105)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2)))/f
```

Fricas [B] time = 2.31827, size = 1354, normalized size = 4.92

$$2\left(\left(8\,Bc^{2}m^{3} + 36\,Bc^{2}m^{2} + 46\,Bc^{2}m + 15\,Bc^{2}\right)\cos\left(fx+e\right)^{4} + 64\,(A+B)c^{2}m - \left(8\,(A-2\,B)c^{2}m^{3} + 4\,(11\,A-28\,B)c^{2}m^{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 64
*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A -
 86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 + (8
*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A - 85
*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 + 2*(79
*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^2*m -
(8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3 + 32*(7
```

```
*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(31*A - 63
*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*m^3 + 60*
(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(f*x + e))*
sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(16*f*m^4 +
128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f
*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 10
5*f)*sin(f*x + e) + 105*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=166

$$-\frac{2Bc^2\cos(e+fx)(a\sin(e+fx)+a)^{m+2}}{a^2f(2m+5)\sqrt{c-c\sin(e+fx)}}+\frac{4c^2(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2c^2(A-3B)\cos(e+fx)(a\sin}{af(2m+3)\sqrt{c-c\sin}}$$

```
[Out] (4*(A - B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c
*Sin[e + f*x]]) - (2*(A - 3*B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m
))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]) - (2*B*c^2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(2 + m))/(a^2*f*(5 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.354095, antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2973, 2740, 2738}

$$-\frac{8c^2(B(3-2m)-A(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sin(e+fx)}}-\frac{2c(B(3-2m)-A(2m+5))\cos(e+fx)\sqrt{c-c\sin(e+}}{f(2m+3)(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-8*c^2*(B*(3 - 2*m) - A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f
*(5 + 2*m)*(3 + 8*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) - (2*c*(B*(3 - 2*m)
- A*(5 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]]
)/(f*(3 + 2*m)*(5 + 2*m)) - (2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(3/2))/(f*(5 + 2*m))
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2740

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+f}{f(5+2m)}$$

$$= -\frac{2c(B(3-2m)-A(5+2m))\cos(e+fx)(a+a\sin(e}{f(3+2m)(5+2m)}$$

$$= -\frac{8c^2(B(3-2m)-A(5+2m))\cos(e+fx)(a+a\sin(}{f(1+2m)(3+2m)(5+2m)\sqrt{c-c\sin(e+f}}$$

Mathematica [A] time = 1.69844, size = 174, normalized size = 1.05

$$\frac{c\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(-2(2m+1)(2Am+5A-2Bm-9B)\sin(e\right.}{f(2m+1)(2m+3)(2m+5)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3 +
```

```
 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)*Si
n[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]))
```

Maple [F] time = 0.325, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(A+B\sin\left(fx+e\right)\right) \left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [B] time = 1.69904, size = 672, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] -2*((a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x +
e) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m*c
^(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*A*e^(2*m*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1))/((4*m^2 + 8*m + 3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2
*(a^m*c^(3/2)*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)/(cos(f*x
 + e) + 1) + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2*(2*m^2 +
9*m)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*c^(3/2)*(2*m + 9
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 3
6*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Fricas [A] time = 2.11756, size = 772, normalized size = 4.65

$$\frac{2\left(\left(4\,Bcm^2+8\,Bcm+3\,Bc\right)\cos\left(fx+e\right)^3+8\,(A+B)cm+\left(4\,Acm^2+12\,(A-B)cm+(5\,A-6\,B)c\right)\cos\left(fx+e\right)^2+4\,(5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m^
2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4*(
A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A +
B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c - (
4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2
+ 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 3
6*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=104

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 +
 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.282108, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2971, 2738}

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 +
 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2971

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist
[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - D
ist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^
2 - b^2, 0]
```

Rule 2738

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^
n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{B\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{a} - (-A$$
$$= \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{2Bc\cos(e}{af(3}$$

Mathematica [A] time = 0.443223, size = 116, normalized size = 1.12

$$\frac{2\sqrt{c-c\sin(e+fx)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m(A(2m+3)+B(2m+1)\sin(e+fx)-2B)}{f(2m+1)(2m+3)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*
m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m \left(A+B\sin\left(fx+e\right)\right)\sqrt{c-c\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [B] time = 1.61937, size = 436, normalized size = 4.19

$$-\frac{2\left(\frac{2\left(\frac{2a^m\sqrt{c}m\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^m\sqrt{c}m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-a^m\sqrt{c}-\frac{a^m\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(4m^2+8m+\frac{(4m^2+8m+3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+3\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\frac{\left(a^m\sqrt{c}+\frac{a^m\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}\right)Ae^{\left(2m\cdots\right.}}{(2m+\cdots}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -2*(2*(2*a^m*sqrt(c)*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c)*m*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2 + 8
*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e
) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)))/f
```

Fricas [A] time = 2.14303, size = 417, normalized size = 4.01

$$-\frac{2\left((2Bm+B)\cos(fx+e)^2-2(A+B)m-(2Am+3A-2B)\cos(fx+e)-\left(2(A+B)m+(2Bm+B)\cos(fx+e)\cdots\right.\right.}{4fm^2+8fm+\left(4fm^2+8fm+3f\right)\cos(fx+e)-\left(4fm^2+8f\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x
+ e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) - 3*
A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m +
(4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x + e)
 + 3*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e
 + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=118

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rubi [A] time = 0.28212, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*
x]])
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + (A+B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{((A+B)\cos(e+fx))\int \sec(e+}{\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(a(A+B)\cos(e+fx))\,\mathrm{Subst}\left(\int\right.}{f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,{}_2F_1\left(1,\frac{1}{2}+\right.}{f(1+}$$

Mathematica [A] time = 2.30926, size = 200, normalized size = 1.69

$$\frac{2^{-2m-\frac{3}{2}}\sin\left(\frac{1}{4}(2e+2fx+\pi)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m\left(2^{2m+1}(A+B)\,{}_2F_1\left(1,2m+1;2\right.\right.}{(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1
, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F
1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2
*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Si
n[e + f*x]])
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\frac{1}{\sqrt{c-c\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c\sin\left(fx+e\right)-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*
x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
+ c), x)
```

3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f(c-c\sin}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3/
2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c*f*(1 + 2*m)*
Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.29977, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2972, 2745, 2667, 68}

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f(c-c\sin}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3/
2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c*f*(1 + 2*m)*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(-\frac{1}{2}+\right.\right.}{2c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(-\frac{1}{2}+\right.\right.\right.}{2c^2\sqrt{}}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{\left(a\left(Bc\left(-\frac{3}{2}-m\right)-Ac\left(-\frac{1}{2}\right.\right.\right.}{2c^2 f}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}} + \frac{(A(1-2m)-B(3+2m))\,c}{}$$

Mathematica [B] time = 10.1848, size = 369, normalized size = 2.75

$$-\frac{\sec^2\left(\frac{1}{4}\left(-e-fx+\frac{\pi}{2}\right)\right)^{2m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a\sin(e+fx)+a)^m\left(\frac{4^{-m}(A-3B)\,{}_2F_1\left(2m,2m;2m+1;\frac{1}{2}\left(1-\tan^2\left(\frac{1}{4}\left(-\right.\right.\right.\right.}{m}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] -((Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3*(a + a*Sin[e + f*x])^m*(((A - 3*B)*Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (
1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2])/(4^m*m) - ((A - 3*B)*Hypergeometric2F1[
1, 2*m, 1 + 2*m, Cos[(-e + Pi/2 - f*x)/2]])/(m*(Sec[(-e + Pi/2 - f*x)/4]^2)
^(2*m)) - (2*(A + B)*Cos[(-e + Pi/2 - f*x)/2]*Hypergeometric2F1[2, 1 + 2*m,
 2 + 2*m, Cos[(-e + Pi/2 - f*x)/2]])/((1 + 2*m)*(Sec[(-e + Pi/2 - f*x)/4]^2
)^(2*m)) + ((A + B)*Hypergeometric2F1[2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(-e
 + Pi/2 - f*x)/4]^2)/2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2))/(4^m*(1 + 2*m)))
)/(8*Sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2))
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\sin\left(fx+e\right)-2\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(3/2), x)
```

3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+f}{4f(c-c}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5/
2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(16*c^2*f*(1 + 2*
m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] time = 0.332324, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2972, 2745, 2667, 68}

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+f}{4f(c-c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2
),x]
```


```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5/
2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2 +
m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(16*c^2*f*(1 + 2*
m)*Sqrt[c - c*Sin[e + f*x]])
```


Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(-\frac{3}{2}+m\right.\right.}{4c^2}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(-\frac{3}{2}+m\right.\right.\right.}{4ac^3\sqrt{\ }}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{\left(a^2\left(Bc\left(-\frac{5}{2}-m\right)-Ac\left(-\frac{3}{2}\right.\right.\right.}{4c^3f\sqrt{\ }}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}} + \frac{(A(3-2m)-B(5+2m))\cos}{}$$

Mathematica [C] time = 23.8037, size = 5387, normalized size = 40.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.293, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}-\left(c^{3}\cos\left(fx+e\right)^{2}-4\,c^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x +
 e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^(5/2), x)
```

3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal. Leaf size=267

$$\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{c^2f(2m+7)\left(4m^2+16m+15\right)}+\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+f}{c^3f(2m+5)(2m+7)\left(4\right.}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m))
/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-3 - m))/(c*f*(5 + 2*m)*(7 + 2*m)) + (2*(3*A - 2*B*(2
+ m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(c
^2*f*(7 + 2*m)*(15 + 16*m + 4*m^2)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(
a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^3*f*(5 + 2*m)*(7 +
2*m)*(3 + 8*m + 4*m^2))
```

Rubi [A] time = 0.42687, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{c^2f(2m+7)\left(4m^2+16m+15\right)}+\frac{2(3A-2B(m+2))\cos(e+fx)(a\sin(e+f}{c^3f(2m+5)(2m+7)\left(4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m))
/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-3 - m))/(c*f*(5 + 2*m)*(7 + 2*m)) + (2*(3*A - 2*B*(2
+ m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(c
^2*f*(7 + 2*m)*(15 + 16*m + 4*m^2)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(
a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^3*f*(5 + 2*m)*(7 +
2*m)*(3 + 8*m + 4*m^2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(7+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(7+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(7+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(7+2m)}$$

Mathematica [A] time = 12.3615, size = 353, normalized size = 1.32

$$-\frac{2^{-m-18}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^{21}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec^{7}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-4 - m),x]
```

```
[Out] -((2^(-18 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^21*Sec[(-e
 + Pi/2 - f*x)/8]^7*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m)*(-
96*A + 58*B - 176*A*m + 64*B*m - 96*A*m^2 + 16*B*m^2 - 16*A*m^3 + 4*(2 + m)
*(-3*A + 2*B*(2 + m))*Cos[2*(-e + Pi/2 - f*x)] + 3*A*Cos[3*(-e + Pi/2 - f*x
)] - 4*B*Cos[3*(-e + Pi/2 - f*x)] - 2*B*m*Cos[3*(-e + Pi/2 - f*x)] + (29 +
32*m + 8*m^2)*(3*A - 2*B*(2 + m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5
+ 2*m)*(7 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^7*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-4 - m))))
```

Maple [F] time = 0.56, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-4-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.18018, size = 506, normalized size = 1.9

$$\frac{\left(4\left(2\,Bm^2-(3\,A-8\,B)m-6\,A+8\,B\right)\cos\left(fx+e\right)^3+\left(8\,Am^3+12\,(4\,A-B)m^2+2\,(47\,A-24\,B)m+60\,A-45\,B\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="fricas")
```

```
[Out] (4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4*
A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m - 3*
A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*m -
 45*A + 60*B)*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*
x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, al
gorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.212 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=191

$$\frac{(2A-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\left(4m^2+8m+3\right)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-}{f(2m+5)}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))
/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-2 - m))/(c*f*(3 + 2*m)*(5 + 2*m)) + ((2*A - B*(3 + 2*
m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2
*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rubi [A] time = 0.309681, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2972, 2743, 2742}

$$\frac{(2A-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{c^2f(2m+5)\left(4m^2+8m+3\right)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-}{f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m))
/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-2 - m))/(c*f*(3 + 2*m)*(5 + 2*m)) + ((2*A - B*(3 + 2*
m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2
*f*(5 + 2*m)*(3 + 8*m + 4*m^2))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(5+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(5+2m)}$$

$$= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(}{f(5+2m)}$$

Mathematica [A] time = 10.0505, size = 269, normalized size = 1.41

$$\frac{2^{-m-13}\cos\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)\csc^{15}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sec^{5}\left(\frac{1}{8}\left(-e-fx+\frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e-fx+\frac{\pi}{2}\right)\right)(a\sin(e+fx)+a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-3 - m),x]
```

```
[Out] (2^(-13 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^15*Sec[(-e +
 Pi/2 - f*x)/8]^5*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(16*
A - 9*B + 24*A*m - 6*B*m + 8*A*m^2 + (2*A - 3*B - 2*B*m)*Cos[2*(-e + Pi/2 -
 f*x)] + 2*(3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 +
2*m)*(5 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^5*Sin[(-e + Pi/2 - f*x)/2]
^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

Maple [F] time = 0.558, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx+e\right)\right)^{m} \left(A + B\sin\left(fx+e\right)\right) \left(c - c\sin\left(fx+e\right)\right)^{-3-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.29649, size = 335, normalized size = 1.75

$$\frac{\left((2\,Bm - 2\,A + 3\,B)\cos\left(fx+e\right)^{3} + \left(4\,Bm^{2} - 4\,(A - 3\,B)m - 6\,A + 9\,B\right)\cos\left(fx+e\right)\sin\left(fx+e\right) + \left(4\,Am^{2} + 4\,(3\,A - \right.\right.}{8\,fm^{3} + 36\,fm^{2} + 46\,fm + 15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B)
*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*x
+ e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8*f*m^3 + 36*f
*m^2 + 46*f*m + 15*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, al
gorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=114

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a)^m(c-}{cf(2m+1)(2m+3)}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))
/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c
- c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

Rubi [A] time = 0.221648, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2972, 2742}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a)^m(c-}{cf(2m+1)(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))
/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c
- c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
 + 1, 0]
```

Rule 2742

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*
(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-2-m}\,dx = \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(}{f(3 + 2m)}$$
$$= \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(}{f(3 + 2m)}$$

Mathematica [A] time = 8.50224, size = 211, normalized size = 1.85

$$-\frac{2^{-m-7}\cos\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right)\csc^9\left(\frac{1}{8}\left(-e - fx + \frac{\pi}{2}\right)\right)\sec^3\left(\frac{1}{8}\left(-e - fx + \frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right)(a\sin(e + fx) + a}{f\left(4m^2 + 8m + 3\right)\left(\cot^2\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-2 - m),x]
```

```
[Out] -((2^(-7 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^9*Sec[(-e +
 Pi/2 - f*x)/8]^3*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m)*(B -
 2*A*(1 + m) + (A - 2*B*(1 + m))*Sin[e + f*x]))/(f*(3 + 8*m + 4*m^2)*(-1 +
Cot[(-e + Pi/2 - f*x)/8]^2)^3*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))
```

Maple [F] time = 0.515, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)^m \left(A + B\sin\left(fx + e\right)\right)\left(c - c\sin\left(fx + e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 2), x)
```

Fricas [A] time = 2.01434, size = 213, normalized size = 1.87

$$\frac{((2\,Bm-A+2\,B)\cos(fx+e)\sin(fx+e)+(2\,Am+2\,A-B)\cos(fx+e))(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c}{4\,fm^2+8\,fm+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="fricas")
```

```
[Out] ((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x +
e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m
+ 3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, al
gorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=163

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)}{}$$

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))
/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2,
 (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 +
m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rubi [A] time = 0.30901, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2972, 2745, 2689, 70, 69}

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))
/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2,
 (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 +
m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))
```

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*m
```

```
 + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-1-m}\,dx = \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin}{f(1 + 2m)}$$

$$= \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin}{f(1 + 2m)}$$

$$= \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin}{f(1 + 2m)}$$

$$= \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin}{f(1 + 2m)}$$

$$= \frac{(A + B)\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin}{f(1 + 2m)}$$

Mathematica [C] time = 11.3356, size = 675, normalized size = 4.14

$$-\frac{2^{-m}(2m - 3)\cos^2\left(\frac{1}{4}\left(-e - fx + \frac{\pi}{2}\right)\right)\cot\left(\frac{1}{4}\left(-e - fx + \frac{\pi}{2}\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right)(a\sin(e + fx) + a)^m (A + B\sin(e}{f\left(4m^2 - 1\right)\left((2m - 3)\left((A + B)\left(\cos\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right) + 1\right)\left(1 - \tan^2\left(\frac{1}{4}\left(-e - fx + \frac{\pi}{2}\right)\right)\right)^{2m} - 4\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] -((((-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/4]^2*Cot[(-e + Pi/2 - f*x)/4]*(a + a*Si
n[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - m)*(8*B*(1 +
2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - (A + B)*((-1 + 2*m)*Hyp
ergeometric2F1[-1/2 - m, -2*m, 1/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (1 +
2*m)*Hypergeometric2F1[1/2 - m, -2*m, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2]*
Tan[(-e + Pi/2 - f*x)/4]^2)))/(2^m*f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^
(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-1 - m))*(-64*B*m*AppellF1[
3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
 f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^4 - 32*B*AppellF1[3/2 - m, -2*m, 2, 5/
2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + P
i/2 - f*x)/4]^4 + (-3 + 2*m)*(-4*B*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/
2]^2 + (A + B)*(1 + Cos[(-e + Pi/2 - f*x)/2])*(1 - Tan[(-e + Pi/2 - f*x)/4]
```

```
^2)^(2*m)))))
```

Maple [F] time = 0.475, size = 0, normalized size = 0.

$$\int \left(a+a\sin \left(fx+e\right)\right)^{m} \left(A+B\sin \left(fx+e\right)\right) \left(c-c\sin \left(fx+e\right)\right)^{-1-m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin \left(fx+e\right)+A\right)\left(a\sin \left(fx+e\right)+a\right)^{m}\left(-c\sin \left(fx+e\right)+c\right)^{-m-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin \left(fx+e\right)+A\right)\left(a\sin \left(fx+e\right)+a\right)^{m}\left(-c\sin \left(fx+e\right)+c\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=158

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\right.}{f(2m+1)}$$

```
[Out] (2^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)) - (B*Cos[
e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)
```

Rubi [A] time = 0.268144, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+1);\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x]
```


```
[Out] (2^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)) - (B*Cos[
e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-m}\, dx = -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx)}{f}$$
$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx)}{f}$$
$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx)}{f}$$
$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx)}{f}$$
$$= \frac{2^{\frac{1}{2}-m} c(A + 2Bm)\cos(e + fx)\, {}_2F_1\left(\frac{1}{2}(1 + 2m), \frac{1}{2}(1 +\right.}{}$$

<u>Mathematica</u> [C] time = 16.8999, size = 2552, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
)^m,x]
```

```
[Out] (2^(2 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/
2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[
1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2]))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a + a*Sin[e + f*x])
^m*((A*Cos[(-e + Pi/2 - f*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Si
n[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin
[e + f*x])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2]
)^(2*m))*Tan[(-e + Pi/2 - f*x)/4])/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(
2*m)*(c - c*Sin[e + f*x])^m*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-((2^(2
 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 -
 m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2
 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/
4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e +
 Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*
Sin[(-e + Pi/2 - f*x)/2]^(2*m))) - (((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2
```

```
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-Appe
llF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e
+ Pi/2 - f*x)/4]^2)/(2^m*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan
[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(2 - m)*m*((A + B)*AppellF1[1/2 - m, -
2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] +
 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(1
+ 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 +
 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(2 - m)*m*((A + B)*Appel
lF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
- f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m
, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2
 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e + Pi/2 - f
*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2^(2 - m)*Co
s[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*((A + B)*(-(((1/2 - m
)*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4
])/(3/2 - m)) - ((1/2 - m)*AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan
[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m))) + 8*B*(((1/2 - m)*m*AppellF1[3/2 - m,
 1 - 2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m) - ((1/2
- m)*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x
)/4])/(3/2 - m) + ((1/2 - m)*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*T
an[(-e + Pi/2 - f*x)/4])/(3/2 - m) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4
, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m)))))/((-1 + 2*m)*
Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)))
```

Maple [F] time = 1.58, size = 0, normalized size = 0.

$$\int \frac{\left(a + a\sin\left(fx + e\right)\right)^{m}\left(A + B\sin\left(fx + e\right)\right)}{\left(c - c\sin\left(fx + e\right)\right)^{m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^
m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algor
ithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=170

$$\frac{c^2 2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2\right.}{f(2m+1)}$$

```
[Out] (2^(1/2 - m)*c^2*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)
) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(2
*f)
```

Rubi [A] time = 0.328651, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - m
),x]
```



```
[Out] (2^(1/2 - m)*c^2*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(-1 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m)
) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(2
*f)
```


Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
```

```
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\,dx = -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx}{2f}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx}{2f}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx}{2f}$$

$$= -\frac{B\cos(e + fx)(a + a\sin(e + fx))^m (c - c\sin(e + fx}{2f}$$

$$= \frac{2^{\frac{1}{2}-m} c^2 (2A - B(1 - 2m))\cos(e + fx)\,{}_2F_1\left(\frac{1}{2}(-1 + 2\right.}{}$$

Mathematica [C] time = 92.8968, size = 3601, normalized size = 21.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(1 - m),x]
```

```
[Out] (2^(5 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(
2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*(a + a*Sin[e + f*x])^m*(c - c*Sin
[e + f*x])^(1 - m)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2*((A*Cos[(-e + Pi/2 - f
*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x])/(Cos[Pi/4 + (e
 - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m)) + (A*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e - Pi/2
 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*
x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e -
 Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + Cos[Pi/4 + (e - P
i/2 + f*x)/2]*((-2*A*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 +
f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^
(2*m) - (2*B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi
```

/2 + f*x)/2])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m)))*Tan[(-e + Pi/2 - f*x)/4])/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/
2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(1 - m))*(1 - Tan[(-e + P
i/2 - f*x)/4]^2)^(2*m)*(-((2^(5 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 2,
3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*
B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(
-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/
4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/
2 - f*x)/2]^(2*m))) - (2^(3 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 -
m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*App
ellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2)/((-1 + 2*m)*Sin[(-e + Pi/2
- f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(5 - m)*m*((A
+ B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2
- m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4
]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Sin[(-e + Pi/
2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e +
 Pi/2 - f*x)/4]^2)^(2*m)) + (2^(5 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 2
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A +
9*B)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + 8*B*(2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m,
 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos
[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e
 + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2
^(5 - m)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*((A + B)*(
-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi
/2 - f*x)/4])/(3/2 - m)) - ((1/2 - m)*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*
x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (A + 9*B)*(-(((1/2 - m)*m*Ap
pellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/
2 - m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e
 + Pi/2 - f*x)/4])/(2*(3/2 - m))) + 8*B*(((1/2 - m)*m*AppellF1[3/2 - m, 1 -

```
 2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m) + (5*(1/2 -
m)*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(
2*(3/2 - m)) + 2*(-(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 4, 5/2 - m, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)
/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)) - (2*(1/2 - m)*AppellF1[3/2 - m,
 -2*m, 5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(3/2 - m)))))/((-1 +
2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))
))
```

Maple [F] time = 0.506, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, alg
orithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.217 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=173

$$\frac{c^3 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m\right.}{3f(2m+1)}$$

```
[Out] (2^(5/2 - m)*c^3*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(3*f*(1 + 2*
m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
(3*f)
```

Rubi [A] time = 0.336136, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m\right.}{3f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - m
),x]
```

```
[Out] (2^(5/2 - m)*c^3*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2
*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(
1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(3*f*(1 + 2*
m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
(3*f)
```

Rule 2973

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
 b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2
```

```
^(-1)] && NeQ[m + n + 1, 0]
```

Rule 2745

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
 + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx}{3f}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx}{3f}$$

$$= \frac{2^{\frac{5}{2}-m}c^3(3A-2B(1-m))\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2\right.}{}$$

Mathematica [C] time = 53.1319, size = 5163, normalized size = 29.84

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(2 - m),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.522, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

Rubi [A] time = 0.273913, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2967, 2854}

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c-c\sin(e+fx))^{-}$$
$$= \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{-3+n}}{f}$$

Mathematica [A] time = 0.52944, size = 63, normalized size = 1.85

$$\frac{a^3B(14\sin(2(e+fx))-\sin(4(e+fx))+14\cos(e+fx)-6\cos(3(e+fx)))(c-c\sin(e+fx))^n}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 +
 n)*Sin[e + f*x]),x]
```

```
[Out] (a^3*B*(c - c*Sin[e + f*x])^n*(14*Cos[e + f*x] - 6*Cos[3*(e + f*x)] + 14*Si
n[2*(e + f*x)] - Sin[4*(e + f*x)]))/(8*f)
```

Maple [F] time = 2.334, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^3\left(c-c\sin\left(fx+e\right)\right)^n\left(B\left(3-n\right)-B\left(4+n\right)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(B(n+4)\sin\left(fx+e\right)+B(n-3)\right)\left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

Fricas [B] time = 1.98062, size = 185, normalized size = 5.44

$$-\frac{\left(3\,Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)+\left(Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(-c\sin\left(fx+e\right)+c\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] -(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) + (B*a^3*cos(f*x + e)^3 - 4
*B*a^3*cos(f*x + e))*sin(f*x + e))*(-c*sin(f*x + e) + c)^n/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(B(n+4)\sin\left(fx+e\right)+B(n-3)\right)\left(a\sin\left(fx+e\right)+a\right)^3\left(-c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] integrate(-(B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c*
sin(f*x + e) + c)^n, x)
```

3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

Rubi [A] time = 0.238354, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2967, 2854}

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*Si
n[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(c+c\sin(e+fx))$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(e+fx))^{-3}}{f}$$

Mathematica [A] time = 1.1436, size = 67, normalized size = 1.97

$$-\frac{a^3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(c(\sin(e+fx)+1))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 +
 n)*Sin[e + f*x]),x]
```

```
[Out] -((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)
```

Maple [F] time = 2.428, size = 0, normalized size = 0.

$$\int \left(a-a\sin\left(fx+e\right)\right)^3\left(c+c\sin\left(fx+e\right)\right)^n\left(B\left(3-n\right)+B\left(4+n\right)\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(B(n+4)\sin\left(fx+e\right)-B(n-3)\right)\left(a\sin\left(fx+e\right)-a\right)^3\left(c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="maxima")
```

```
[Out] -integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

Fricas [B] time = 2.02568, size = 182, normalized size = 5.35

$$\frac{\left(3\,Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)-\left(Ba^3\cos\left(fx+e\right)^3-4\,Ba^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(c\sin\left(fx+e\right)+c\right)^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="fricas")
```

```
[Out] (3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4*
B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**3*(c+c*sin(f*x+e))**n*(B*(3-n)+B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(B(n+4)\sin\left(fx+e\right)-B(n-3)\right)\left(a\sin\left(fx+e\right)-a\right)^3\left(c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)
),x, algorithm="giac")
```

```
[Out] integrate(-(B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*s
in(f*x + e) + c)^n, x)
```

3.220 $\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^3 (B(-3 + m) - B(4 + m)\sin(e + fx))\,dx$

Optimal. Leaf size=33

$$\frac{a^3 B c^3 \cos^7(e + fx)(a\sin(e + fx) + a)^{m-3}}{f}$$

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

Rubi [A] time = 0.237485, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2967, 2854}

$$\frac{a^3 B c^3 \cos^7(e + fx)(a\sin(e + fx) + a)^{m-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a+a\sin(e+f$$
$$= \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))}{f}$$

Mathematica [A] time = 1.12281, size = 66, normalized size = 2.

$$\frac{Bc^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)(a(\sin(e+fx)+1))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4
+ m)*Sin[e + f*x]),x]
```

```
[Out] (B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/f
```

Maple [F] time = 2.378, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^3\left(B\left(m-3\right)-B\left(4+m\right)\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(m-3)-B*(4+m)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(m-3)-B*(4+m)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B(m+4)\sin\left(fx+e\right)-B(m-3)\right)\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

Fricas [B] time = 2.09549, size = 184, normalized size = 5.58

$$-\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)-\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] -(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 - 4
*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3*(B*(-3+m)-B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B(m+4)\sin\left(fx+e\right)-B(m-3)\right)\left(c\sin\left(fx+e\right)-c\right)^3\left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*si
n(f*x + e) + a)^m, x)
```

3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$

Optimal. Leaf size=35

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

Rubi [A] time = 0.238397, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2967, 2854}

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*S
in[e + f*x]),x]
```

```
[Out] -((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

Rule 2967

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2854

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; Fr
eeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c
*(m + p + 1), 0]
```

Rubi steps

$$\int (a - a\sin(e+fx))^m (c + c\sin(e+fx))^3 (B(-3+m) + B(4+m)\sin(e+fx))\,dx = \left(a^3c^3\right)\int \cos^6(e+fx)(a - a\sin(e+f$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a - a\sin(e+fx}{f}$$

Mathematica [A] time = 0.544851, size = 61, normalized size = 1.74

$$\frac{Bc^3(-14\sin(2(e+fx)) + \sin(4(e+fx)) - 14\cos(e+fx) + 6\cos(3(e+fx)))(a - a\sin(e+fx))^m}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4
+ m)*Sin[e + f*x]),x]
```

```
[Out] (B*c^3*(a - a*Sin[e + f*x])^m*(-14*Cos[e + f*x] + 6*Cos[3*(e + f*x)] - 14*S
in[2*(e + f*x)] + Sin[4*(e + f*x)]))/(8*f)
```

Maple [F] time = 2.238, size = 0, normalized size = 0.

$$\int \left(a - a\sin\left(fx+e\right)\right)^m \left(c + c\sin\left(fx+e\right)\right)^3 \left(B(m-3) + B(4+m)\sin\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(m-3)+B*(4+m)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(m-3)+B*(4+m)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B(m+4)\sin\left(fx+e\right) + B(m-3)\right)\left(c\sin\left(fx+e\right) + c\right)^3 \left(-a\sin\left(fx+e\right) + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

Fricas [B] time = 2.03624, size = 184, normalized size = 5.26

$$\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)+\left(Bc^3\cos\left(fx+e\right)^3-4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(-a\sin\left(fx+e\right)+a\right)^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="fricas")
```

```
[Out] (3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) + (B*c^3*cos(f*x + e)^3 - 4*
B*c^3*cos(f*x + e))*sin(f*x + e))*(-a*sin(f*x + e) + a)^m/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**3*(B*(-3+m)+B*(4+m)*sin(f*x
+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B(m+4)\sin\left(fx+e\right)+B(m-3)\right)\left(c\sin\left(fx+e\right)+c\right)^3\left(-a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e
)),x, algorithm="giac")
```

```
[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*s
in(f*x + e) + a)^m, x)
```

3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$

Optimal. Leaf size=36

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

```
[Out] (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

Rubi [A] time = 0.131828, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2970}

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

Rule 2970

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0]
 && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2
^(-1)]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx=\frac{B\cos(e+fx)(a+a\sin(e+fx))^n}{f}$$

Mathematica [A] time = 0.463766, size = 36, normalized size = 1.

$$\frac{B\cos(e+fx)(a(\sin(e+fx)+1))^m(c-c\sin(e+fx))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 +
 m + n)*Sin[e + f*x]),x]
```

```
[Out] (B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f
```

Maple [F] time = 2.552, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\left(B\left(m-n\right)-B\left(m+n+1\right)\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(m+n+1)*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(m+n+1)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(B(m+n+1)\sin\left(fx+e\right)-B(m-n)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] -integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m*
(-c*sin(f*x + e) + c)^n, x)
```

Fricas [A] time = 2.05146, size = 88, normalized size = 2.44

$$\frac{\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{n}B\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n*(B*(m-n)-B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$

Optimal. Leaf size=37

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

```
[Out] -((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

Rubi [A] time = 0.123455, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.022, Rules used = {2970}

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n
)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

Rule 2970

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0]
 && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2
^(-1)]
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx=-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m}{f}$$

Mathematica [A] time = 0.463715, size = 37, normalized size = 1.

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c(\sin(e+fx)+1))^n}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 +
 m + n)*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^n*(a - a*Sin[e + f*x])^m)/f)
```

Maple [F] time = 2.543, size = 0, normalized size = 0.

$$\int \left(a-a\sin\left(fx+e\right)\right)^m\left(c+c\sin\left(fx+e\right)\right)^n\left(B\left(m-n\right)+B\left(m+n+1\right)\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(m+n+1)*sin(f*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(m+n+1)*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B(m+n+1)\sin\left(fx+e\right)+B(m-n)\right)\left(-a\sin\left(fx+e\right)+a\right)^m\left(c\sin\left(fx+e\right)+c\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="maxima")
```

```
[Out] integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m*
(c*sin(f*x + e) + c)^n, x)
```

Fricas [A] time = 2.07714, size = 89, normalized size = 2.41

$$-\frac{\left(-a\sin\left(fx+e\right)+a\right)^m\left(c\sin\left(fx+e\right)+c\right)^nB\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="fricas")
```

```
[Out] -(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**n*(B*(m-n)+B*(1+m+n)*sin(f*
x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+
e)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\cos(c}{12d}$$

```
[Out] (a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5*
d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[c +
 d*x]^5)/(3*d)
```

Rubi [A] time = 0.185963, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 2633, 2635, 8}

$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\cos(c}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5*
d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[c +
 d*x]^5)/(3*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin^3(c+dx)+2a^3A\sin^4(c+dx)-2a^3A\sin^6(c+dx\right.$$
$$= \left(a^3A\right)\int \sin^3(c+dx)\,dx-\left(a^3A\right)\int \sin^7(c+dx)\,dx+\left(2a^3A\right.$$
$$= -\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx}{3d}$$
$$= -\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos^7(c+dx}{7d}$$
$$= \frac{3}{4}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos}{7}$$
$$= \frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\cos}{7}$$

Mathematica [A] time = 0.15218, size = 87, normalized size = 0.62

$$\frac{a^3A(-210\sin(2(c+dx))-210\sin(4(c+dx))+70\sin(6(c+dx))-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c-}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Co
s[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c
+ d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.031, size = 158, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^3A\cos(dx+c)}{7}\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6\,(\sin(dx+c))^4}{5}+\frac{8\,(\sin(dx+c))^2}{5}\right)-2\,a^3A\left(-1/6\,\left((\sin(dx+c))^5+5/4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/7*a^3*A*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*
x+c)-2*a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+
c)+5/16*d*x+5/16*c)+2*a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+
3/8*d*x+3/8*c)-1/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A] time = 0.992001, size = 212, normalized size = 1.51

$$-\frac{96\left(5\cos(dx+c)^7-21\cos(dx+c)^5+35\cos(dx+c)^3-35\cos(dx+c)\right)Aa^3-1120\left(\cos(dx+c)^3-3\cos(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*
cos(d*x + c))*A*a^3 - 1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 35*(4*
sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*
c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a
^3)/d
```

Fricas [A] time = 2.08523, size = 269, normalized size = 1.92

$$-\frac{120\,Aa^3\cos(dx+c)^7-504\,Aa^3\cos(dx+c)^5+560\,Aa^3\cos(dx+c)^3-105\,Aa^3dx-35\left(8\,Aa^3\cos(dx+c)^5-14\,Aa^3\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*cos
(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d*x
 + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 15.4764, size = 440, normalized size = 3.14

$$\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{8} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2} - \frac{5Aa^3 \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/8 - 15*A*a**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/8 + 3*A*a**3*x*sin(c + d*x)**4/4 - 15*A*a**3*x*sin(c + d*x)**2*
cos(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 - 5*A*a**3
*x*cos(c + d*x)**6/8 + 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*sin(c + d*x)**
6*cos(c + d*x)/d + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 2*A*a**3*
sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**
3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin(c + d*
x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 5*A*a
**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)
**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d*x)**3/(3*
d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, True))
```

Giac [A] time = 1.13997, size = 177, normalized size = 1.26

$$\frac{1}{8}Aa^3x - \frac{Aa^3\cos(7dx+7c)}{448d} + \frac{7Aa^3\cos(5dx+5c)}{320d} - \frac{5Aa^3\cos(3dx+3c)}{192d} - \frac{13Aa^3\cos(dx+c)}{64d} + \frac{Aa^3\sin(6dx+}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

[Out] 1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c)
/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A*a
^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2*d*
x + 2*c)/d

3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin(c+}{1}$$

```
[Out] (3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)/
(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.168042, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 2635, 8, 2633}

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin(c+}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)/
(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin^2(c+dx)+2a^3A\sin^3(c+dx)-2a^3A\sin^5(c+
$$

$$= \left(a^3A\right)\int \sin^2(c+dx)\,dx-\left(a^3A\right)\int \sin^6(c+dx)\,dx+\left(2a^3
$$

$$= -\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^3A\cos(c+dx)\sin^5(c+d
$$

$$= \frac{1}{2}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{a^3A\,\mathrm{co}
$$

$$= \frac{1}{2}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{3a^3A\,\mathrm{c}
$$

$$= \frac{3}{16}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{3a^3A
$$

Mathematica [A] time = 0.107905, size = 77, normalized size = 0.64

$$\frac{a^3A(-15\sin(2(c+dx))-45\sin(4(c+dx))+5\sin(6(c+dx))-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5
*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

Maple [A] time = 0.032, size = 136, normalized size = 1.1

$$\frac{1}{d}\left(-a^3A\left(-\frac{\cos(dx+c)}{6}\left((\sin(dx+c))^5+\frac{5\,(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{2\,a^3A\cos(dx+c)}{5}\left(\frac{8}{3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c
)+5/16*d*x+5/16*c)+2/5*a^3*A*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
-2/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)+a^3*A*(-1/2*cos(d*x+c)*sin(d*x+c)+1/
2*d*x+1/2*c))
```

Maxima [A] time = 0.955482, size = 186, normalized size = 1.54

$$\frac{128\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)Aa^3+640\left(\cos(dx+c)^3-3\cos(dx+c)\right)Aa^3-5\left(4\sin(2\,dx\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 +
 640*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 + 60
*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d*x
+ 2*c - sin(2*d*x + 2*c))*A*a^3)/d
```

Fricas [A] time = 1.95713, size = 227, normalized size = 1.88

$$\frac{96\,Aa^3\cos(dx+c)^5-160\,Aa^3\cos(dx+c)^3+45\,Aa^3dx+5\left(8\,Aa^3\cos(dx+c)^5-26\,Aa^3\cos(dx+c)^3+9\,Aa^3\cos(dx\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x +
5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c))
*sin(d*x + c))/d
```

Sympy [A] time = 9.33865, size = 359, normalized size = 2.97

$$\begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{16} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{Aa^3x\sin^2(c+dx)}{2} - \frac{5Aa^3x\cos^6(c+dx)}{16} + \frac{Aa^3x\cos^2(c+d}{2} \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-5*A*a**3*x*sin(c + d*x)**6/16 - 15*A*a**3*x*sin(c + d*x)**4*cos
(c + d*x)**2/16 - 15*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + A*a**3*x
*sin(c + d*x)**2/2 - 5*A*a**3*x*cos(c + d*x)**6/16 + A*a**3*x*cos(c + d*x)*
*2/2 + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 2*A*a**3*sin(c + d*x
)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*A*
a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*A*a**3*sin(c + d*x)**2*cos(c
 + d*x)/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a**3*sin(c + d
*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**3*cos(c
+ d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**2,
 True))
```

Giac [A] time = 1.12209, size = 153, normalized size = 1.26

$$\frac{3}{16}Aa^3x + \frac{Aa^3\cos(5dx+5c)}{40d} - \frac{Aa^3\cos(3dx+3c)}{24d} - \frac{Aa^3\cos(dx+c)}{4d} + \frac{Aa^3\sin(6dx+6c)}{192d} - \frac{3Aa^3\sin(4dx+4c}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)/
d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^3*
sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d
```

3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

```
[Out] (a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*
x]^3)/(2*d)
```

Rubi [A] time = 0.116473, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2966, 2638, 2635, 8, 2633}

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d)
 - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*
x]^3)/(2*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(a^3A\sin(c+dx)+2a^3A\sin^2(c+dx)-2a^3A\sin^4(c+dx\right.$$
$$= \left(a^3A\right)\int \sin(c+dx)\,dx - \left(a^3A\right)\int \sin^5(c+dx)\,dx + \left(2a^3A\right.$$
$$= -\frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cos(c+dx)\sin(c+dx)}{d} + \frac{a^3A\cos(}{}$$
$$= a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos(c}{}$$
$$= \frac{1}{4}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos(}{}$$

Mathematica [A] time = 0.482205, size = 55, normalized size = 0.57

$$\frac{a^3A(-90\cos(c+dx)-25\cos(3(c+dx))+3(-5\sin(4(c+dx))+\cos(5(c+dx))+20dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*d*x + Cos[5*(c + d*x
)] - 5*Sin[4*(c + d*x)])))/(240*d)
```

Maple [A] time = 0.026, size = 117, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^3A\cos(dx+c)}{5}\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)-2a^3A\left(-1/4\left((\sin(dx+c))^3+3/2\sin(dx+c)\right)\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a^3*A*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-2*a^3*A*(-1/4
*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2*a^3*A*(-1/2*cos(
d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-a^3*A*cos(d*x+c))
```

Maxima [A] time = 1.02804, size = 151, normalized size = 1.57

$$\frac{16\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)Aa^3-15(12dx+12c+\sin(4dx+4c)-8\sin(2dx+2c))Aa^3}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 -
15*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d
*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*cos(d*x + c))/d
```

Fricas [A] time = 2.17571, size = 188, normalized size = 1.96

$$\frac{12Aa^3\cos(dx+c)^5-40Aa^3\cos(dx+c)^3+15Aa^3dx-15\left(2Aa^3\cos(dx+c)^3-Aa^3\cos(dx+c)\right)\sin(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 15
*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 6.07396, size = 267, normalized size = 2.78

$$\begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2} + Aa^3x\sin^2(c+dx) - \frac{3Aa^3x\cos^4(c+dx)}{4} + Aa^3x\cos^2(c+dx) + \frac{Aa^3\sin^4(c+dx)\cos(c+dx)}{d} \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 + A*
a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**3*s
in(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*x)**
3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c + d*x)
*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*x)/d,
Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))
```

Giac [A] time = 1.11934, size = 104, normalized size = 1.08

$$\frac{1}{4}Aa^3x + \frac{Aa^3\cos(5dx+5c)}{80d} - \frac{5Aa^3\cos(3dx+3c)}{48d} - \frac{3Aa^3\cos(dx+c)}{8d} - \frac{Aa^3\sin(4dx+4c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/d
 - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d
```

3.227 $\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{4d}+\frac{5a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3Ax$$

```
[Out] (5*a^3*A*x)/8 - (5*a^3*A*Cos[c + d*x]^3)/(12*d) + (5*a^3*A*Cos[c + d*x]*Sin
[c + d*x])/(8*d) - (A*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(4*d)
```

Rubi [A] time = 0.105893, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2736, 2678, 2669, 2635, 8}

$$-\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{4d}+\frac{5a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*x)/8 - (5*a^3*A*Cos[c + d*x]^3)/(12*d) + (5*a^3*A*Cos[c + d*x]*Sin
[c + d*x])/(8*d) - (A*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(4*d)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2678

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos
[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g,
m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ[2
*m, 2*p]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= (aA)\int \cos^2(c+dx)(a+a\sin(c+dx))^2\,dx\\ &= -\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2A\right)\int \cos^2(c+dx)(a+a\\ &= -\frac{5a^3A\cos^3(c+dx)}{12d}-\frac{A\cos^3(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^3A\right)\\ &= -\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{A\cos^3(c+dx)\left(a\right.}{}\\ &= \frac{5}{8}a^3Ax-\frac{5a^3A\cos^3(c+dx)}{12d}+\frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d}-\frac{A\cos^3(c}{}\end{aligned}$$

Mathematica [A] time = 0.357282, size = 54, normalized size = 0.66

$$\frac{a^3A(24\sin(2(c+dx))-3\sin(4(c+dx))-48\cos(c+dx)-16\cos(3(c+dx))+60dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x)
] - 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.026, size = 89, normalized size = 1.1

$$\frac{1}{d}\left(-a^3A\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^3A\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}-2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+2
/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)-2*a^3*A*cos(d*x+c)+a^3*A*(d*x+c))
```

Maxima [A] time = 0.960309, size = 116, normalized size = 1.41

$$-\frac{64\left(\cos(dx+c)^3-3\cos(dx+c)\right)Aa^3+3\left(12dx+12c+\sin(4dx+4c)-8\sin(2dx+2c)\right)Aa^3-96(dx+c)Aa^3+1}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(64*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3*c
os(d*x + c))/d
```

Fricas [A] time = 1.94899, size = 155, normalized size = 1.89

$$-\frac{16Aa^3\cos(dx+c)^3-15Aa^3dx+3\left(2Aa^3\cos(dx+c)^3-5Aa^3\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3 -
 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.45875, size = 196, normalized size = 2.39

$$\begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{8} - \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} - \frac{3Aa^3x\cos^4(c+dx)}{8} + Aa^3x + \frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Aa^3\sin^2(c+dx)\cos(c+dx)}{d} + \\ x(-A\sin(c)+A)(a\sin(c)+a)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(c
 + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c + d
*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*A*a
**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d) - 2
*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3, Tr
ue))
```

Giac [A] time = 1.12431, size = 104, normalized size = 1.27

$$\frac{5}{8}Aa^3x - \frac{Aa^3\cos(3dx+3c)}{6d} - \frac{Aa^3\cos(dx+c)}{2d} - \frac{Aa^3\sin(4dx+4c)}{32d} + \frac{Aa^3\sin(2dx+2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/3
2*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d
```

3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+a^3Ax$$

```
[Out] a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*A
*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

Rubi [A] time = 0.104153, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2966, 3770, 2635, 8, 2633}

$$-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*A
*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(2a^3A + a^3A\csc(c+dx) - 2a^3A\sin^2(c+dx) - a^3A\sin^3(\right.$$
$$= 2a^3Ax + \left(a^3A\right)\int \csc(c+dx)\,dx - \left(a^3A\right)\int \sin^3(c+dx)\,dx$$
$$= 2a^3Ax - \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx)\sin(c+}{d}$$
$$= a^3Ax - \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3A\cos(c+dx)}{d} - \frac{a^3A}{}$$

Mathematica [A] time = 0.153143, size = 74, normalized size = 0.97

$$\frac{a^3A\left(9\cos(c+dx) - \cos(3(c+dx)) + 6\left(\sin(2(c+dx)) + 2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2c + 2dx\right)\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(-2*c + 2*d*x - 2*Log[Cos[(c
+ d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12*d)
```

Maple [A] time = 0.05, size = 99, normalized size = 1.3

$$\frac{A\cos(dx+c)(\sin(dx+c))^2a^3}{3d} + \frac{2a^3A\cos(dx+c)}{3d} + \frac{a^3A\cos(dx+c)\sin(dx+c)}{d} + a^3Ax + \frac{a^3Ac}{d} + \frac{a^3A\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/3/d*A*cos(d*x+c)*sin(d*x+c)^2*a^3+2/3*a^3*A*cos(d*x+c)/d+a^3*A*cos(d*x+c)
*sin(d*x+c)/d+a^3*A*x+1/d*a^3*A*c+1/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.969253, size = 115, normalized size = 1.51

$$-\frac{2\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)Aa^{3}+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-12\left(dx+c\right)Aa^{3}+6\,Aa^{3}\log\left(\cot\left(dx+c\right)+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(2*d*x + 2*c - sin(2*d*
x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*log(cot(d*x + c) + csc(d*x +
 c)))/d
```

Fricas [A] time = 2.0146, size = 247, normalized size = 3.25

$$-\frac{2\,Aa^{3}\cos\left(dx+c\right)^{3}-6\,Aa^{3}dx-6\,Aa^{3}\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,Aa^{3}\cos\left(dx+c\right)+3\,Aa^{3}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x +
 c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^3*
log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15356, size = 144, normalized size = 1.89

$$\frac{3\,(dx+c)Aa^3+3\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Aa^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*A*a^
3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1/2
*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=79

$$\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{1}{2}a^3Ax$$

```
[Out] -(a^3*A*x)/2 - (2*a^3*A*ArcTanh[Cos[c + d*x]])/d + (2*a^3*A*Cos[c + d*x])/d
 - (a^3*A*Cot[c + d*x])/d + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.178912, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2950, 2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{1}{2}a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```


```
[Out] -(a^3*A*x)/2 - (2*a^3*A*ArcTanh[Cos[c + d*x]])/d + (2*a^3*A*Cos[c + d*x])/d
 - (a^3*A*Cot[c + d*x])/d + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```


Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = (aA)\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$$

$$= \frac{A\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-\right.}{a}$$

$$= \left(a^3A\right)\int \csc^2(c+dx)\,dx - \left(a^3A\right)\int \sin^2(c+dx)\,dx + \left(2a^3\right.$$

$$= -\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^3A\cos(c+dx)}{d} + \frac{a^3A\cos}{}$$

$$= -\frac{1}{2}a^3Ax - \frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^3A\cos(c+dx)}{d}$$

Mathematica [A] time = 0.186276, size = 77, normalized size = 0.97

$$\frac{a^3A\left(-8\sin(c)\sin(dx)+\sin(2(c+dx))+8\cos(c)\cos(dx)-4\cot(c+dx)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\cos\left(\frac{1}{2}(c+d\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (a^3*A*(-2*c - 2*d*x + 8*Cos[c]*Cos[d*x] - 4*Cot[c + d*x] - 8*Log[Cos[(c +
d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] - 8*Sin[c]*Sin[d*x] + Sin[2*(c + d*x)]))
/(4*d)
```

Maple [A] time = 0.046, size = 95, normalized size = 1.2

$$\frac{a^3A\cos(dx+c)\sin(dx+c)}{2d}-\frac{a^3Ax}{2}-\frac{a^3Ac}{2d}+2\frac{a^3A\cos(dx+c)}{d}+2\frac{a^3A\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{a^3A\cot(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/2*a^3*A*x-1/2/d*a^3*A*c+2*a^3*A*cos(d*x
+c)/d+2/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3*A*cot(d*x+c)/d
```

Maxima [A] time = 0.99272, size = 112, normalized size = 1.42

$$-\frac{(2dx+2c-\sin(2dx+2c))Aa^3+4Aa^3(\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1))-8Aa^3\cos(dx+c)+\frac{4Aa^3}{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) - 8*A*a^3*cos(d*x + c) + 4*A*a^3/tan(d*x + c))/
d
```

Fricas [A] time = 1.95138, size = 297, normalized size = 3.76

$$-\frac{Aa^3\cos(dx+c)^3 + 2\,Aa^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2\,Aa^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + Aa^3\,c}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x + c)
 + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18799, size = 207, normalized size = 2.62

$$-\frac{(dx+c)Aa^3 - 4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Aa^3}{\left(\mathrm{ta}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*((d*x + c)*A*a^3 - 4*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - A*a^3*tan(
1/2*d*x + 1/2*c) + (4*A*a^3*tan(1/2*d*x + 1/2*c) + A*a^3)/tan(1/2*d*x + 1/2
*c) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - A*
a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

```
[Out] -2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d -
 (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rubi [A] time = 0.120552, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2966, 3767, 8, 3768, 3770, 2638}

$$\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d -
 (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= \int \left(-2a^3A+2a^3A\csc^2(c+dx)+a^3A\csc^3(c+dx)-a^3A\sin\right.\\&= -2a^3Ax+\left(a^3A\right)\int \csc^3(c+dx)\,dx-\left(a^3A\right)\int \sin(c+dx)\,d\\&= -2a^3Ax+\frac{a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}+\\&= -2a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}-\frac{2a}{}\end{aligned}$$

Mathematica [A] time = 0.034722, size = 142, normalized size = 1.82

$$-\frac{a^3A\sin(c)\sin(dx)}{d}+\frac{a^3A\cos(c)\cos(dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3A\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A*
Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*Lo
```

```
g[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*Sin[
c]*Sin[d*x])/d
```

Maple [A] time = 0.058, size = 94, normalized size = 1.2

$$\frac{a^3A\cos(dx+c)}{d} - 2a^3Ax - 2\frac{Aa^3c}{d} - 2\frac{a^3A\cot(dx+c)}{d} - \frac{a^3A\cot(dx+c)\csc(dx+c)}{2d} + \frac{a^3A\ln(\csc(dx+c)-\cot($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] a^3*A*cos(d*x+c)/d-2*a^3*A*x-2/d*a^3*A*c-2*a^3*A*cot(d*x+c)/d-1/2*a^3*A*cot
(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.97063, size = 122, normalized size = 1.56

$$-\frac{8(dx+c)Aa^3 - Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1)\right) - 4Aa^3\cos(dx+c) + \frac{8Aa^3}{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/4*(8*(d*x + c)*A*a^3 - A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 4*A*a^3*cos(d*x + c) + 8*A*a^3
/tan(d*x + c))/d
```

Fricas [B] time = 2.028, size = 377, normalized size = 4.83

$$-\frac{8Aa^3dx\cos(dx+c)^2 - 4Aa^3\cos(dx+c)^3 - 8Aa^3dx - 8Aa^3\cos(dx+c)\sin(dx+c) + 2Aa^3\cos(dx+c) + \left(Aa^3 c\right.}{4\left(d\cos(dx+c)^2 - a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x - 8
*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x +
c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^3)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22309, size = 185, normalized size = 2.37

$$\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)Aa^3+4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{6\,Aa}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(ta
n(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x +
1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

```
[Out] -(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a^
3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

Rubi [A] time = 0.130712, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 3770, 3768, 3767}

$$-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a^
3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A-2a^3A\csc(c+dx)+2a^3A\csc^3(c+dx)+a^3A\csc\right.$$

$$= -a^3Ax+\left(a^3A\right)\int \csc^4(c+dx)\,dx-\left(2a^3A\right)\int \csc(c+dx)\,d$$

$$= -a^3Ax+\frac{2a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)\csc(c}{d}$$

$$= -a^3Ax+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A}{}$$

Mathematica [A] time = 0.461471, size = 141, normalized size = 1.81

$$-\frac{a^3A\left(-8\tan\left(\frac{1}{2}(c+dx)\right)+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-6\sec^2\left(\frac{1}{2}(c+dx)\right)+24\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-24\,\mathrm{lo}\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] -(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 24*Log
[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 - 8*Cs
c[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 8*T
an[(c + d*x)/2]))/(24*d)
```

Maple [A] time = 0.057, size = 103, normalized size = 1.3

$$-a^3Ax-\frac{Aa^3c}{d}-\frac{a^3A\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{a^3A\cot(dx+c)\csc(dx+c)}{d}-\frac{2\,a^3A\cot(dx+c)}{3\,d}-\frac{a^3A\cot(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] -a^3*A*x-1/d*a^3*A*c-1/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3*A*cot(d*x+c)*c
sc(d*x+c)/d-2/3*a^3*A*cot(d*x+c)/d-1/3/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 0.988819, size = 158, normalized size = 2.03

$$-\frac{6\,(dx+c)Aa^3 - 3\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - 6\,Aa^3(\log\left(\cos\left(dx+c\right)+1\right) - \mathrm{lo}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6*(6*(d*x + c)*A*a^3 - 3*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*A*a^3*(log(cos(d*x + c) +
1) - log(cos(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3
)/d
```

Fricas [B] time = 1.93814, size = 435, normalized size = 5.58

$$-\frac{4\,Aa^3\cos\left(dx+c\right)^3 - 6\,Aa^3\cos\left(dx+c\right) - 3\left(Aa^3\cos\left(dx+c\right)^2 - Aa^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) + 3\left(Aa^3\,\mathrm{c}\right.}{6\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c)
^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x + c
)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*cos(d
*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17242, size = 203, normalized size = 2.6

$$\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)Aa^3-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,Aa^3\tan\left(\frac{1}{2}\,dx+\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(d
*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2*d
*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1/2*
c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] (5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (3
*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x]^
3)/(4*d)
```

Rubi [A] time = 0.147784, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2966, 3770, 3767, 8, 3768}

$$-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (3
*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x]^
3)/(4*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A\csc(c+dx) - 2a^3A\csc^2(c+dx) + 2a^3A\csc^4(c+d\right.$$

$$= -\left(\left(a^3A\right)\int \csc(c+dx)\,dx\right) + \left(a^3A\right)\int \csc^5(c+dx)\,dx - \left(2a\right.$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot(c}{}$$

$$= \frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot(}{}$$

Mathematica [B] time = 0.0689217, size = 210, normalized size = 2.44

$$a^3A\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3\sec^2\left(\frac{1}{2}(c+dx\right.}{32d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d
*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[
(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2
```

```
]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

Maple [A] time = 0.058, size = 109, normalized size = 1.3

$$-\frac{5\,a^3A\ln(\csc(dx+c)-\cot(dx+c))}{8\,d}+\frac{2\,a^3A\cot(dx+c)}{3\,d}-\frac{2\,a^3A\cot(dx+c)(\csc(dx+c))^2}{3\,d}-\frac{a^3A\cot(dx+c)(\csc}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] -5/8/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))+2/3*a^3*A*cot(d*x+c)/d-2/3/d*a^3*A*c
ot(d*x+c)*csc(d*x+c)^2-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a^3*A*cot(d*
x+c)*csc(d*x+c)/d
```

Maxima [A] time = 0.973548, size = 196, normalized size = 2.28

$$\frac{3\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)+24\,Aa^3(\log(\cos(dx+c)+1)-\text{lo}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 24
*A*a^3*(log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 96*A*a^3/tan(d*x +
 c) - 32*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3)/d
```

Fricas [B] time = 1.98415, size = 431, normalized size = 5.01

$$-\frac{32\,Aa^3\cos(dx+c)^3\sin(dx+c)-18\,Aa^3\cos(dx+c)^3+30\,Aa^3\cos(dx+c)-15\left(Aa^3\cos(dx+c)^4-2\,Aa^3\cos(dx\right.}{48\left(d\cos(dx+c)^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30*
A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*
a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d
*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*co
s(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24556, size = 235, normalized size = 2.73

$$\frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-48\,A}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 2
4*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 48*A*a^3*tan(1/2*d*x + 1/2*c) + (250*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 48*A*a
^3*tan(1/2*d*x + 1/2*c)^3 - 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$-\frac{a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3A\cot(c+dx)}{4d}$$

```
[Out] (a^3*A*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (a^3
*A*Cot[c + d*x]^5)/(5*d) + (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^3*A
*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.234037, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2950, 2709, 3767, 8, 3768, 3770}

$$-\frac{a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3A\cot(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```


```
[Out] (a^3*A*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (a^3
*A*Cot[c + d*x]^5)/(5*d) + (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^3*A
*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d)
```


Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \left(a^3A^3\right)\int \frac{\cot^6(c+dx)}{(A-A\sin(c+dx))^2}\,dx$$

$$= \frac{a^3\int\left(-A^4\csc^2(c+dx)-2A^4\csc^3(c+dx)+2A^4\csc^5(c+d\right.}{A^3}$$

$$= -\left(\left(a^3A\right)\int\csc^2(c+dx)\,dx\right)+\left(a^3A\right)\int\csc^6(c+dx)\,dx-\left(2\right.$$

$$= \frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot^5(c}{5d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot^5(c}{5d}$$

Mathematica [B] time = 0.074181, size = 268, normalized size = 2.55

$$a^3A\left(-\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{30d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{16d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{16d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((7*Cot[(c + d*x)/2])/(30*d) + Csc[(c + d*x)/2]^2/(16*d) - (19*Cot[(c
 + d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - Csc[(c + d*x)/2]^4/(32*d) - (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + Log[Cos[(c + d*x)/2]]/(4*d) - Log
[Sin[(c + d*x)/2]]/(4*d) - Sec[(c + d*x)/2]^2/(16*d) + Sec[(c + d*x)/2]^4/(
32*d) - (7*Tan[(c + d*x)/2])/(30*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d))
```

Maple [A] time = 0.06, size = 132, normalized size = 1.3

$$\frac{7a^3A\cot(dx+c)}{15d}+\frac{a^3A\cot(dx+c)\csc(dx+c)}{4d}-\frac{a^3A\ln(\csc(dx+c)-\cot(dx+c))}{4d}-\frac{a^3A\cot(dx+c)(\csc(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 7/15*a^3*A*cot(d*x+c)/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/4/d*a^3*A*ln(cs
c(d*x+c)-cot(d*x+c))-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d-1/5/d*a^3*A*cot(d*
x+c)*csc(d*x+c)^4-4/15/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2
```

Maxima [A] time = 0.994365, size = 236, normalized size = 2.25

$$\frac{15\,Aa^3\left(\frac{2\left(3\cos(dx+c)^3-5\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-60\,Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-\log(\cos(d\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log
(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 10*ta
n(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Fricas [B] time = 2.09352, size = 520, normalized size = 4.95

$$\frac{56\,Aa^3\cos(dx+c)^5 - 80\,Aa^3\cos(dx+c)^3 + 15\left(Aa^3\cos(dx+c)^4 - 2\,Aa^3\cos(dx+c)^2 + Aa^3\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{120\,(d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(56*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 + 15*(A*a^3*cos(d*
x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(A*a^3*cos(d*x + c)^3 + A*a^3*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20684, size = 235, normalized size = 2.24

$$\frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 15\,Aa^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 + 25\,Aa^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 90\,A}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 2
5*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) -
 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a
^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*tan(
1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal. Leaf size=130

$$-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c+d}{24}$$

```
[Out] (3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d)
- (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc[c
+ d*x]^5)/(6*d)
```

Rubi [A] time = 0.19665, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 3768, 3770, 3767}

$$-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}+\frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c+d}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] (3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d)
- (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc[c
+ d*x]^5)/(6*d)
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \int \left(-a^3A\csc^3(c+dx)-2a^3A\csc^4(c+dx)+2a^3A\csc^6(c+\right.$$

$$= -\left(\left(a^3A\right)\int \csc^3(c+dx)\,dx\right)+\left(a^3A\right)\int \csc^7(c+dx)\,dx-($$

$$= \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx}{6d}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{2a^3A\cot}{5}$$

$$= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{2a^3A\cot}{5}$$

$$= \frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{2a^3A\cot}{5}$$

Mathematica [B] time = 0.0797904, size = 306, normalized size = 2.35

$$a^3A\left(-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d}+\frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^6\left(\frac{1}{2}(c+\right.}{384d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

```
[Out] a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Cot[
```

```
(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (3*L
og[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Sec[(c
 + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/(384*
d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2
40*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

Maple [A] time = 0.062, size = 155, normalized size = 1.2

$$\frac{3\,a^3A\cot(dx+c)\csc(dx+c)}{16\,d}-\frac{3\,a^3A\ln(\csc(dx+c)-\cot(dx+c))}{16\,d}+\frac{4\,a^3A\cot(dx+c)}{15\,d}+\frac{2\,a^3A\cot(dx+c)(\csc(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

```
[Out] 3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-3/16/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))+4
/15*a^3*A*cot(d*x+c)/d+2/15/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2-2/5/d*a^3*A*cot
(d*x+c)*csc(d*x+c)^4-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d-5/24*a^3*A*cot(d*x
+c)*csc(d*x+c)^3/d
```

Maxima [A] time = 0.990973, size = 279, normalized size = 2.15

$$\frac{5\,Aa^3\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)-120\,Aa^3\left(\frac{2\,\cos(dx+c}{\cos(dx+c)^2-}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(5*A*a^3*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c))
/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(d*
x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 120*A*a^3*(2*cos(d*x + c)/(cos(d*
x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 320*(3*tan
(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3 - 64*(15*tan(d*x + c)^4 + 10*tan(d*x
+ c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.95144, size = 602, normalized size = 4.63

$$-\frac{90\,Aa^3\cos\left(dx+c\right)^5-80\,Aa^3\cos\left(dx+c\right)^3-90\,Aa^3\cos\left(dx+c\right)-45\left(Aa^3\cos\left(dx+c\right)^6-3\,Aa^3\cos\left(dx+c\right)^4+3\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d*
x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*
x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6 -
3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x +
 c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(d*x +
 c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21561, size = 327, normalized size = 2.52

$$\frac{5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^3\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1920*(5*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
45*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^
3*tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 240*A
*a^3*tan(1/2*d*x + 1/2*c) + (882*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 240*A*a^3*t
an(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*tan(1/2*
d*x + 1/2*c)^3 - 45*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*tan(1/2*d*x + 1
/2*c) - 5*A*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx}{5a^3d(\sin(c+dx)+}$$

```
[Out] (-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A*C
os[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*a^3
*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.208172, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2966, 2638, 2635, 8, 2650, 2648}

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx}{5a^3d(\sin(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d*
x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A*C
os[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*a^3
*d*(1 + Sin[c + d*x]))
```


Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int\left(-\frac{9A}{a^3}+\frac{4A\sin(c+dx)}{a^3}-\frac{A\sin^2(c+dx)}{a^3}+\frac{2A}{a^3(1+\sin(c+dx))^3}-\frac{9}{a^3(1+\mathrm{si}}\right.$$

$$= -\frac{9Ax}{a^3}-\frac{A\int\sin^2(c+dx)\,dx}{a^3}+\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(4A)\int\sin(c+dx)\,dx}{a^3}$$

$$= -\frac{9Ax}{a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}$$

$$= -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}$$

$$= -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}$$

Mathematica [A] time = 0.927193, size = 254, normalized size = 1.97

$$\frac{A\left(-11400dx\sin\left(c+\frac{dx}{2}\right)-5700dx\sin\left(c+\frac{3dx}{2}\right)+1830\sin\left(2c+\frac{3dx}{2}\right)-4234\sin\left(2c+\frac{5dx}{2}\right)+1140dx\sin\left(3c+\frac{5dx}{2}\right)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-11400*d*x*Cos[(d*x)/2] + 12060*Cos[c + (d*x)/2] - 14090*Cos[c + (3*d*x
)/2] + 5700*d*x*Cos[2*c + (3*d*x)/2] + 1140*d*x*Cos[2*c + (5*d*x)/2] + 1050
*Cos[3*c + (5*d*x)/2] + 165*Cos[3*c + (7*d*x)/2] + 15*Cos[5*c + (9*d*x)/2]
+ 19780*Sin[(d*x)/2] - 11400*d*x*Sin[c + (d*x)/2] - 5700*d*x*Sin[c + (3*d*x
)/2] + 1830*Sin[2*c + (3*d*x)/2] - 4234*Sin[2*c + (5*d*x)/2] + 1140*d*x*Sin
[3*c + (5*d*x)/2] + 165*Sin[4*c + (7*d*x)/2] - 15*Sin[4*c + (9*d*x)/2]))/(4
80*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [B] time = 0.112, size = 257, normalized size = 2.

$$-\frac{A}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-8\,\frac{A\left(\tan\left(1/2\,dx+c/2\right)\right)^2}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^2}+\frac{A}{da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-8/d*A/a^3/(1+tan
(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^2+1/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^2*tan(1/2*d*x+1/2*c)-8/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2-19/d*A/a^3*arct
an(tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(tan
(1/2*d*x+1/2*c)+1)^4+4/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-10/d*A/a^3/(tan(1
/2*d*x+1/2*c)+1)^2-18/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.49411, size = 965, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/15*(A*((1325*sin(d*x + c)/(cos(d*x + c) + 1) + 2673*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4329*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 3575*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 227
5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 975*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + 195*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 304)/(a^3 + 5*a^3*sin(d*x
+ c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 20*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 26*a^3*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 26*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 20*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 12*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) +
1)^9) + 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 6*A*((105*sin(d*
x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 200*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
))/d
```

Fricas [B] time = 2.07252, size = 656, normalized size = 5.09

$$-\frac{15\,A\cos\left(dx+c\right)^{5}+90\,A\cos\left(dx+c\right)^{4}+\left(285\,Adx+683\,A\right)\cos\left(dx+c\right)^{3}-1140\,Adx+\left(855\,Adx-526\,A\right)\cos\left(dx+c\right.}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos(
d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*x
+ 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 1140*A
*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d*x +
 c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos
(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1601, size = 211, normalized size = 1.64

$$-\frac{\frac{285\,(dx+c)A}{a^3}+\frac{30\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+615\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1025\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+685\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+164\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(285*(d*x + c)*A/a^3 + 30*(A*tan(1/2*d*x + 1/2*c)^3 + 8*A*tan(1/2*d*x
 + 1/2*c)^2 - A*tan(1/2*d*x + 1/2*c) + 8*A)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2
*a^3) + 4*(135*A*tan(1/2*d*x + 1/2*c)^4 + 615*A*tan(1/2*d*x + 1/2*c)^3 + 10
25*A*tan(1/2*d*x + 1/2*c)^2 + 685*A*tan(1/2*d*x + 1/2*c) + 164*A)/(a^3*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

```
[Out] (4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + S
in[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (10
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.18774, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2966, 2638, 2650, 2648}

$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + S
in[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (10
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(\frac{4A}{a^3}-\frac{A\sin(c+dx)}{a^3}-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{7A}{a^3(1+\sin(c+dx))^2}-\frac{}{a^3(1}\right.$$

$$=\frac{4Ax}{a^3}-\frac{A\int\sin(c+dx)\,dx}{a^3}-\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(7A)\int\frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}$$

$$=\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{7A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{}{a^3}$$

$$=\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{}{3}$$

$$=\frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{}{1}$$

Mathematica [B] time = 0.789342, size = 228, normalized size = 2.21

$$-\frac{A\left(-1200dx\sin\left(c+\frac{dx}{2}\right)-600dx\sin\left(c+\frac{3dx}{2}\right)+405\sin\left(2c+\frac{3dx}{2}\right)-491\sin\left(2c+\frac{5dx}{2}\right)+120dx\sin\left(3c+\frac{5dx}{2}\right)+15\,\mathrm{s}\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(A*(-1200*d*x*Cos[(d*x)/2] + 1665*Cos[c + (d*x)/2] - 1675*Cos[c + (3*d*x)/
2] + 600*d*x*Cos[2*c + (3*d*x)/2] + 120*d*x*Cos[2*c + (5*d*x)/2] + 75*Cos[3
*c + (5*d*x)/2] + 15*Cos[3*c + (7*d*x)/2] + 2495*Sin[(d*x)/2] - 1200*d*x*Si
n[c + (d*x)/2] - 600*d*x*Sin[c + (3*d*x)/2] + 405*Sin[2*c + (3*d*x)/2] - 49
1*Sin[2*c + (5*d*x)/2] + 120*d*x*Sin[3*c + (5*d*x)/2] + 15*Sin[4*c + (7*d*x
```

```
)/2]))/(120*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^5)
```

Maple [A] time = 0.102, size = 155, normalized size = 1.5

$$2\,{\frac {A}{d{a}^{3} \left(1+ \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2} \right) }}+8\,{\frac {A\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{d{a}^{3}}}+{\frac {16\,A}{5\,d{a}^{3}} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) +1 \right) ^{-5}}-8\,{\frac {A}{d{a}^{3} \left(\tan \left(1/2\,dx+c \right. \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)+8/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))+16/
5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+4/3/d
*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+6/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+8/d*A/a^
3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.50767, size = 733, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c
) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos
(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
```

```
a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^3))/d
```

Fricas [B] time = 1.96004, size = 585, normalized size = 5.68

$$\frac{15\,A\cos\left(dx+c\right)^{4}+\left(60\,Adx+149\,A\right)\cos\left(dx+c\right)^{3}-240\,Adx+\left(180\,Adx-103\,A\right)\cos\left(dx+c\right)^{2}-3\left(40\,Adx+81\,A\right)\cos}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x +
 (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) + (1
5*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*(40
*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*
x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14841, size = 153, normalized size = 1.49

$$\frac{2\left(\frac{30\,(dx+c)A}{a^{3}}+\frac{15\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{3}}+\frac{60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+285\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+505\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+335\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+79\,A}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A*
tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x +
 1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c) +
 1)^5))/d
```

3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{Ax}{a^3}$$

```
[Out] -((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x]))
```

Rubi [A] time = 0.173079, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2966, 2650, 2648}

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{Ax}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x]))
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{A}{a^3}+\frac{2A}{a^3(1+\sin(c+dx))^3}-\frac{5A}{a^3(1+\sin(c+dx))^2}+\frac{4A}{a^3(1+\sin(c+dx))}\right) d$$

$$= -\frac{Ax}{a^3}+\frac{(2A)\int\frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(4A)\int\frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(5A)\int\frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}$$

$$= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{5A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{4A\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

$$= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{7A\cos(c+dx)}{3a^3d(1+\sin(c+dx)}$$

$$= -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx)}$$

Mathematica [B] time = 0.762797, size = 189, normalized size = 2.12

$$\frac{A\left(-50dx\sin\left(c+\frac{dx}{2}\right)-25dx\sin\left(c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{3dx}{2}\right)-26\sin\left(2c+\frac{5dx}{2}\right)+5dx\sin\left(3c+\frac{5dx}{2}\right)+110\cos\left(c+\frac{dx}{2}\right.\right.}{20a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-50*d*x*Cos[(d*x)/2] + 110*Cos[c + (d*x)/2] - 90*Cos[c + (3*d*x)/2] + 2
5*d*x*Cos[2*c + (3*d*x)/2] + 5*d*x*Cos[2*c + (5*d*x)/2] + 150*Sin[(d*x)/2]
- 50*d*x*Sin[c + (d*x)/2] - 25*d*x*Sin[c + (3*d*x)/2] + 40*Sin[2*c + (3*d*x
)/2] - 26*Sin[2*c + (5*d*x)/2] + 5*d*x*Sin[3*c + (5*d*x)/2]))/(20*a^3*d*(Co
s[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.099, size = 131, normalized size = 1.5

$$-2\,\frac{A\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}-\frac{16\,A}{5\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}+8\,\frac{A}{da^3\left(\tan\left(1/2\,dx+c/2\right)+1\right)^4}-4\,\frac{A}{da^3\left(\tan\left(1/2\,dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -2/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5
+8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4-4/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-2/d*
A/a^3/(tan(1/2*d*x+1/2*c)+1)^2-2/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.48544, size = 529, normalized size = 5.94

$$-\frac{2\left(A\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2A\left(\frac{5\sin}{\cos(d}\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -2/15*(A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5))/d
```

Fricas [B] time = 1.76422, size = 508, normalized size = 5.71

$$-\frac{(5\,Adx+13\,A)\cos(dx+c)^3-20\,Adx+3\,(5\,Adx-2\,A)\cos(dx+c)^2-(10\,Adx+21\,A)\cos(dx+c)-\left(20\,Adx-(5\,A\right.}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\cos(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d*
x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*co
s(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3
*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16008, size = 126, normalized size = 1.42

$$-\frac{\frac{5\,(dx+c)A}{a^3}+\frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*tan(1/2*d*x + 1/2*c)^4 + 25*A*tan(1/2*d*x
 + 1/2*c)^3 + 55*A*tan(1/2*d*x + 1/2*c)^2 + 35*A*tan(1/2*d*x + 1/2*c) + 8*A)
/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

```
[Out] (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(15
*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*
x]))
```

Rubi [A] time = 0.138005, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2966, 2650, 2648}

$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(15
*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*
x]))
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{2A}{a^3(1+\sin(c+dx))^3}+\frac{3A}{a^3(1+\sin(c+dx))^2}-\frac{A}{a^3(1+\sin(c+dx))}\right)dx$$
$$= -\frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3}+\frac{(3A)\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))^2}+\frac{A\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{(4A)\int$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-\frac{(4A)\int \frac{1}{1+\sin(c+dx)}\,dx}{15a^3}$$
$$= \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}-\frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

Mathematica [A] time = 0.459339, size = 107, normalized size = 1.3

$$-\frac{A\left(15\sin\left(2c+\frac{3dx}{2}\right)-4\sin\left(2c+\frac{5dx}{2}\right)+15\cos\left(c+\frac{dx}{2}\right)-5\cos\left(c+\frac{3dx}{2}\right)+25\sin\left(\frac{dx}{2}\right)\right)}{30a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 15*Sin[
2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Maple [A] time = 0.093, size = 71, normalized size = 0.9

$$4\,\frac{A}{da^3}\left(-1/2\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}+4/5\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-5}+5/3\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-3}-2\,\left(\tan\left(1/2\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 4/d*A/a^3*(-1/2/(tan(1/2*d*x+1/2*c)+1)^2+4/5/(tan(1/2*d*x+1/2*c)+1)^5+5/3/(
tan(1/2*d*x+1/2*c)+1)^3-2/(tan(1/2*d*x+1/2*c)+1)^4)
```

Maxima [B] time = 1.00779, size = 470, normalized size = 5.73

$$\frac{2\left(\frac{2A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 2/15*(2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3 + 5*a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Fricas [B] time = 1.85775, size = 386, normalized size = 4.71

$$\frac{4A\cos(dx+c)^3+7A\cos(dx+c)^2-3A\cos(dx+c)-\left(4A\cos(dx+c)^2-3A\cos(dx+c)-6A\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d+\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)-4a^3d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*cos
(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos
(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [A] time = 34.0405, size = 461, normalized size = 5.62

$$\begin{cases} -\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d} + \frac{}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\right.} \\ \frac{x(-A\sin(c)+A)\sin(c)}{(a\sin(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(
c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2 +
d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 +
150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d
*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 +
 d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3
 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150
*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*ta
n(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)/(a*sin(c)
 + a)**3, True))
```

Giac [A] time = 1.16554, size = 85, normalized size = 1.04

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(1
/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=58

$$-\frac{A\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^3}-\frac{aA\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}$$

```
[Out] -(a*A*Cos[c + d*x]^3)/(5*d*(a + a*Sin[c + d*x])^4) - (A*Cos[c + d*x]^3)/(15
*d*(a + a*Sin[c + d*x])^3)
```

Rubi [A] time = 0.114368, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2736, 2672, 2671}

$$-\frac{A\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^3}-\frac{aA\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a*A*Cos[c + d*x]^3)/(5*d*(a + a*Sin[c + d*x])^4) - (A*Cos[c + d*x]^3)/(15
*d*(a + a*Sin[c + d*x])^3)
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c +
d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ
[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rule 2672

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2671

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= (aA)\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx\\ &= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}+\frac{1}{5}A\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx\\ &= -\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}\end{aligned}$$

Mathematica [A] time = 0.238575, size = 92, normalized size = 1.59

$$\frac{A\left(\sin\left(2c+\frac{5dx}{2}\right)-15\cos\left(c+\frac{dx}{2}\right)+5\cos\left(c+\frac{3dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)\right)}{30a^3d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c
+ (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^5)
```

Maple [A] time = 0.087, size = 86, normalized size = 1.5

$$2\,\frac{A}{da^3}\left(-8/5\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-5}-\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-1}+3\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}+4\,\left(\tan\left(1/2\,dx+c/2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d*A/a^3*(-8/5/(tan(1/2*d*x+1/2*c)+1)^5-1/(tan(1/2*d*x+1/2*c)+1)+3/(tan(1/
2*d*x+1/2*c)+1)^2+4/(tan(1/2*d*x+1/2*c)+1)^4-14/3/(tan(1/2*d*x+1/2*c)+1)^3)
```

Maxima [B] time = 1.01554, size = 522, normalized size = 9.

$$-\frac{2\left(\frac{A\left(\frac{20\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+7\right)}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{a^3+\frac{5\,a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\cdots}\cdots}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Fricas [B] time = 1.86094, size = 381, normalized size = 6.57

$$\frac{A\cos(dx+c)^3-2\,A\cos(dx+c)^2+3\,A\cos(dx+c)-\left(A\cos(dx+c)^2+3\,A\cos(dx+c)+6\,A\right)\sin(dx+c)+\cdots}{15\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d+\left(a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a^3d\right)\sin\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*x
 + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x
 + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [A] time = 14.1702, size = 571, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((6*A*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3
*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/
2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 30*A*tan(c/2 + d*
x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 15
0*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*t
an(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2
+ d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**
3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d
) + 20*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/
2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2
)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 2*A/(15*a**3*d*tan(c/2 + d
*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 +
 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d),
Ne(d, 0)), (x*(-A*sin(c) + A)/(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.12946, size = 107, normalized size = 1.84

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,A\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*tan
(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x +
 1/2*c) + 1)^5)
```

3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d}$$

```
[Out] -((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin
[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.164468, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2966, 3770, 2650, 2648}

$$\frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin
[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(\frac{A\csc(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{A}{a^3(1+\sin(c+dx))^2} - \frac{A}{a^3(1+\sin(c\right.$$

$$= \frac{A\int \csc(c+dx)\,dx}{a^3} - \frac{A\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} - \frac{A\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\sin(}}{a^3}$$

$$= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{A\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2} +$$

$$= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} +$$

$$= -\frac{A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} +$$

Mathematica [B] time = 1.0052, size = 313, normalized size = 3.19

$$\frac{(A-A\sin(c+dx))\left(2\sin\left(\frac{dx}{2}\right)(-19\sin(c+dx)+4\cos(2(c+dx))-17)+\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(3\cos\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2*Cos[c/2] - 2*Sin[c/2] + 3*Cos[c/
2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c/2]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2 - 5*Log[Cos[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4 + 5*Log[Sin[(c + d*x)/2]]*(Cos[c/2] + Sin
```

```
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + 2*Sin[(d*x)/2]*(-17 + 4*C
os[2*(c + d*x)] - 19*Sin[c + d*x]))*(A - A*Sin[c + d*x]))/(5*a^3*d*(Cos[c/2
] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^5)
```

Maple [A] time = 0.153, size = 130, normalized size = 1.3

$$\frac{16\,A}{5\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}-8\,\frac{A}{da^3\,(\tan(1/2\,dx+c/2)+1)^4}+12\,\frac{A}{da^3\,(\tan(1/2\,dx+c/2)+1)^3}-10\,\frac{A}{da^3\,(\tan(1/2\,dx+c/}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+12
/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-10/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+8/d*A
/a^3/(tan(1/2*d*x+1/2*c)+1)+1/d*A/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.01314, size = 585, normalized size = 5.97

$$\frac{A\left(\frac{2\left(\frac{115\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{135\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+32\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2A\left(\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30}{(\cos}\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(}{(\cos(dx+c}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) +
2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
```

```
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5))/d
```

Fricas [B] time = 2.06006, size = 817, normalized size = 8.34

$$\frac{16\,A\cos\left(dx+c\right)^{3}-22\,A\cos\left(dx+c\right)^{2}-42\,A\cos\left(dx+c\right)-5\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}-2\,A\cos\left(dx+c\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A*
cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2
- 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) +
 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x +
 c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c) - 4*A
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4
*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x +
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{A\left(\int -\frac{\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx+\int \frac{\sin\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
 d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*si
n(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.17805, size = 134, normalized size = 1.37

$$\frac{\frac{5\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{2\left(20\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+45\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)^
4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(1/
2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=113

$$-\frac{A\cot(c+dx)}{a^3d}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{104A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)}+\frac{31A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)^2}-\frac{2A\cot(c+dx)}{5a^3d(\csc(c+dx)+1}$$

```
[Out] (4*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (A*Cot[c + d*x])/(a^3*d) - (2*A*Cot[c
 + d*x])/(5*a^3*d*(1 + Csc[c + d*x])^3) + (31*A*Cot[c + d*x])/(15*a^3*d*(1
+ Csc[c + d*x])^2) - (104*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.398275, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2950, 2709, 3770, 3767, 8, 3777, 3922, 3919, 3794}

$$-\frac{A\cot(c+dx)}{a^3d}+\frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{104A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)}+\frac{31A\cot(c+dx)}{15a^3d(\csc(c+dx)+1)^2}-\frac{2A\cot(c+dx)}{5a^3d(\csc(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (A*Cot[c + d*x])/(a^3*d) - (2*A*Cot[c
 + d*x])/(5*a^3*d*(1 + Csc[c + d*x])^3) + (31*A*Cot[c + d*x])/(15*a^3*d*(1
+ Csc[c + d*x])^2) - (104*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x]))
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = (aA)\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$

$$= \frac{A\int \left(\frac{9}{a^2} - \frac{4\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a^2} - \frac{2}{a^2(1+\csc(c+dx))^3} + \frac{9}{a^2(1+\csc(c+dx))^2} - \frac{16}{a^2(1+\csc(c+}\right.}{a}$$

$$= \frac{9Ax}{a^3} + \frac{A\int \csc^2(c+dx)\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\csc(c+dx))^3}\,dx}{a^3} - \frac{(4A)\int \csc(c+dx)\,dx}{a^3}$$

$$= \frac{9Ax}{a^3} + \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{3A\cot(c+dx)}{a^3d(1+\csc(c+}$$

$$= \frac{2Ax}{a^3} + \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} +$$

$$= \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A}{15a^3d(1}$$

$$= \frac{4A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A}{15a^3d(1}$$

Mathematica [A] time = 3.09343, size = 167, normalized size = 1.48

$$-\frac{A\left(-15\tan\left(\frac{1}{2}(c+dx)\right) + 15\cot\left(\frac{1}{2}(c+dx)\right) + 120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-3}{\left(\sin\left(\frac{1}{2}(\right.\right.}\right.}{30a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c + d*x
)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*x)] - 35
4*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[(c + d*x)
/2]))/(30*a^3*d)
```

Maple [A] time = 0.169, size = 169, normalized size = 1.5

$$\frac{A}{2\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16\,A}{5\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5}+8\,\frac{A}{da^3\left(\tan\left(1/2\,dx+c/2\right)+1\right)^4}-\frac{44\,A}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*A/a^3*tan(1/2*d*x+1/2*c)-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/
a^3/(tan(1/2*d*x+1/2*c)+1)^4-44/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+14/d*A/a
^3/(tan(1/2*d*x+1/2*c)+1)^2-18/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)-1/2/d*A/a^3/t
an(1/2*d*x+1/2*c)-4/d*A/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.02189, size = 701, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/30*(3*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a^
3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3
 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + 2*A*(2*(115*sin(d*x + c)/(cos
(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3
 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [B] time = 2.10721, size = 1065, normalized size = 9.42

$$\frac{94\,A\cos\left(dx+c\right)^{4}+222\,A\cos\left(dx+c\right)^{3}-115\,A\cos\left(dx+c\right)^{2}-237\,A\cos\left(dx+c\right)+30\,\left(A\cos\left(dx+c\right)^{4}-2\,A\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 - 2
37*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x
 + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*c
os(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30*(A*
cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c)
 - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x
 + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 128*A*co
s(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x
 + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x
 + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*
cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{A\left(\int -\frac{\csc^{2}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx+\int \frac{\sin\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(
c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3
+ 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Giac [A] time = 1.16707, size = 197, normalized size = 1.74

$$-\frac{\frac{120\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{15\left(8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+605\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+385\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+104\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c)
/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(13
5*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/2*d
*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^5))/d
```

3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=138

$$\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19A\tanh^{-1}(\cos(c+d$$

```
[Out] (-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*C
ot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[
c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (164*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.22469, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2966, 3770, 3767, 8, 3768, 2650, 2648}

$$\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19A\tanh^{-1}(\cos(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```



```
[Out] (-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*C
ot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[
c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (164*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```


Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(\frac{9A\csc(c+dx)}{a^3} - \frac{4A\csc^2(c+dx)}{a^3} + \frac{A\csc^3(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))}\right.$$
$$= \frac{A\int \csc^3(c+dx)\,dx}{a^3} - \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} - \frac{(4A)\int \csc^2(c+dx)\,dx}{a^3} - \frac{(5A}{}$$
$$= -\frac{9A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx)}$$
$$= -\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{4A\cot(c+dx)}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{}{5}$$
$$= -\frac{19A\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{4A\cot(c+dx)}{a^3d} - \frac{A\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{}{5}$$

Mathematica [A] time = 4.23625, size = 245, normalized size = 1.78

$$\frac{A\left(-240\tan\left(\frac{1}{2}(c+dx)\right)+240\cot\left(\frac{1}{2}(c+dx)\right)-15\csc^2\left(\frac{1}{2}(c+dx)\right)+15\sec^2\left(\frac{1}{2}(c+dx)\right)+1140\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/2
]] + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*x)
/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^4 - (464*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*x)/2]
)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120*a^3*d
)
```

Maple [A] time = 0.197, size = 209, normalized size = 1.5

$$\frac{A}{8da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - 2\frac{A\tan(1/2\,dx+c/2)}{da^3} + \frac{16A}{5da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5} - 8\frac{A}{da^3(\tan(1/2\,dx+c/2)+1)^4} + \frac{52A}{3da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/8/d*A/a^3*tan(1/2*d*x+1/2*c)^2-2/d*A/a^3*tan(1/2*d*x+1/2*c)+16/5/d*A/a^3/
(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+52/3/d*A/a^3/(t
an(1/2*d*x+1/2*c)+1)^3-18/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+32/d*A/a^3/(tan(
1/2*d*x+1/2*c)+1)-1/8/d*A/a^3/tan(1/2*d*x+1/2*c)^2+2/d*A/a^3/tan(1/2*d*x+1/
2*c)+19/2/d*A/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.02398, size = 840, normalized size = 6.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a
^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)/(cos(d*
x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9285*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)
^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*
x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [B] time = 2.04622, size = 1327, normalized size = 9.62

$$\frac{896\,A\cos(dx+c)^5 - 1222\,A\cos(dx+c)^4 - 3218\,A\cos(dx+c)^3 + 1168\,A\cos(dx+c)^2 + 2292\,A\cos(dx+c) - 285\,\big(A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
+ 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c)^5 + 3*A
*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c
) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d
*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) + 285*(A*cos
(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2
+ 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x +
 c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) +
 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*cos(d*x + c
)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos(d*x + c)^
5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^
2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*
x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17636, size = 243, normalized size = 1.76

$$\frac{\frac{1140\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\left(114\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{15\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6}+\frac{16\left(240\,A\tan\cdots\right.}{\cdots}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x +
1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 15
*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(2
40*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan(1/2
*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*x + 1
/2*c) + 1)^5))/d
```

3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{A\cot^3(c+dx)}{3a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{18A}{}$$

```
[Out] (18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Cot
[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Rubi [A] time = 0.246471, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2966, 3770, 3767, 8, 3768, 2650, 2648}

$$-\frac{A\cot^3(c+dx)}{3a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{18A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Cot
[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*Cos
[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d*(1
 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```


Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \int \left(-\frac{16A\csc(c+dx)}{a^3} + \frac{9A\csc^2(c+dx)}{a^3} - \frac{4A\csc^3(c+dx)}{a^3} + \frac{A\csc^4(c+dx)}{a^3}\right.$$

$$= \frac{A\int \csc^4(c+dx)\,dx}{a^3} + \frac{(2A)\int \frac{1}{(1+\sin(c+dx))^3}\,dx}{a^3} - \frac{(4A)\int \csc^3(c+dx)\,dx}{a^3} + \frac{(7A}{}$$

$$= \frac{16A\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx}$$

$$= \frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} + \frac{2A\cot(c+}{}$$

$$= \frac{18A\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} + \frac{2A\cot(c+}{}$$

Mathematica [B] time = 6.2232, size = 348, normalized size = 2.27

$$A\left(\frac{29\tan\left(\frac{1}{2}(c+dx)\right)}{6d} - \frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{6d} + \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{186\sin\left(\frac{1}{2}(c\right.}{5d\left(\sin\left(\frac{1}{2}(c+dx)\right)+c\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[Si
n[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*Sin[(c
+ d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c + d*x)/
2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)))/a^3
```

Maple [A] time = 0.208, size = 249, normalized size = 1.6

$$\frac{A}{24\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{A}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + \frac{39\,A}{8\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{16\,A}{5\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5} + 8\,\frac{}{da^3\,(\tan(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d*A/a^3*tan(1/2*d*x+1/2*c)^3-1/2/d*A/a^3*tan(1/2*d*x+1/2*c)^2+39/8/d*A
/a^3*tan(1/2*d*x+1/2*c)-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(ta
n(1/2*d*x+1/2*c)+1)^4-20/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+22/d*A/a^3/(tan(1
/2*d*x+1/2*c)+1)^2-50/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)-1/24/d*A/a^3/tan(1/2*d
*x+1/2*c)^3+1/2/d*A/a^3/tan(1/2*d*x+1/2*c)^2-39/8/d*A/a^3/tan(1/2*d*x+1/2*c
)-18/d*A/a^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.0227, size = 953, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 25
80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a
^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((20*si
n(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)/(a^3*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 9
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [B] time = 2.20219, size = 1563, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4 -
 2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135*(
A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x +
c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(
d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4
*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x + c)^6
 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A
*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c)
 - 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 578*A*cos(d*
x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A*cos(d*x +
c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*cos(d*x + c)^
5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*cos(d*x + c)^
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*
x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*
x + c) + 4*a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1754, size = 288, normalized size = 1.88

$$-\frac{\frac{2160\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{5\left(792\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-117\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{48\left(125\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+445\,A\tan\left(\frac{1}{2}\,d\right.\right.}{a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/(a
^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*tan(1
/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x + 1/2*
c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x + 1/2
*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2*c))/a
^9)/d
```

3.244 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=327

$$-\frac{a\left(5Ad\left(16c^2d+3c^3+12cd^2+4d^3\right)-B\left(-52c^2d^2-15c^3d+3c^4-60cd^3-16d^4\right)\right)\cos(e+fx)}{30df}-\frac{a\left(5Ad\left(6c^2+20cd+\right.\right.}{}$$

```
[Out] (a*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c*d^2
 + 3*d^3))*x)/8 - (a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) - B*(3*c^
4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/(30*d*f) - (a
*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45*d^3)
)*Cos[e + f*x]*Sin[e + f*x])/(120*f) - (a*(4*(5*A + 4*B)*d^2 - 3*c*(B*c - 5
*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d*f) + (a*(B*c - 5*(A
 + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a*B*Cos[e + f*x]*
(c + d*Sin[e + f*x])^4)/(5*d*f)
```

Rubi [A] time = 0.579065, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2753, 2734}

$$-\frac{a\left(5Ad\left(16c^2d+3c^3+12cd^2+4d^3\right)-B\left(-52c^2d^2-15c^3d+3c^4-60cd^3-16d^4\right)\right)\cos(e+fx)}{30df}-\frac{a\left(5Ad\left(6c^2+20cd+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (a*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c*d^2
 + 3*d^3))*x)/8 - (a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) - B*(3*c^
4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/(30*d*f) - (a
*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45*d^3)
)*Cos[e + f*x]*Sin[e + f*x])/(120*f) - (a*(4*(5*A + 4*B)*d^2 - 3*c*(B*c - 5
*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d*f) + (a*(B*c - 5*(A
 + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a*B*Cos[e + f*x]*
(c + d*Sin[e + f*x])^4)/(5*d*f)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \int (c+d\sin(e+fx))^3\left(aA+(aA+aB)\sin(e+fx)+a\right.$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df} + \frac{\int (c+d\sin(e+\right.}{}$$

$$= \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} - \frac{aI}{}$$

$$= -\frac{a\left(4(5A+4B)d^2-3c(Bc-5(A+B)d)\right)\cos(e+fx)(c}{60df}$$

$$= \frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A\left(8c^3+12c^2d+1\right.\right.$$

Mathematica [A] time = 2.02801, size = 267, normalized size = 0.82

$$a(\sin(e+fx)+1)\left(15\left(-8\left(Ad\left(3c^2+3cd+d^2\right)+B(c+d)^3\right)\sin(2(e+fx))+4fx\left(A\left(12c^2d+8c^3+12cd^2+3d^3\right)+B\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(8
*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d) +
 B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)] +
 15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c
*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2*(e +
 f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)])))/(480*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)
```

Maple [A] time = 0.069, size = 422, normalized size = 1.3

$$\frac{1}{f}\left(-Ac^3a\cos\left(fx+e\right)+3\,Ac^2da\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-Acd^2a\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-A*c^3*a*cos(f*x+e)+3*A*c^2*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-A*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+A*d^3*a*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*c^3*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1
/2*f*x+1/2*e)-B*c^2*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*c*d^2*a*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*d^3*a*(8/3+sin(f*x
+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*c^3*a*(f*x+e)-3*A*c^2*d*a*cos(f*x+e)+3
*A*c*d^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*A*d^3*a*(2+sin(f*
x+e)^2)*cos(f*x+e)-B*c^3*a*cos(f*x+e)+3*B*c^2*d*a*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)-B*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^3*a*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A] time = 1.01289, size = 548, normalized size = 1.68

$$\frac{480\left(fx+e\right)Aac^3+120\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Bac^3+360\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aac^2d+480\left(\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^3
 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 - 3
*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*d +
 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d^2 +
45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2 + 160*
(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x +
 4*e) - 8*sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e
)^3 + 15*cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*cos(f*x +
 e) - 1440*A*a*c^2*d*cos(f*x + e))/f
```

Fricas [A] time = 2.22948, size = 608, normalized size = 1.86

$$-\frac{24\,Bad^3\cos\left(fx+e\right)^5-40\left(3\,Bac^2d+3\,(A+B)acd^2+(A+2\,B)ad^3\right)\cos\left(fx+e\right)^3-15\left(4\,(2\,A+B)ac^3+12\,(A+B)ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 + (
A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3*(A
 + B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2*(3*
B*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*c^2*d
 + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 7.87179, size = 996, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f*
x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*x)*
*2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*cos(
e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d**2*co
s(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*
A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f*x)**3/(3
*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2 - B*a*c**
3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*B*a*c**2*d*
x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*c**2*d*sin(e
 + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) -
2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)**4/8 + 9*B*a*c
*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*cos(e + f*x)**4/
8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a*c*d**2*sin(e +
 f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) -
 2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*x)**4/8 + 3*B*a*d*
*3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*cos(e + f*x)**4/8 - B
*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*B*a*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*d**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a*d**3*cos(e + f*x)**5/(15*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a), True))
```

Giac [A] time = 1.15537, size = 424, normalized size = 1.3

$$-\frac{Bad^3\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(8\,Aac^3+4\,Bac^3+12\,Aac^2d+12\,Bac^2d+12\,Aacd^2+9\,Bacd^2+3\,Aad^3+3\,Bad^3\right)x+\frac{(12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2*
d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x +
1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3)*c
os(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*c^2*
d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/f + 1
/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c^3 + 3
*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*s
in(2*f*x + 2*e)/f
```

3.245 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=213

$$-\frac{a\left(4Ad\left(c^2+3cd+d^2\right)-B\left(-4c^2d+c^3-8cd^2-4d^3\right)\right)\cos(e+fx)}{6df}+\frac{1}{8}ax\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right.$$

```
[Out] (a*(4*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/8 - (a*(4*A*d
*(c^2 + 3*c*d + d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/(
6*d*f) - (a*(3*(4*A + 3*B)*d^2 - 2*c*(B*c - 4*(A + B)*d))*Cos[e + f*x]*Sin[
e + f*x])/(24*f) + (a*(B*c - 4*(A + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*d*f) - (a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f)
```

Rubi [A] time = 0.360404, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2753, 2734}

$$-\frac{a\left(4Ad\left(c^2+3cd+d^2\right)-B\left(-4c^2d+c^3-8cd^2-4d^3\right)\right)\cos(e+fx)}{6df}+\frac{a\left(-8cd(A+B)-3d^2(4A+3B)+2Bc^2\right)\sin(e+}{24f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*(4*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/8 - (a*(4*A*d
*(c^2 + 3*c*d + d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/(
6*d*f) + (a*(2*B*c^2 - 8*(A + B)*c*d - 3*(4*A + 3*B)*d^2)*Cos[e + f*x]*Sin[
e + f*x])/(24*f) + (a*(B*c - 4*(A + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])
^2)/(12*d*f) - (a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (c+d\sin(e+fx))^2\left(aA+(aA+aB)\sin(e+fx)+a\right.$$

$$= -\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df} + \frac{\int (c+d\sin(e+\right.}{}$$

$$= \frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^2}{12df} - \frac{aI}{}$$

$$= \frac{1}{8}a\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)x - \frac{a(}{}$$

Mathematica [A] time = 1.10146, size = 185, normalized size = 0.87

$$\frac{a(\sin(e+fx)+1)\left(3\left(4fx\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)-8\left(Ad(2c+d)+B(c+d)^2\right)\sin(2(e+fx))+Bd\right.\right.}{96f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d +
3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4*A
*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2 +
A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)])))/(96*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [A] time = 0.056, size = 274, normalized size = 1.3

$$\frac{1}{f}\left(-Ac^2a\cos\left(fx+e\right)+2\,Acda\left(-1/2\,\sin\left(fx+e\right)\cos\left(fx+e\right)+1/2\,fx+e/2\right)-\frac{Ad^2a\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+\right.}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-A*c^2*a*cos(f*x+e)+2*A*c*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-1/3*A*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*c^2*a*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2/3*B*c*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^2*a*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+A*c^2*a*(f*x+e)-2*A*c
*d*a*cos(f*x+e)+A*d^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*c^2*a*
cos(f*x+e)+2*B*c*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*d^2*a
*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 0.986382, size = 356, normalized size = 1.67

$$\frac{96\left(fx+e\right)Aac^2+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Bac^2+48\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aacd+64\left(\cos\left(fx+e\right)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
```

```
x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a
*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e) - 96
*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

Fricas [A] time = 2.0495, size = 402, normalized size = 1.89

$$\frac{8\left(2\,Bacd+(A+B)ad^{2}\right)\cos\left(fx+e\right)^{3}+3\left(4\,(2\,A+B)ac^{2}+8\,(A+B)acd+(4\,A+3\,B)ad^{2}\right)fx-24\left((A+B)ac^{2}+2\,(A-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2 +
 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)*a
*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*a*c
^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 3.64629, size = 571, normalized size = 2.68

$$\begin{cases} Aac^{2}x-\frac{Aac^{2}\cos\left(e+fx\right)}{f}+Aacdx\sin^{2}\left(e+fx\right)+Aacdx\cos^{2}\left(e+fx\right)-\frac{Aacd\sin\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{2Aacd\cos\left(e+fx\right)}{f}+\frac{Aad^{2}x\sin^{2}}{2} \\ x\left(A+B\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)^{2}\left(a\sin\left(e\right)+a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**2
 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a*
c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f*x)
**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)*cos
(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e + f*x
)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e + f*x)/
(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*d*x*cos
(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*sin(e + f
```

```
*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*x*sin(e +
 f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**2*x*
cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - B*a*d**
2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
 d*sin(e))**2*(a*sin(e) + a), True))
```

Giac [A] time = 1.13262, size = 267, normalized size = 1.25

$$\frac{Bad^2\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(8\,Aac^2+4\,Bac^2+8\,Aacd+8\,Bacd+4\,Aad^2+3\,Bad^2\right)x+\frac{\left(2\,Bacd+Aad^2+Bad^2\right)\cos\left(3\,f\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d +
8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^2)
*cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d +
3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*a*c*
d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f
```

3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=111

$$-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}ax(A(2c+d)+B(c+d))-\frac{Bd}{}$$

```
[Out] (a*(B*(c + d) + A*(2*c + d))*x)/2 - (a*(3*A*(c + d) + B*(3*c + d))*Cos[e +
f*x])/(3*f) - (a*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (
B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(3*a*f)
```

Rubi [A] time = 0.15674, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2968, 3023, 2734}

$$-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}-\frac{a(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}ax(A(2c+d)+B(c+d))-\frac{Bd}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(B*(c + d) + A*(2*c + d))*x)/2 - (a*(3*A*(c + d) + B*(3*c + d))*Cos[e +
f*x])/(3*f) - (a*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (
B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(3*a*f)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))\left(Ac+(Bc+Ad)\sin(e+fx)+B\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af} + \frac{\int (a+a\sin(e+}{}$$

$$= \frac{1}{2}a(B(c+d)+A(2c+d))x - \frac{a(3A(c+d)+B(3c+d))}{3f}$$

Mathematica [A] time = 0.426629, size = 104, normalized size = 0.94

$$\frac{a(-3(4A(c+d)+B(4c+3d))\cos(e+fx)+12Acfx-3Ad\sin(2(e+fx))+6Adfx-3Bc\sin(2(e+fx))+6Bcfx-3B}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4*
c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] - 3*
A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)]))/(12*f)
```

Maple [A] time = 0.047, size = 147, normalized size = 1.3

$$\frac{1}{f}\left(-\frac{Bad\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+Aad\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\sin\left(fx+e\right)\cos\left(f\right.}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*B*a*d*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a*d*(-1/2*sin(f*x+e)*cos(f*x+
e)+1/2*f*x+1/2*e)+B*a*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+B*a*d*(-
1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*a*c*cos(f*x+e)-A*a*d*cos(f*x+e)-
B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

Maxima [A] time = 0.957769, size = 193, normalized size = 1.74

$$\frac{12\left(fx+e\right)Aac+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Bac+3\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aad+4\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(\right.\right.}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 1/12*(12*(f*x + e)*A*a*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c + 3*(2*
f*x + 2*e - sin(2*f*x + 2*e))*A*a*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))*B
*a*d + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*d - 12*A*a*c*cos(f*x + e) - 1
2*B*a*c*cos(f*x + e) - 12*A*a*d*cos(f*x + e))/f
```

Fricas [A] time = 1.94864, size = 225, normalized size = 2.03

$$\frac{2\,Bad\cos\left(fx+e\right)^{3}+3\left(\left(2\,A+B\right)ac+\left(A+B\right)ad\right)fx-3\left(Bac+\left(A+B\right)ad\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-6\left(\left(A+B\right)ac+\left(A\right.\right.}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a*
c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d)*
cos(f*x + e))/f
```

Sympy [A] time = 1.35109, size = 277, normalized size = 2.5

$$\begin{cases} Aacx - \frac{Aac\cos(e+fx)}{f} + \frac{Aadx\sin^2(e+fx)}{2} + \frac{Aadx\cos^2(e+fx)}{2} - \frac{Aad\sin(e+fx)\cos(e+fx)}{2f} - \frac{Aad\cos(e+fx)}{f} + \frac{Bacx\sin^2(e+fx)}{2} + \frac{Bacx\cos^2 \dots}{2} \\ x(A+B\sin(e))(c+d\sin(e))(a\sin(e)+a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*a
*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*cos(
e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a*c*
sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e + f*
x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*x)/f
- B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))
```

Giac [A] time = 1.21923, size = 136, normalized size = 1.23

$$\frac{Bad\cos(3fx+3e)}{12f} + \frac{1}{2}(2Aac + Bac + Aad + Bad)x - \frac{(4Aac + 4Bac + 4Aad + 3Bad)\cos(fx+e)}{4f} - \frac{(Bac + Aad + \dots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x - 1
/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c + A*
a*d + B*a*d)*sin(2*f*x + 2*e)/f
```

3.247 $\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx$

Optimal. Leaf size=48

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] (a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rubi [A] time = 0.0231044, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2734}

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
f*x])/(2*f)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx=\frac{1}{2}a(2A+B)x-\frac{a(A+B)\cos(e+fx)}{f}-\frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

Mathematica [A] time = 0.0975212, size = 45, normalized size = 0.94

$$\frac{a(-4(A+B)\cos(e+fx)+4Afx-B\sin(2(e+fx))+2Be+2Bfx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*Cos[e + f*x] - B*Sin[2*(e + f*x)]
))/(4*f)
```

Maple [A] time = 0.037, size = 59, normalized size = 1.2

$$\frac{1}{f}\left(Ba\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aa\cos\left(fx+e\right)-Ba\cos\left(fx+e\right)+Aa\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] 1/f*(B*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*a*cos(f*x+e)-B*a*cos(
f*x+e)+A*a*(f*x+e))
```

Maxima [A] time = 0.944981, size = 77, normalized size = 1.6

$$\frac{4\left(fx+e\right)Aa+\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Ba-4\,Aa\cos\left(fx+e\right)-4\,Ba\cos\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*x
 + e) - 4*B*a*cos(f*x + e))/f
```

Fricas [A] time = 1.88905, size = 113, normalized size = 2.35

$$\frac{(2\,A+B)afx-Ba\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A+B)a\cos\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((2*A + B)*a*f*x - B*a*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*a*cos(f*x
+ e))/f
```

Sympy [A] time = 0.612363, size = 94, normalized size = 1.96

$$\begin{cases} Aax - \frac{Aa\cos\left(e+fx\right)}{f} + \frac{Bax\sin^2\left(e+fx\right)}{2} + \frac{Bax\cos^2\left(e+fx\right)}{2} - \frac{Ba\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} - \frac{Ba\cos\left(e+fx\right)}{f} & \text{for } f \neq 0 \\ x\left(A + B\sin\left(e\right)\right)\left(a\sin\left(e\right) + a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*cos
(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f,
Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))
```

Giac [A] time = 1.16977, size = 65, normalized size = 1.35

$$\frac{1}{2}\,(2\,Aa + Ba)x - \frac{Ba\sin\left(2\,fx + 2\,e\right)}{4\,f} - \frac{(Aa + Ba)\cos\left(fx + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e)
/f
```

3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=98

$$\frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d^2}-\frac{aB\cos(e+fx)}{df}$$

```
[Out] -((a*(B*c - (A + B)*d)*x)/d^2) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a*B*Cos[e + f*x
])/(d*f)
```

Rubi [A] time = 0.273027, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d^2}-\frac{aB\cos(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```


```
[Out] -((a*(B*c - (A + B)*d)*x)/d^2) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(d + c*Tan
[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*Sqrt[c^2 - d^2]*f) - (a*B*Cos[e + f*x
])/(d*f)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{c+d\sin(e+fx)}\,dx$$

$$= -\frac{aB\cos(e+fx)}{df} + \frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2} - \frac{aB\cos(e+fx)}{df} + \frac{(a(c-d)(Bc-Ad))\int \frac{1}{c+d\sin(e+}}{d^2}$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2} - \frac{aB\cos(e+fx)}{df} + \frac{(2a(c-d)(Bc-Ad))\,\mathrm{Subst}\Big(\int}{d}$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2} - \frac{aB\cos(e+fx)}{df} - \frac{(4a(c-d)(Bc-Ad))\,\mathrm{Subst}\Big(\int}{}$$

$$= -\frac{a(Bc-(A+B)d)x}{d^2} + \frac{2a(c-d)(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f} - \frac{aB}{}$$

Mathematica [C] time = 0.650054, size = 196, normalized size = 2.

$$\frac{a(\sin(e+fx)+1)\left(\frac{2(c-d)(\cos(e)-i\sin(e))(Bc-Ad)\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e+\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{f\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + Adx + Bx(d-c) + \frac{Bd\sin(e)\sin(fx)}{f}\right.}{d^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x])/f + (2*(c - d)*(B*c - A*d)
*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)
/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(
Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/f)*(
1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Maple [B] time = 0.117, size = 294, normalized size = 3.

$$-2\frac{Aac}{df\sqrt{c^2-d^2}}\arctan\left(1/2\frac{2c\tan\left(1/2fx+e/2\right)+2d}{\sqrt{c^2-d^2}}\right)+2\frac{Aa}{f\sqrt{c^2-d^2}}\arctan\left(1/2\frac{2c\tan\left(1/2fx+e/2\right)+2d}{\sqrt{c^2-d^2}}\right)+2\frac{}{fd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))*A*c+2/f*a/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c
^2-d^2)^(1/2))*A+2/f*a/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(
1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/f*a/d*B/(1+tan(1/2*f*x+1/2*e)^2)
+2/f*a/d*A*arctan(tan(1/2*f*x+1/2*e))-2/f*a/d^2*B*arctan(tan(1/2*f*x+1/2*e)
)*c+2/f*a/d*B*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98764, size = 680, normalized size = 6.94

$$\left[-\frac{2\,Bad\cos\left(fx+e\right)+2\,(Bac-(A+B)ad)fx-(Bac-Aad)\sqrt{-\frac{c-d}{c+d}}\log\left(-\frac{\left(2\,c^2-d^2\right)\cos\left(fx+e\right)^2-2\,cd\sin\left(fx+e\right)-c^2-d^2-2\left(\left(c^2+cd\right)\cos\right.}{d^2\cos\left(fx+e\right)^2-2\,cd\sin\left(f\right.}\right.}{2\,d^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x - (B*a*c - A*a*d)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d)*f*x
 + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt(
(c - d)/(c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21573, size = 190, normalized size = 1.94

$$-\frac{\frac{(Bac-Aad-Bad)(fx+e)}{d^2}+\frac{2\,Ba}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\left(Bac^2-Aacd-Bacd+Aad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] -((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 +
1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sq
rt(c^2 - d^2)*d^2))/f
```

3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{2a\left(d^2(A+B)(c-d)-Bc\left(c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{aBx}{d^2}$$

```
[Out] (a*B*x)/d^2 + (2*a*((A + B)*(c - d)*d^2 - B*c*(c^2 - d^2))*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d
)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.325147, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2968, 3021, 2735, 2660, 618, 204}

$$\frac{2a\left(d^2(A+B)(c-d)-Bc\left(c^2-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2f\left(c^2-d^2\right)^{3/2}}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}+\frac{aBx}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a*B*x)/d^2 + (2*a*((A + B)*(c - d)*d^2 - B*c*(c^2 - d^2))*ArcTan[(d + c*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d
)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\int \frac{-a(A+B)(c-d)d-aB\left(c^2-d^2\right)\sin(e+fx)}{c+d\sin(e+fx)}\,dx}{d\left(c^2-d^2\right)}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(a\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\right)\int \frac{}{c+d\,\mathrm{s}}}{d^2(c+d)}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\left(2a\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\right)\text{Subs}}{d^2(c}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))} - \frac{\left(4a\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\right)\text{Subs}}{}$$

$$= \frac{aBx}{d^2} + \frac{2a\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c+d)\sqrt{c^2-d^2}\,f} + \frac{a(Bc-A}{d(c+d)f(c}$$

Mathematica [C] time = 1.31534, size = 217, normalized size = 1.75

$$\frac{a(\sin(e+fx)+1)\left(\frac{2(\cos(e)-i\sin(e))\left(Ad^2-B\left(c^2+cd-d^2\right)\right)\tan^{-1}\left(\frac{(\cos(e)-i\sin(e))\sec\left(\frac{fx}{2}\right)\left(c\sin\left(\frac{fx}{2}\right)+d\cos\left(e+\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)}{f(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{\csc(e)(Ad-Bc)(c\cos(e)+d\sin(fx))}{f(c+d)(c+d\sin(e+fx))} + \right.}{d^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[(
f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c
^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-(B*c) + A*d)*Csc[e]*(c*Cos[e]
 + d*Sin[f*x]))/((c + d)*f*(c + d*Sin[e + f*x]))))/(d^2*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2)
```

Maple [B] time = 0.141, size = 434, normalized size = 3.5

$$-2\,\frac{da\tan\left(1/2\,fx+e/2\right)A}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)(c+d)\,c}+2\,\frac{a\tan\left(1/2\,fx+e/2\right)B}{f\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^{2}+2\,\tan\left(1/2\,fx+e/2\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/f*a*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/2*
f*x+1/2*e)*A+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*
tan(1/2*f*x+1/2*e)*B-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)/(c+d)*A+2/f*a/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B
*c+2/f*a/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))*A-2/f*a/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+
1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2
*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2/f*a/(c+d)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2/f*a*B/d^2*
arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25826, size = 1416, normalized size = 11.42

$$\left[\frac{2\left(Bac^{3}d+Bac^{2}d^{2}-Bacd^{3}-Bad^{4}\right)fx\sin\left(fx+e\right)+2\left(Bac^{4}+Bac^{3}d-Bac^{2}d^{2}-Bacd^{3}\right)fx+\left(Bac^{3}+Bac^{2}d-(A+B\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) +
2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*
d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))
*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^
2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c^3*d -
 A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c*d
^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f), ((B*a*
c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a*c^4 + B*
a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (A + B)*a*c
*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqrt(c^2 - d^2
)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (B*a*c^3*d
 - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c
*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27083, size = 275, normalized size = 2.22

$$\frac{\frac{(fx+e)Ba}{d^2} - \frac{2\left(Bac^2+Bacd-Aad^2-Bad^2\right)\left(\pi\left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^2+d^3\right)\sqrt{c^2-d^2}} + \frac{2\left(Bacd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Aad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Bac^2-Aacd\right)}{\left(c^2d+cd^2\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] ((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2*e)
 - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(c*ta
n(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=176

$$\frac{a(2Ac-Ad+Bc-2Bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{2df(c-d)(c+d)^2(c+d\sin(e+fx))}+\frac{a(Bc-Ad)\cos}{2df(c+d)(c+d\sin}$$

```
[Out] (a*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c + d)*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2
))*Cos[e + f*x])/(2*(c - d)*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.420982, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{a(2Ac-Ad+Bc-2Bd)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{f(c+d)\left(c^2-d^2\right)^{3/2}}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{2df(c-d)(c+d)^2(c+d\sin(e+fx))}+\frac{a(Bc-Ad)\cos}{2df(c+d)(c+d\sin}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d
^2]])/((c + d)*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - (a*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2
))*Cos[e + f*x])/(2*(c - d)*d*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{aA+(aA+aB)\sin(e+fx)+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\int \frac{-2a(A+B)(c-d)d-a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}}{2d\left(c^2-d^2\right)}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d+B\left(c^2+2cd-2d^2\right)\right)\cos(}{2(c-d)d(c+d)^2f(c+d\sin(e+fx}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d+B\left(c^2+2cd-2d^2\right)\right)\cos(}{2(c-d)d(c+d)^2f(c+d\sin(e+fx}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d+B\left(c^2+2cd-2d^2\right)\right)\cos(}{2(c-d)d(c+d)^2f(c+d\sin(e+fx}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a\left(A(c-2d)d+B\left(c^2+2cd-2d^2\right)\right)\cos(}{2(c-d)d(c+d)^2f(c+d\sin(e+fx}$$

$$= \frac{a(2Ac+Bc-Ad-2Bd)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c-d)(c+d)^2\sqrt{c^2-d^2}f} + \frac{a(Bc-Ad)\cos(e+}{2d(c+d)f(c+d\sin(e}$$

<u>Mathematica</u> [C] time = 2.63203, size = 345, normalized size = 1.96

$$\frac{a(\sin(e+fx)+1)\left(\frac{d\csc(e)\left(\left(Ad^2(d-2c)+Bc\left(2c^2+2cd-3d^2\right)\right)\sin(2e+fx)-d\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+2fx)+\sin(fx)\left(Bc\left(2c^2+6cd-5d^2\right)-Ad\left(-4c^2+\right.\right.\right.}{d^2(c+d\sin(e+fx))^2}\right.}{4f(c-d)(c+d)^2\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]*
(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^2]
*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(
Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2
```

```
*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2))*Cos
[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d^2))*S
in[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e + f*x]))
/(d^2*(c + d*Sin[e + f*x])^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^2)
```

Maple [B] time = 0.164, size = 2021, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^3+c^2*d-c*
d^2-d^3)*tan(1/2*f*x+1/2*e)^3*B-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c^2*tan(1/2*f*x+1/2*e)^2*A-3/f*a/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/
2*f*x+1/2*e)^2*A*d^2-2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^3+c^2*d-c*d^2-d^3)*c^2*tan(1/2*f*x+1/2*e)^2*B-2/f*a/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A*c^2+1/f*a/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*tan(
1/2*f*x+1/2*e)^2*B*d+4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^3+c^2*d-c*d^2-d^3)/c*tan(1/2*f*x+1/2*e)^2*A*d^3+2/f*a/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c^2*tan(1/2*f*x
+1/2*e)^2*A*d^4-3/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c
/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3*A*d-2/f*a/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3
*B*d+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^3+c^2*d
-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*A*d^3-6/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/
2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*B*d+2/f*a/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*t
an(1/2*f*x+1/2*e)^2*A*d+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c*tan(1/2*f*x+1/2*e)^2*B*d^3+2/f*a/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*
x+1/2*e)^3*A*d^3-5/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*
c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*A*d+1/f*a/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A*d^2-2/f*a/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*B*c^2-2/f*a/
(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*
d)/(c^2-d^2)^(1/2))*B*d+1/f*a/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(
1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+1/f*a/(c*tan(1/2*f*x+
```

```
1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*B*c*d+2/f*a/(c^3
+c^2*d-c*d^2-d^3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*A*c+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^3*A*d^2-1/f*a/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/
2*e)*B+2/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d
-c*d^2-d^3)*A*c*d-4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)^2*B*d^2-1/f*a/(c^3+c^2*d-c*d^2-d^
3)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*A*d+6/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c
*d^2-d^3)*tan(1/2*f*x+1/2*e)*A*d^2+4/f*a/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)*B*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.47855, size = 2151, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*
c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A
+ 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 -
 (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2
)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))
*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
```

```
 2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)
*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d
^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3
*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 -
 c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)*a*c^3*d - (
2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*si
n(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A
+ 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2
*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-
(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*(A + B)*a*c^4 - (
2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*d^3 + A*a*d^4)*cos
(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos
(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)
*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*
d^6 + d^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29728, size = 802, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d -
c*d^2 - d^3)*sqrt(c^2 - d^2)) + (B*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 3*A*a*c^3
*d*tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c^2*
```

```
d^2*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a*c^4
*tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^3*d*ta
n(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*d^2*tan
(1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*A*a*c*d^3*ta
n(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*d^4*tan(1
/2*f*x + 1/2*e)^2 - B*a*c^4*tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*tan(1/2*f*x
+ 1/2*e) - 6*B*a*c^3*d*tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*tan(1/2*f*x + 1
/2*e) + 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*
e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2*d^2)/((c^5 +
 c^4*d - c^3*d^2 - c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1
/2*e) + c)^2))/f
```

3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=464

$$\frac{a^2\left(6Ad\left(-44c^2d^2-10c^3d+c^4-40cd^3-12d^4\right)-B\left(47c^3d^2+208c^2d^3-12c^4d+2c^5+216cd^4+64d^5\right)\right)\cos(e+fx)}{60d^2f}+$$

```
[Out] (a^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c
*d^2 + 11*d^3))*x)/16 + (a^2*(6*A*d*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3
 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 + 208*c^2*d^3 + 216*c*d^4 + 6
4*d^5))*Cos[e + f*x])/(60*d^2*f) + (a^2*(6*A*d*(2*c^3 - 20*c^2*d - 57*c*d^2
 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c*d^3 + 165*d^4))*Cos[e
 + f*x]*Sin[e + f*x])/(240*d*f) + (a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2) - B*(
2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)
/(120*d^2*f) + (a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^2*(2*B*c - 6*A*d - 7*B*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rubi [A] time = 0.95209, antiderivative size = 464, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^2\left(6Ad\left(-44c^2d^2-10c^3d+c^4-40cd^3-12d^4\right)-B\left(47c^3d^2+208c^2d^3-12c^4d+2c^5+216cd^4+64d^5\right)\right)\cos(e+fx)}{60d^2f}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (a^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c
*d^2 + 11*d^3))*x)/16 + (a^2*(6*A*d*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3
 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 + 208*c^2*d^3 + 216*c*d^4 + 6
4*d^5))*Cos[e + f*x])/(60*d^2*f) + (a^2*(6*A*d*(2*c^3 - 20*c^2*d - 57*c*d^2
 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c*d^3 + 165*d^4))*Cos[e
 + f*x]*Sin[e + f*x])/(240*d*f) + (a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2) - B*(
2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)
/(120*d^2*f) + (a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^2*(2*B*c - 6*A*d - 7*B*d)*C
os[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2
```

```
*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
```

```
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+f}{6df}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+f}{6df}$$

$$= \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f}$$

$$= \frac{a^2\left(6A(c-10d)d-B\left(2c^2-12cd+55d^2\right)\right)\cos(e+f}{120d^2f}$$

$$= \frac{a^2\left(6Ad\left(c^2-10cd-12d^2\right)-B\left(2c^3-12c^2d+51cd^2\right.\right.}{120d^2f}$$

$$= \frac{1}{16}a^2\left(6A\left(4c^3+8c^2d+7cd^2+2d^3\right)+B\left(16c^3+42c\right.\right.$$

Mathematica [A] time = 3.14323, size = 437, normalized size = 0.94

$$-\frac{a^2\cos(e+fx)\left(60\left(6A\left(8c^2d+4c^3+7cd^2+2d^3\right)+B\left(42c^2d+16c^3+36cd^2+11d^3\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3
 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] +
Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*B*c^2*d +
 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*(5*c^2 + 1
0*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2*(e + f*x)]
 + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Sin[e + f*x] +
 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c^2*d*Sin[e +
f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x] + 540*A*d^3*Si
n[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e + f*x)] - 90*A*c*
d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*d^3*Sin[3*(e + f
*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)])))/(480*f*Sqrt[
```

```
Cos[e + f*x]^2])
```

Maple [A] time = 0.077, size = 745, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(A*a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*a^2*c^2*d*(2+si
n(f*x+e)^2)*cos(f*x+e)+3*A*a^2*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)-1/5*A*a^2*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)-1/3*B*a^2*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^2*c^2*d*(-1/4*(si
n(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^2*c*d^2*(8/3+s
in(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d^3*(-1/6*(sin(f*x+e)^5+5/4*
sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-2*A*a^2*c^3*cos(f
*x+e)+6*A*a^2*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*A*a^2*c*d^
2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*a^2*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e
))*cos(f*x+e)+3/8*f*x+3/8*e)+2*B*a^2*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*
x+1/2*e)-2*B*a^2*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^2*c*d^2*(-1/4*(sin
(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^2*d^3*(8/3+sin(
f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*a^2*c^3*(f*x+e)-3*A*a^2*c^2*d*cos(f
*x+e)+3*A*a^2*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*A*a^2*d^
3*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^3*cos(f*x+e)+3*B*a^2*c^2*d*(-1/2*sin(
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+B*
a^2*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Maxima [A] time = 1.02168, size = 977, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2
*c^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e -
```

```
 sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c^2*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(cos(f*x +
 e)^3 - 3*cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*
a^2*c^2*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d^2 + 90*(12*f*x
 + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*(2*f*x +
 2*e - sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x +
 e)^3 + 15*cos(f*x + e))*B*a^2*c*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e)
)*B*a^2*c*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))
*B*a^2*c*d^2 - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*
A*a^2*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^3 + 60*(12*f*x +
12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^3 - 128*(3*cos(f*x +
e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^3 + 5*(4*sin(2*f*x + 2*
e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*d^3
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^3 - 19
20*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e) - 2880*A*a^2*c^2*d*c
os(f*x + e))/f
```

Fricas [A] time = 2.58673, size = 844, normalized size = 1.82

$$-\frac{48\left(3\,Ba^2cd^2+(A+2\,B)a^2d^3\right)\cos\left(fx+e\right)^5-80\left(Ba^2c^3+3\,(A+2\,B)a^2c^2d+3\,(2\,A+3\,B)a^2cd^2+(3\,A+4\,B)a^2d^3\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*c
^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^3)
*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d + 6*(
7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^3 +
3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x + e) +
 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2*c*d^2
 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 + 6*(8*A
+ 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3)*cos(f*
x + e))*sin(f*x + e))/f
```

Sympy [A] time = 11.8445, size = 1865, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((A*a**2*c**3*x*sin(e + f*x)**2/2 + A*a**2*c**3*x*cos(e + f*x)**2/
2 + A*a**2*c**3*x - A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**3*cos(e + f*x)/f + 3*A*a**2*c**2*d*x*sin(e + f*x)**2 + 3*A*a**2*c**2*d*x
*cos(e + f*x)**2 - 3*A*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**
2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a**2*c**2*d*cos(e + f*x)**3/f -
3*A*a**2*c**2*d*cos(e + f*x)/f + 9*A*a**2*c*d**2*x*sin(e + f*x)**4/8 + 9*A*
a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**2*c*d**2*x*sin(e +
 f*x)**2/2 + 9*A*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**2*c*d**2*x*cos(e
+ f*x)**2/2 - 15*A*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*A*a**
2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**2*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 3*A*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a
**2*c*d**2*cos(e + f*x)**3/f + 3*A*a**2*d**3*x*sin(e + f*x)**4/4 + 3*A*a**2
*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**2*d**3*x*cos(e + f*x)**4
/4 - A*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*d**3*sin(e + f*x
)**3*cos(e + f*x)/(4*f) - 4*A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*
f) - A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*d**3*sin(e + f*x
)*cos(e + f*x)**3/(4*f) - 8*A*a**2*d**3*cos(e + f*x)**5/(15*f) - 2*A*a**2*d
**3*cos(e + f*x)**3/(3*f) + B*a**2*c**3*x*sin(e + f*x)**2 + B*a**2*c**3*x*c
os(e + f*x)**2 - B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c**3*s
in(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c**3*cos(e + f*x)**3/(3*f) - B*a**2*c
**3*cos(e + f*x)/f + 9*B*a**2*c**2*d*x*sin(e + f*x)**4/8 + 9*B*a**2*c**2*d*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**2*c**2*d*x*sin(e + f*x)**2/2 +
 9*B*a**2*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**2*c**2*d*x*cos(e + f*x)**2/2
- 15*B*a**2*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*B*a**2*c**2*d*sin
(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 3*B*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*c**2*d*c
os(e + f*x)**3/f + 9*B*a**2*c*d**2*x*sin(e + f*x)**4/4 + 9*B*a**2*c*d**2*x*
sin(e + f*x)**2*cos(e + f*x)**2/2 + 9*B*a**2*c*d**2*x*cos(e + f*x)**4/4 - 3
*B*a**2*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**2*c*d**2*sin(e + f*
x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 3*B*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**2*c*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(4*f) - 8*B*a**2*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*
a**2*c*d**2*cos(e + f*x)**3/f + 5*B*a**2*d**3*x*sin(e + f*x)**6/16 + 15*B*a
**2*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**2*d**3*x*sin(e + f*x
)**4/8 + 15*B*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*d**
3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*B*a**2*d**3*x*cos(e + f*x)**6/16
```

```
+ 3*B*a**2*d**3*x*cos(e + f*x)**4/8 - 11*B*a**2*d**3*sin(e + f*x)**5*cos(e
+ f*x)/(16*f) - 2*B*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**
3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*B*a**2*d**3*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 8*B*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*B
*a**2*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*d**3*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 16*B*a**2*d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.26544, size = 640, normalized size = 1.38

$$-\frac{Ba^2d^3\sin\left(6fx+6e\right)}{192f}+\frac{1}{16}\left(24Aa^2c^3+16Ba^2c^3+48Aa^2c^2d+42Ba^2c^2d+42Aa^2cd^2+36Ba^2cd^2+12Aa^2d^3+11\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 + 4
8*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a^2
*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*cos
(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d + 24*
A*a^2*c*d^2 + 27*B*a^2*c*d^2 + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x + 3*e)
/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2*d + 3
6*A*a^2*c*d^2 + 33*B*a^2*c*d^2 + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*x + e)/
f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c*d^2 + 12*B*a^2*c*d^2 + 4*A*a^2*d^3 + 5*
B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3 + 96*A*a^
2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c*d^2 + 96*B*a^2*c*d^2 + 32*A*a^2*d^3 +
 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f
```

3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=336

$$\frac{a^2\left(5Ad\left(-8c^2d+c^3-20cd^2-8d^3\right)-2B\left(16c^2d^2-5c^3d+c^4+40cd^3+18d^4\right)\right)\cos(e+fx)}{30d^2f}+\frac{a^2\left(5Ad(c-8d)-2B\left(c^2-\right.\right.}{}$$

```
[Out] (a^2*(12*A*c^2 + 8*B*c^2 + 16*A*c*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/8 +
(a^2*(5*A*d*(c^3 - 8*c^2*d - 20*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^
2*d^2 + 40*c*d^3 + 18*d^4))*Cos[e + f*x])/(30*d^2*f) + (a^2*(5*A*d*(2*c^2 -
 16*c*d - 21*d^2) - B*(4*c^3 - 20*c^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*
Sin[e + f*x])/(120*d*f) + (a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2
))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d^2*f) + (a^2*(2*B*(c - 3*d) -
5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d^2*f) - (B*Cos[e + f*x]*(a
^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(5*d*f)
```

Rubi [A] time = 0.703095, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^2\left(5Ad\left(-8c^2d+c^3-20cd^2-8d^3\right)-2B\left(16c^2d^2-5c^3d+c^4+40cd^3+18d^4\right)\right)\cos(e+fx)}{30d^2f}+\frac{a^2\left(5Ad(c-8d)-2B\left(c^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```


```
[Out] (a^2*(12*A*c^2 + 8*B*c^2 + 16*A*c*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/8 +
(a^2*(5*A*d*(c^3 - 8*c^2*d - 20*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^
2*d^2 + 40*c*d^3 + 18*d^4))*Cos[e + f*x])/(30*d^2*f) + (a^2*(5*A*d*(2*c^2 -
 16*c*d - 21*d^2) - B*(4*c^3 - 20*c^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*
Sin[e + f*x])/(120*d*f) + (a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2
))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d^2*f) + (a^2*(2*B*(c - 3*d) -
5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d^2*f) - (B*Cos[e + f*x]*(a
^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(5*d*f)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx}{5df}$$
$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx}{5df}$$
$$= \frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f}$$
$$= \frac{a^2\left(5A(c-8d)d-2B\left(c^2-5cd+18d^2\right)\right)\cos(e+fx)(c}{60d^2f}$$
$$= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad^2+6Bd^2\right)$$

Mathematica [A] time = 1.52556, size = 296, normalized size = 0.88

$$-\frac{a^2\cos(e+fx)\left(60\left(A\left(12c^2+16cd+7d^2\right)+2B\left(4c^2+7cd+3d^2\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(-8\left(10Ad(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16*c*d +
7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(480*
A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2 - 8*(10*A
*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*d^2*Cos[4*
(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 480*A*c*d*Si
n[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] + 270*B*d^2*Si
n[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f*x)] - 30*B*d
^2*Sin[3*(e + f*x)])))/(240*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.066, size = 496, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(A*a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*A*a^2*c*d*(2+
sin(f*x+e)^2)*cos(f*x+e)+A*a^2*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(
f*x+e)+3/8*f*x+3/8*e)-1/3*B*a^2*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*B*a^2*c*d
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^2*d^
2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*A*a^2*c^2*cos(f*x+e)+4*A
*a^2*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*A*a^2*d^2*(2+sin(f*
x+e)^2)*cos(f*x+e)+2*B*a^2*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4
/3*B*a^2*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+2*B*a^2*d^2*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+A*a^2*c^2*(f*x+e)-2*A*a^2*c*d*cos(f
*x+e)+A*a^2*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^2*cos(f*
x+e)+2*B*a^2*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*d^2*(
2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 0.992176, size = 645, normalized size = 1.92

$$\frac{120\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^2c^2+480\left(fx+e\right)Aa^2c^2+160\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Ba^2c^2+240\left(2fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2
*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*
c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e)^3
- 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(
2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c*d +
320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x +
 e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2 + 30*(12*f
*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 960*A*a^2*c^
2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos(f*x + e))/f
```

Fricas [A] time = 2.26476, size = 574, normalized size = 1.71

$$-\frac{24\,Ba^2d^2\cos\left(fx+e\right)^5-40\left(Ba^2c^2+2\,(A+2\,B)a^2cd+(2\,A+3\,B)a^2d^2\right)\cos\left(fx+e\right)^3-15\left(4\,(3\,A+2\,B)a^2c^2+2\,(8\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/120*(24*B*a^2*d^2*cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d +
 (2*A + 3*B)*a^2*d^2)*cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A +
 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A + B)
*a^2*c*d + (A + B)*a^2*d^2)*cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*B)*a
^2*d^2)*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d + (9*
A + 10*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 5.84911, size = 1129, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((A*a**2*c**2*x*sin(e + f*x)**2/2 + A*a**2*c**2*x*cos(e + f*x)**2/
2 + A*a**2*c**2*x - A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*
c**2*cos(e + f*x)/f + 2*A*a**2*c*d*x*sin(e + f*x)**2 + 2*A*a**2*c*d*x*cos(e
 + f*x)**2 - 2*A*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A*a**2*c*d*sin
(e + f*x)*cos(e + f*x)/f - 4*A*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*A*a**2*c*
d*cos(e + f*x)/f + 3*A*a**2*d**2*x*sin(e + f*x)**4/8 + 3*A*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*d**2*x*sin(e + f*x)**2/2 + 3*A*a**2*
d**2*x*cos(e + f*x)**4/8 + A*a**2*d**2*x*cos(e + f*x)**2/2 - 5*A*a**2*d**2*
sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a**2*d**2*sin(e + f*x)**2*cos(e +
f*x)/f - 3*A*a**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*d**2*sin
(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a**2*d**2*cos(e + f*x)**3/(3*f) + B*a**2
*c**2*x*sin(e + f*x)**2 + B*a**2*c**2*x*cos(e + f*x)**2 - B*a**2*c**2*sin(e
 + f*x)**2*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a
**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e + f*x)/f + 3*B*a**2*c*d*
x*sin(e + f*x)**4/4 + 3*B*a**2*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*
a**2*c*d*x*sin(e + f*x)**2 + 3*B*a**2*c*d*x*cos(e + f*x)**4/4 + B*a**2*c*d*
```

```
x*cos(e + f*x)**2 - 5*B*a**2*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a
**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*c*d*sin(e + f*x)*cos(e +
f*x)**3/(4*f) - B*a**2*c*d*sin(e + f*x)*cos(e + f*x)/f - 8*B*a**2*c*d*cos(e
 + f*x)**3/(3*f) + 3*B*a**2*d**2*x*sin(e + f*x)**4/4 + 3*B*a**2*d**2*x*sin(
e + f*x)**2*cos(e + f*x)**2/2 + 3*B*a**2*d**2*x*cos(e + f*x)**4/4 - B*a**2*
d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**2*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - 4*B*a**2*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*
d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*d**2*sin(e + f*x)*cos(e + f*
x)**3/(4*f) - 8*B*a**2*d**2*cos(e + f*x)**5/(15*f) - 2*B*a**2*d**2*cos(e +
f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a
)**2, True))
```

Giac [A] time = 1.30365, size = 420, normalized size = 1.25

$$-\frac{Ba^2d^2\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(12\,Aa^2c^2+8\,Ba^2c^2+16\,Aa^2cd+14\,Ba^2cd+7\,Aa^2d^2+6\,Ba^2d^2\right)x+\frac{\left(4\,Ba^2c^2+8\,Aa^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/80*B*a^2*d^2*cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*A
*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^2
+ 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*cos(3*f*x + 3*e)/
f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12*A*a
^2*d^2 + 11*B*a^2*d^2)*cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2 + 2*B
*a^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d +
 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*sin(2*f*x + 2*e)/f
```

3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=166

$$-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\cos(e+fx)}{6f}-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x(12Ac+8Ad$$

```
[Out] (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*x)/8 - (a^2*(12*A*c + 8*B*c + 8*A*d +
 7*B*d)*Cos[e + f*x])/(6*f) - (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*Cos[e +
 f*x]*Sin[e + f*x])/(24*f) - ((4*B*c + 4*A*d - B*d)*Cos[e + f*x]*(a + a*Sin
[e + f*x])^2)/(12*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```

Rubi [A] time = 0.270738, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2751, 2644}

$$-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\cos(e+fx)}{6f}-\frac{a^2(12Ac+8Ad+8Bc+7Bd)\sin(e+fx)\cos(e+fx)}{24f}+\frac{1}{8}a^2x(12Ac+8Ad$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```


```
[Out] (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*x)/8 - (a^2*(12*A*c + 8*B*c + 8*A*d +
 7*B*d)*Cos[e + f*x])/(6*f) - (a^2*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*Cos[e +
 f*x]*Sin[e + f*x])/(24*f) - ((4*B*c + 4*A*d - B*d)*Cos[e + f*x]*(a + a*Sin
[e + f*x])^2)/(12*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*a*f)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^2\left(Ac+(Bc+Ad)\sin(e+fx)+\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af} + \frac{\int (a+a\sin(e}{}$$

$$= -\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^2}{12f} -$$

$$= \frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x - \frac{a^2(12Ac+8Bc+}{}$$

Mathematica [A] time = 0.747391, size = 160, normalized size = 0.96

$$-\frac{a^2\cos(e+fx)\left(6(12Ac+8Ad+8Bc+7Bd)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8(Ad+B(c+2d))\sin^2(e+fx)+3\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -(a^2*Cos[e + f*x]*(6*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*ArcSin[Sqrt[1 - Sin[
e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(6*A*c + 5*B*c + 5*A*d + 4*B*d
```

```
) + 3*(4*A*c + 8*B*c + 8*A*d + 7*B*d)*Sin[e + f*x] + 8*(A*d + B*(c + 2*d))*
Sin[e + f*x]^2 + 6*B*d*Sin[e + f*x]^3)))/(24*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.051, size = 278, normalized size = 1.7

$$\frac{1}{f}\left(Aa^2c\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{Aa^2d\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}-\frac{Ba^2c\left(2+\left(\sin\left(fx+e\right)\right)^2\right)}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(A*a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*A*a^2*d*(2+sin(
f*x+e)^2)*cos(f*x+e)-1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*A*a^2*c*cos(f*x+e
)+2*A*a^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*B*a^2*c*(-1/2*sin(
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*B*a^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+A*
a^2*c*(f*x+e)-A*a^2*d*cos(f*x+e)-B*a^2*c*cos(f*x+e)+B*a^2*d*(-1/2*sin(f*x+e
)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A] time = 0.966241, size = 362, normalized size = 2.18

$$\frac{24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c+96\left(fx+e\right)Aa^2c+32\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba^2c+48\left(2\,fx+2\,e-\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c +
32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*x
+ 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*
a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d +
 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x + e) - 9
6*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f
```

Fricas [A] time = 2.03709, size = 343, normalized size = 2.07

$$\frac{8\left(Ba^2c+(A+2B)a^2d\right)\cos\left(fx+e\right)^3+3\left(4(3A+2B)a^2c+(8A+7B)a^2d\right)fx-48\left((A+B)a^2c+(A+B)a^2d\right)\cos\left(f\right.}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*c
 + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x + e)
 + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)*co
s(f*x + e))*sin(f*x + e))/f
```

Sympy [A] time = 2.22361, size = 571, normalized size = 3.44

$$\begin{cases}\frac{Aa^2cx\sin^2\left(e+fx\right)}{2}+\frac{Aa^2cx\cos^2\left(e+fx\right)}{2}+Aa^2cx-\frac{Aa^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aa^2c\cos\left(e+fx\right)}{f}+Aa^2dx\sin^2\left(e+fx\right)+Aa^2dx\cos^2 \\ x\left(A+B\sin\left(e\right)\right)\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A*
a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e + f*
x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*d*s
in(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*
a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*sin(e +
 f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(e + f*
x)/f - B*a**2*c*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x)**3/(3
*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*B*a**2*d
*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/2 + 3*B*a
**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a**2*d*sin(e
 + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e + f*x)/f -
 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e + f*x)*cos(
e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin
(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.24558, size = 232, normalized size = 1.4

$$\frac{Ba^2d\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{1}{8}\left(12\,Aa^2c+8\,Ba^2c+8\,Aa^2d+7\,Ba^2d\right)x+\frac{\left(Ba^2c+Aa^2d+2\,Ba^2d\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(8\,Aa^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d +
 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f - 1
/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*(A*
a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f
```

3.254 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=94

$$-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3A+2B)-\frac{B\cos(e+fx)(a\sin(e+fx)+a}{3f}$$

```
[Out] (a^2*(3*A + 2*B)*x)/2 - (2*a^2*(3*A + 2*B)*Cos[e + f*x])/(3*f) - (a^2*(3*A
+ 2*B)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rubi [A] time = 0.0607967, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2751, 2644}

$$-\frac{2a^2(3A+2B)\cos(e+fx)}{3f}-\frac{a^2(3A+2B)\sin(e+fx)\cos(e+fx)}{6f}+\frac{1}{2}a^2x(3A+2B)-\frac{B\cos(e+fx)(a\sin(e+fx)+a}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*x)/2 - (2*a^2*(3*A + 2*B)*Cos[e + f*x])/(3*f) - (a^2*(3*A
+ 2*B)*Cos[e + f*x]*Sin[e + f*x])/(6*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*
x])^2)/(3*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))\,dx = -\frac{B\cos(e + fx)(a + a\sin(e + fx))^2}{3f} + \frac{1}{3}(3A + 2B)\int (a + a\sin(e + fx))^2$$
$$= \frac{1}{2}a^2(3A + 2B)x - \frac{2a^2(3A + 2B)\cos(e + fx)}{3f} - \frac{a^2(3A + 2B)\cos(e + fx)\,\mathrm{s}}{6f}$$

Mathematica [A] time = 0.319729, size = 106, normalized size = 1.13

$$-\frac{a^2\cos(e + fx)\left(6(3A + 2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right) + \sqrt{\cos^2(e + fx)}\left(3(A + 2B)\sin(e + fx) + 2(6A + 5B) + 2B\sin^2(e + fx\right.\right.}{6f\sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a^2*Cos[e + f*x]*(6*(3*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] +
Sqrt[Cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*Sin[e + f*x] + 2*B*Sin[e
+ f*x]^2)))/(6*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.04, size = 117, normalized size = 1.2

$$\frac{1}{f}\left(Aa^2\left(-\frac{\sin(fx + e)\cos(fx + e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - \frac{Ba^2\left(2 + (\sin(fx + e))^2\right)\cos(fx + e)}{3} - 2\,Aa^2\cos(fx + e) + 2\,Ba^2\,(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

```
[Out] 1/f*(A*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*(2+sin(f*x+
e)^2)*cos(f*x+e)-2*A*a^2*cos(f*x+e)+2*B*a^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)+A*a^2*(f*x+e)-B*a^2*cos(f*x+e))
```

Maxima [A] time = 0.976525, size = 154, normalized size = 1.64

$$\frac{3\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2 + 12\,(fx + e)Aa^2 + 4\left(\cos(fx + e)^3 - 3\,\cos(fx + e)\right)Ba^2 + 6\,(2\,fx + 2\,e - \sin(2\,fx}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(co
s(f*x + e)^3 - 3*cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))*B
*a^2 - 24*A*a^2*cos(f*x + e) - 12*B*a^2*cos(f*x + e))/f
```

Fricas [A] time = 1.9591, size = 176, normalized size = 1.87

$$\frac{2\,Ba^2\cos\left(fx+e\right)^3+3\,(3\,A+2\,B)a^2fx-3\,(A+2\,B)a^2\cos\left(fx+e\right)\sin\left(fx+e\right)-12\,(A+B)a^2\cos\left(fx+e\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*a^2*cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*cos(f
*x + e)*sin(f*x + e) - 12*(A + B)*a^2*cos(f*x + e))/f
```

Sympy [A] time = 0.918068, size = 199, normalized size = 2.12

$$\begin{cases}\frac{Aa^2x\sin^2\left(e+fx\right)}{2}+\frac{Aa^2x\cos^2\left(e+fx\right)}{2}+Aa^2x-\frac{Aa^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aa^2\cos\left(e+fx\right)}{f}+Ba^2x\sin^2\left(e+fx\right)+Ba^2x\cos^2\left(e+f\right. \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^2\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**2
*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B*a
**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**2*c
os(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f*x)*
*3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
a)**2, True))
```

Giac [A] time = 1.2568, size = 119, normalized size = 1.27

$$\frac{Ba^2\cos\left(3\,fx+3\,e\right)}{12\,f}+\frac{1}{2}\left(3\,Aa^2+2\,Ba^2\right)x-\frac{\left(8\,Aa^2+7\,Ba^2\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(Aa^2+2\,Ba^2\right)\sin\left(2\,fx+2\,e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/12*B*a^2*cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 +
7*B*a^2)*cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*sin(2*f*x + 2*e)/f
```

3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=171

$$-\frac{2a^2(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{2d^3}+\frac{a^2(-2Ad+2Bc-3Bd)\cos}{2d^2f}$$

```
[Out] -(a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/(2*d^3) - (2*a^2*(c
- d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*S
qrt[c^2 - d^2]*f) + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(2*d^2*f) -
(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```

Rubi [A] time = 0.52178, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2976, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^2(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\sqrt{c^2-d^2}}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{2d^3}+\frac{a^2(-2Ad+2Bc-3Bd)\cos}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```



```
[Out] -(a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/(2*d^3) - (2*a^2*(c
- d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*S
qrt[c^2 - d^2]*f) + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(2*d^2*f) -
(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
```

```
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} + \frac{\int \frac{(a+a\sin(e+fx))(a(Bc+2Ad)-a(2Bc-2}{c+d\sin(e+fx)}}{2d}$$

$$= -\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df} + \frac{\int \frac{a^2(Bc+2Ad)+\left(a^2(Bc+2Ad)-a^2(2Bc-2}{}}{}$$

$$= \frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f} - \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2df}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} + \frac{a^2(2Bc-2Ad-3Bd)\cos}{2d^2f}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} + \frac{a^2(2Bc-2Ad-3Bd)\cos}{2d^2f}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} + \frac{a^2(2Bc-2Ad-3Bd)\cos}{2d^2f}$$

$$= -\frac{a^2\left(2A(c-2d)d-B\left(2c^2-4cd+3d^2\right)\right)x}{2d^3} - \frac{2a^2(c-d)^2(Bc-Ad)\tan}{d^3\sqrt{c^2-}}$$

Mathematica [A] time = 0.629303, size = 177, normalized size = 1.04

$$\frac{a^2(\sin(e+fx)+1)^2\left(2(e+fx)\left(2Ad(2d-c)+B\left(2c^2-4cd+3d^2\right)\right)-\frac{8(c-d)^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-4d(Ad-Bc+\right.}{4d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2))
*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] - B*
d^2*Sin[2*(e + f*x)]))/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [B] time = 0.143, size = 713, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*A*c^2-4/f*a^2/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*A*c+2/f*a^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2
*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-2/f*a^2/d^3/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+4/f*a^2/d^2/(c^2-d^2)^(
1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a^2
/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*B*c+1/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3-2/f*a^2/d/
(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2+2/f*a^2/d^2/(1+tan(1/2*f*
x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2*c-4/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2
*B*tan(1/2*f*x+1/2*e)^2-1/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+
1/2*e)-2/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*A+2/f*a^2/d^2/(1+tan(1/2*f*x+1/
2*e)^2)^2*B*c-4/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B-2/f*a^2/d^2*arctan(tan
(1/2*f*x+1/2*e))*A*c+4/f*a^2/d*arctan(tan(1/2*f*x+1/2*e))*A+2/f*a^2/d^3*arc
tan(tan(1/2*f*x+1/2*e))*B*c^2-4/f*a^2/d^2*arctan(tan(1/2*f*x+1/2*e))*B*c+3/
f*a^2/d*arctan(tan(1/2*f*x+1/2*e))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.29636, size = 1029, normalized size = 6.02

$$\left[-\frac{Ba^2d^2\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(2\,Ba^2c^2-2\,(A+2\,B)a^2cd+(4\,A+3\,B)a^2d^2\right)fx+\left(Ba^2c^2-(A+B)a^2cd+Aa^2d^2\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x + (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)
*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
+ e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*c
os(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f),
 -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2
*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^
2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))
/((c - d)*cos(f*x + e))) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/
(d^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26906, size = 424, normalized size = 2.48

$$\frac{\frac{\left(2\,Ba^2c^2-2\,Aa^2cd-4\,Ba^2cd+4\,Aa^2d^2+3\,Ba^2d^2\right)\left(fx+e\right)}{d^3}-\frac{4\left(Ba^2c^3-Aa^2c^2d-2\,Ba^2c^2d+2\,Aa^2cd^2+Ba^2cd^2-Aa^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{c^2-d^2}}\right.\right.}{\sqrt{c^2-d^2}\,d^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)*
(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^2
+ B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3) + 2*
(B*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^
2*d*tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*tan(1/2*f*x + 1/2*e)^2 - B*a^2*d*tan
(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((tan(1/2*f*x + 1/2*
e)^2 + 1)^2*d^2))/f
```

3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=198

$$-\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}+\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2f(c+d)}-\frac{a^2x(-Ad+2}{d^3}$$

```
[Out] -((a^2*(2*B*c - A*d - 2*B*d)*x)/d^3) - (2*a^2*(c - d)*(A*d*(c + 2*d) - B*(2
*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3
*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(A*d - B*(2*c + d))*Cos[e + f*x])/(d^2*(
c + d)*f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(d*(c + d)*
f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.580826, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2975, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)\sqrt{c^2-d^2}}+\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2f(c+d)}-\frac{a^2x(-Ad+2}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((a^2*(2*B*c - A*d - 2*B*d)*x)/d^3) - (2*a^2*(c - d)*(A*d*(c + 2*d) - B*(2
*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3
*(c + d)*Sqrt[c^2 - d^2]*f) + (a^2*(A*d - B*(2*c + d))*Cos[e + f*x])/(d^2*(
c + d)*f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(d*(c + d)*
f*(c + d*Sin[e + f*x]))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))(-a(B(c-d)-\ldots}{c+d\,\mathrm{s}\ldots}}{d(\ldots}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))} + \int \frac{-a^2(B(c-d)-2Ad)+\left(-a^2(B(c-\ldots\right.}{\ldots}$$

$$= \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e\ldots\right.}{d(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\ldots}{d(\ldots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\ldots}{d(\ldots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\ldots}{d(\ldots}$$

$$= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} - \frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\ldots}{d^3(c+d)\sqrt{c^2-d^2}\,f}$$

Mathematica [A] time = 1.00535, size = 192, normalized size = 0.97

$$\frac{a^2(\sin(e+fx)+1)^2\left(\frac{2(c-d)\left(B\left(2c^2+2cd-d^2\right)-Ad(c+2d)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}+(e+fx)(Ad-2Bc+2Bd)-\frac{d(d-c)(Ad-Bc)\cos(e+fx)}{(c+d)(c+d\sin(e+fx))}\ldots\right.}{d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(-
(A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/
Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c + d
```

```
)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^3*f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Maple [B] time = 0.161, size = 848, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x
+1/2*e)*A-2/f*a^2*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)*A-2/f*a^2/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/2*e)*B+2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B+2/f*a^2/d/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c-2/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+
2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2/f*a^2/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^2+2/f*a^2/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/
2*f*x+1/2*e)*d+c)/(c+d)*B*c-2/f*a^2/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2/f*a^2/d/(c+d)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+4/f*a^2
/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*A+4/f*a^2/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)
+2*d)/(c^2-d^2)^(1/2))*B*c^3-6/f*a^2/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*
c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2/f*a^2/(c+d)/(c^2-d^2)^(1/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B-2/f*a^2/d^2*B/
(1+tan(1/2*f*x+1/2*e)^2)+2/f*a^2/d^2*A*arctan(tan(1/2*f*x+1/2*e))-4/f*a^2/d
^3*B*arctan(tan(1/2*f*x+1/2*e))*c+4/f*a^2/d^2*B*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.56487, size = 1589, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c
^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)
*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((
2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*
d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c +
 d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B*a^2*c
^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A*a^2*c*
d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(
f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*B*a^2*
c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A - 2*B)*a^2
*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*d^2 - (2*A
+ B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
 d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c^2*d - A*a^2*
c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d^2 - (A + 2*B)
*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/((c*d
^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35175, size = 672, normalized size = 3.39

$$\frac{\frac{2\left(2\,Ba^2c^3-Aa^2c^2d-Aa^2cd^2-3\,Ba^2cd^2+2\,Aa^2d^3+Ba^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^3+d^4\right)\sqrt{c^2-d^2}}-\frac{2\left(Ba^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Aa^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3 +
 B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*
x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B*a^
2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B*a^2
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a^2*c*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a^2*c*d^
2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d^3*tan(1
/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan(1/2*f*x
+ 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*
tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^2*d - 2*B*a
^2*d)*(f*x + e)/d^3)/f
```

3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$\frac{a^2\left(3Ad^3-B\left(4c^2d+2c^3+cd^2-4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(Bc-$$

```
[Out] (a^2*B*x)/d^3 + (a^2*(3*A*d^3 - B*(2*c^3 + 4*c^2*d + c*d^2 - 4*d^3))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*
f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - (a^2*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f
*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.622593, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2975, 2968, 3021, 2735, 2660, 618, 204}

$$\frac{a^2\left(3Ad^3-B\left(4c^2d+2c^3+cd^2-4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f(c+d)^2\sqrt{c^2-d^2}}-\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{2d^2f(c+d)^2(c+d\sin(e+fx))}+\frac{(Bc-$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```


```
[Out] (a^2*B*x)/d^3 + (a^2*(3*A*d^3 - B*(2*c^3 + 4*c^2*d + c*d^2 - 4*d^3))*ArcTan
[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^3*(c + d)^2*Sqrt[c^2 - d^2]*
f) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - (a^2*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f
*x])/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
```

```
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))(-a(Bc-3Ad}{(c+d\sin(}}{2d(c}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{-a^2(Bc-3Ad-2Bd)+\left(2a^2B(c}{}}{}$$

$$= \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2-B\left(2c^2+3cd\right.\right.}{2d^2(c+d)^2f(c+}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2-B\left(2\right.\right.}{2d^2(c+d}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2-B\left(2\right.\right.}{2d^2(c+d}$$

$$= \frac{a^2Bx}{d^3} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(3Ad^2-B\left(2\right.\right.}{2d^2(c+d}$$

$$= \frac{a^2Bx}{d^3} - \frac{a^2\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}f}$$

Mathematica [A] time = 1.38937, size = 226, normalized size = 1.05

$$\frac{a^2(\sin(e+fx)+1)^2\left(-\frac{2\left(B\left(4c^2d+2c^3+cd^2-4d^3\right)-3Ad^3\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} - \frac{d\left(Ad(c+4d)+B\left(-3c^2-4cd+2d^2\right)\right)\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))} - \frac{d(d-c)(Ad-Bc)\,\text{co}}{(c+d)(c+d\sin(e}\right.}{2d^3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2*
d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c +
 d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*
(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2))*C
os[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^4)
```

Maple [B] time = 0.187, size = 1916, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 7/f*a^2/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*
d+d^2)*tan(1/2*f*x+1/2*e)*B-2/f*a^2/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arc
tan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3-4/f*a^2/d^2/(c^
2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*B*c^2+1/f*a^2/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*B-2/f*a^2*d^2/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*
B-2/f*a^2*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*
d+d^2)/c*tan(1/2*f*x+1/2*e)^3*A-1/f*a^2/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*a
rctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/f*a^2*d^3/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/
2*f*x+1/2*e)^2*A-2/f*a^2*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+2/f*a^2/d^2/(c*tan(1/2*f*x+1/2
*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*
B+4/f*a^2/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+
d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-8/f*a^2*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A+8/f*a^2*d/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^2*B-12/f*a^2*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-4/f*a^2*d/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-4/f*a^2*d/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*
x+1/2*e)^3*A-1/f*a^2*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/
(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A+3/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*ta
n(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B+1/f*a^2/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2
*f*x+1/2*e)^3*A+4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
```

```
/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*B-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*A+12/f*a^
2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan
(1/2*f*x+1/2*e)*B-1/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+2/f*a^2/d^2/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3+4/f*a^2/d/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2-1/f*a^2*d/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A+3/f*a^2/
(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2))*A+4/f*a^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B-4/f*a^2/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c-1/f*a^2/(c*tan(1/2*f*x+1/2*e)^
2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c+2/f*a^2*B/d^3*arctan(tan(
1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.76706, size = 3087, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*c
os(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*d^
4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*a^2
*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a^2*c
^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x + e)^
```

```
2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*a^2*c*
d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c
*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)
) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3 - (A + 4*B
)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) - 2*(4*(B*a^2
*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x + (3*B*a^2*c^
4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A - 4*B)*a^2*c*d
^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^5 + 2*c^3*d^6
 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5 - 2*c^2*d^7 - c
*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2*d^7 - 2*c*d^8 -
 d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d
^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a
^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d
+ 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (
2*B*a^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(
f*x + e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*
B)*a^2*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(s
qrt(c^2 - d^2)*cos(f*x + e))) - (2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3
*B)*a^2*c^3*d^3 - (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*
cos(f*x + e) - (4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*
c*d^5)*f*x + (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2
*d^4 + (A - 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e
))/((c^4*d^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2
*c^4*d^5 - 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d
^5 - c^2*d^7 - 2*c*d^8 - d^9)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.40316, size = 949, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^2
*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*ta
n(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c
^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/2*f*
x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^3*tan
(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c^5*ta
n(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^2*c^3*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*
c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 8
*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2
- 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e) -
 A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)
 - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan(1/2*f*x + 1/
2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B*a^2*c^4*d - 4
*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2
*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.258 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=604

$$-\frac{a^3\left(7Ad\left(107c^3d^2+472c^2d^3-18c^4d+2c^5+456cd^4+136d^5\right)-3B\left(51c^4d^2-189c^3d^3-920c^2d^4-14c^5d+2c^6-952cd\right.\right.}{420d^3f}$$

```
[Out] (a^3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 7
8*c*d^2 + 23*d^3))*x)/16 - (a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 189
*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/(420*d^3*f) -
(a^3*(7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4
*c^5 - 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e
+ f*x]*Sin[e + f*x])/(1680*d^2*f) - (a^3*(7*A*d*(2*c^3 - 18*c^2*d + 111*c*d
^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*d^3 - 864*d^4))*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(840*d^3*f) - (a^3*(7*A*d*(2*c^2 - 18*c
*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3))*Cos[e + f*x]*(c
 + d*Sin[e + f*x])^3)/(840*d^3*f) - (a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 9
1*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(210*d^3*f) - (a*B
*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(7*d*f) + ((3*
B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f
*x])^4)/(42*d^2*f)
```

Rubi [A] time = 1.48579, antiderivative size = 604, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$-\frac{a^3\left(7Ad\left(107c^3d^2+472c^2d^3-18c^4d+2c^5+456cd^4+136d^5\right)-3B\left(51c^4d^2-189c^3d^3-920c^2d^4-14c^5d+2c^6-952cd\right.\right.}{420d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```



```
[Out] (a^3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 7
8*c*d^2 + 23*d^3))*x)/16 - (a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 189
*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/(420*d^3*f) -
(a^3*(7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4
*c^5 - 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e
```

```
+ f*x]*Sin[e + f*x])/(1680*d^2*f) - (a^3*(7*A*d*(2*c^3 - 18*c^2*d + 111*c*d
^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*d^3 - 864*d^4))*C
os[e + f*x]*(c + d*Sin[e + f*x])^2)/(840*d^3*f) - (a^3*(7*A*d*(2*c^2 - 18*c
*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3))*Cos[e + f*x]*(c
 + d*Sin[e + f*x])^3)/(840*d^3*f) - (a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 9
1*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(210*d^3*f) - (a*B
*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(7*d*f) + ((3*
B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f
*x])^4)/(42*d^2*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

```
&& IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx)}{7df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx)}{7df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx)}{7df}$$

$$= -\frac{a^3\left(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2\right)\cos(e}{210d^3f}$$

$$= -\frac{a^3\left(7Ad\left(2c^2-18cd+115d^2\right)-B\left(6c^3-42c^2d+17\right.\right.}{840d^3}$$

$$= -\frac{a^3\left(7Ad\left(2c^3-18c^2d+111cd^2+136d^3\right)-B\left(6c^4-4\right.\right.}{}$$

$$= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A\left(40c^3+9\right.\right.$$

Mathematica [A] time = 4.76325, size = 528, normalized size = 0.87

$$-\frac{a^3\cos(e+fx)\left(420\left(A\left(90c^2d+40c^3+78cd^2+23d^3\right)+3B\left(26c^2d+10c^3+23cd^2+7d^3\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40
*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
```

```
] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2*d + 32676
*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*d^3 - (112*
A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*c^2*d + 2912
*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) + B*(14*c^2 + 4
2*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)] + 5040*A*c^3*
Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e + f*x] + 22050
*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*B*c*d^2*Sin[e +
f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] - 210*B*c^3*Sin[3*
(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*Sin[3*(e + f*x)] -
 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e + f*x)] - 980*A*d^3*
Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*c*d^2*Sin[5*(e + f*x
)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e + f*x)])))/(3360*f*Sqrt
[Cos[e + f*x]^2])
```

Maple [A] time = 0.092, size = 1077, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/f*(-B*a^3*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+9*B*a^3*c*d^2*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*A*a^3*c^2*d*cos(f*x+e)+3*A
*a^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*
B*a^3*c*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e
)+5/16*f*x+5/16*e)+3*B*a^3*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
+9*B*a^3*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e
)-1/3*A*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^3*(-1/4*(sin(f*x+e)^3+3
/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*A*a^3*c^3*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)+A*a^3*c^3*(f*x+e)+3*B*a^3*d^3*(-1/6*(sin(f*x+e)^5+5/4*
sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-3*A*a^3*c^3*cos(f
*x+e)-B*a^3*c^3*cos(f*x+e)+B*a^3*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*co
s(f*x+e)+3/8*f*x+3/8*e)+A*a^3*d^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8
*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*A*a^3*d^3*(-1/4*(sin(f*x+e)^3+3/
2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*B*a^3*c^3*(-1/2*sin(f*x+e)*cos(f*
x+e)+1/2*f*x+1/2*e)-9/5*B*a^3*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos
(f*x+e)-3*A*a^3*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)-3*B*a^3*c^2*d*(2+sin(f*x+
e)^2)*cos(f*x+e)-3/5*A*a^3*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*
x+e)-3/5*B*a^3*c^2*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*A*a^3
*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+9*A*a^3*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/7*B*a^3*d^3*(16/5+sin(f*x+e)^6+6/5*si
```

```
n(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-3/5*A*a^3*d^3*(8/3+sin(f*x+e)^4+4/3
*sin(f*x+e)^2)*cos(f*x+e)+9*A*a^3*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x
+1/2*e)+3*A*a^3*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^3*
(2+sin(f*x+e)^2)*cos(f*x+e)-3/5*B*a^3*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^
2)*cos(f*x+e)-1/3*A*a^3*d^3*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.08227, size = 1426, normalized size = 2.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 1/6720*(2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^3 + 5040*(2*f*x + 2*
e - sin(2*f*x + 2*e))*A*a^3*c^3 + 6720*(f*x + e)*A*a^3*c^3 + 6720*(cos(f*x
+ e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 210*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*B*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a
^3*c^3 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2*d + 630*(12*f*x
+ 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2*d + 15120*(2*f*x
+ 2*e - sin(2*f*x + 2*e))*A*a^3*c^2*d - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x
 + e)^3 + 15*cos(f*x + e))*B*a^3*c^2*d + 20160*(cos(f*x + e)^3 - 3*cos(f*x
+ e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x +
2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2*d - 134
4*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c*d^2 + 20
160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*x + 12*e + s
in(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 1
5*cos(f*x + e))*B*a^3*c*d^2 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*
c*d^2 + 105*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48
*sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
 + 15*cos(f*x + e))*A*a^3*d^3 + 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^
3*d^3 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*d^3 + 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin
(2*f*x + 2*e))*A*a^3*d^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*c
os(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*d^3 - 1344*(3*cos(f*x + e)^5 - 10*co
s(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^3 + 105*(4*sin(2*f*x + 2*e)^3 + 60*
f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^3 + 210*(12*
f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^3 - 20160*A*a^3
*c^3*cos(f*x + e) - 6720*B*a^3*c^3*cos(f*x + e) - 20160*A*a^3*c^2*d*cos(f*x
```

```
+ e))/f
```

Fricas [A] time = 2.69028, size = 1019, normalized size = 1.69

$$\frac{240\,Ba^3d^3\cos\left(fx+e\right)^7-1008\left(Ba^3c^2d+(A+3\,B)a^3cd^2+(A+2\,B)a^3d^3\right)\cos\left(fx+e\right)^5+560\left((A+3\,B)a^3c^3+3\,(3\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c*
d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A +
 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x +
e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*A +
23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 + 3*(A
 + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e) - 35*
(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c^3 + 18
*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^3*d^3)*c
os(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c^2*d + 3*
(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*sin(f*x + e)
)/f
```

Sympy [A] time = 22.4938, size = 2878, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x)
**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9
*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x*sin(
e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d*x*cos
(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*A*
```

a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*
A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f + 27*A*a**
3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d**2*x*cos(e +
 f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*d**2*sin(e +
f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*a**3*c*d**2*sin
(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*c*d**2*
cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f + 5*A*a**3*d**3*x
*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 +
 9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*x*sin(e + f*x)**2*cos(e
 + f*x)**4/16 + 9*A*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*A*a**
3*d**3*x*cos(e + f*x)**6/16 + 9*A*a**3*d**3*x*cos(e + f*x)**4/8 - 11*A*a**3
*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*A*a**3*d**3*sin(e + f*x)**4*c
os(e + f*x)/f - 5*A*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*
a**3*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**3*sin(e + f*x)**
2*cos(e + f*x)**3/f - A*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**3
*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*A*a**3*d**3*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 8*A*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*A*a**3*d**3*cos
(e + f*x)**3/(3*f) + 3*B*a**3*c**3*x*sin(e + f*x)**4/8 + 3*B*a**3*c**3*x*si
n(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**3*x*sin(e + f*x)**2/2 + 3*B*a
3*c3*x*cos(e + f*x)**4/8 + 3*B*a**3*c**3*x*cos(e + f*x)**2/2 - 5*B*a**3
*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3*c**3*sin(e + f*x)**2*co
s(e + f*x)/f - 3*B*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*
c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c**3*cos(e + f*x)**3/f - B*
a**3*c**3*cos(e + f*x)/f + 27*B*a**3*c**2*d*x*sin(e + f*x)**4/8 + 27*B*a**3
*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**2*d*x*sin(e + f*x
)**2/2 + 27*B*a**3*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**3*c**2*d*x*cos(e + f
*x)**2/2 - 3*B*a**3*c**2*d*sin(e + f*x)**4*cos(e + f*x)/f - 45*B*a**3*c**2*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a**3*c**2*d*sin(e + f*x)**2*cos(
e + f*x)**3/f - 9*B*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 27*B*a**3*
c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*d*sin(e + f*x)*co
s(e + f*x)/(2*f) - 8*B*a**3*c**2*d*cos(e + f*x)**5/(5*f) - 6*B*a**3*c**2*d*
cos(e + f*x)**3/f + 15*B*a**3*c*d**2*x*sin(e + f*x)**6/16 + 45*B*a**3*c*d**
2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 27*B*a**3*c*d**2*x*sin(e + f*x)**4
/8 + 45*B*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 27*B*a**3*c*d*
*2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*c*d**2*x*cos(e + f*x)**6
/16 + 27*B*a**3*c*d**2*x*cos(e + f*x)**4/8 - 33*B*a**3*c*d**2*sin(e + f*x)*
*5*cos(e + f*x)/(16*f) - 9*B*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5
*B*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 45*B*a**3*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 12*B*a**3*c*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/f - 3*B*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 15*B*a**3*c*d*
*2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 27*B*a**3*c*d**2*sin(e + f*x)*cos(

```
e + f*x)**3/(8*f) - 24*B*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*a**3*c*d**
2*cos(e + f*x)**3/f + 15*B*a**3*d**3*x*sin(e + f*x)**6/16 + 45*B*a**3*d**3*
x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**3*d**3*x*sin(e + f*x)**4/8 +
45*B*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**3*d**3*x*sin(e
 + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*d**3*x*cos(e + f*x)**6/16 + 3*B*a*
*3*d**3*x*cos(e + f*x)**4/8 - B*a**3*d**3*sin(e + f*x)**6*cos(e + f*x)/f -
33*B*a**3*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*B*a**3*d**3*sin(e +
f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*B*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 5*B*a**3*d**3*sin(e +
 f*x)**3*cos(e + f*x)/(8*f) - 8*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) - 4*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 15*B*a**3*d**3*s
in(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**3*d**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 16*B*a**3*d**3*cos(e + f*x)**7/(35*f) - 8*B*a**3*d**3*cos(e +
f*x)**5/(5*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a
)**3, True))
```

Giac [A] time = 1.34768, size = 764, normalized size = 1.26

$$\frac{Ba^3d^3\cos\left(7\,fx+7\,e\right)}{448\,f}+\frac{1}{16}\left(40\,Aa^3c^3+30\,Ba^3c^3+90\,Aa^3c^2d+78\,Ba^3c^2d+78\,Aa^3cd^2+69\,Ba^3cd^2+23\,Aa^3d^3+21\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 90
*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^3*
d^3 + 21*B*a^3*d^3)*x - 1/320*(12*B*a^3*c^2*d + 12*A*a^3*c*d^2 + 36*B*a^3*c
*d^2 + 12*A*a^3*d^3 + 19*B*a^3*d^3)*cos(5*f*x + 5*e)/f + 1/192*(16*A*a^3*c^
3 + 48*B*a^3*c^3 + 144*A*a^3*c^2*d + 204*B*a^3*c^2*d + 204*A*a^3*c*d^2 + 22
8*B*a^3*c*d^2 + 76*A*a^3*d^3 + 81*B*a^3*d^3)*cos(3*f*x + 3*e)/f - 1/64*(240
*A*a^3*c^3 + 208*B*a^3*c^3 + 624*A*a^3*c^2*d + 552*B*a^3*c^2*d + 552*A*a^3*
c*d^2 + 504*B*a^3*c*d^2 + 168*A*a^3*d^3 + 155*B*a^3*d^3)*cos(f*x + e)/f - 1
/192*(3*B*a^3*c*d^2 + A*a^3*d^3 + 3*B*a^3*d^3)*sin(6*f*x + 6*e)/f + 1/64*(2
*B*a^3*c^3 + 6*A*a^3*c^2*d + 18*B*a^3*c^2*d + 18*A*a^3*c*d^2 + 27*B*a^3*c*d
^2 + 9*A*a^3*d^3 + 11*B*a^3*d^3)*sin(4*f*x + 4*e)/f - 1/64*(48*A*a^3*c^3 +
64*B*a^3*c^3 + 192*A*a^3*c^2*d + 192*B*a^3*c^2*d + 192*A*a^3*c*d^2 + 189*B*
a^3*c*d^2 + 63*A*a^3*d^3 + 61*B*a^3*d^3)*sin(2*f*x + 2*e)/f
```

3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=463

$$-\frac{a^3\left(2Ad\left(72c^2d^2-15c^3d+2c^4+180cd^3+76d^4\right)-B\left(37c^3d^2-112c^2d^3-12c^4d+2c^5-304cd^4-136d^5\right)\right)\cos(e+fx)}{60d^3f}$$

```
[Out] (a^3*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/16 -
(a^3*(2*A*d*(2*c^4 - 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5
 - 12*c^4*d + 37*c^3*d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x]
)/(60*d^3*f) - (a^3*(2*A*d*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*
c^4 - 24*c^3*d + 76*c^2*d^2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*
x])/(240*d^2*f) - (a^3*(2*A*d*(2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2
*d + 41*c*d^2 - 136*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^3*f)
+ (a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*d + 21*d^2))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^3)/(40*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c
 + d*Sin[e + f*x])^3)/(6*d*f) + ((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3
+ a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(30*d^2*f)
```

Rubi [A] time = 1.128, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2976, 2968, 3023, 2753, 2734}

$$-\frac{a^3\left(2Ad\left(72c^2d^2-15c^3d+2c^4+180cd^3+76d^4\right)-B\left(37c^3d^2-112c^2d^3-12c^4d+2c^5-304cd^4-136d^5\right)\right)\cos(e+fx)}{60d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```


```
[Out] (a^3*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/16 -
(a^3*(2*A*d*(2*c^4 - 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5
 - 12*c^4*d + 37*c^3*d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x]
)/(60*d^3*f) - (a^3*(2*A*d*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*
c^4 - 24*c^3*d + 76*c^2*d^2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*
x])/(240*d^2*f) - (a^3*(2*A*d*(2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2
*d + 41*c*d^2 - 136*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^3*f)
+ (a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*d + 21*d^2))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^3)/(40*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c
 + d*Sin[e + f*x])^3)/(6*d*f) + ((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3
```

```
+ a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(30*d^2*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
```

```
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx)}{6df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx)}{6df}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx)}{6df}$$

$$= \frac{a^3\left(2A(2c-11d)d-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx}{40d^3f}$$

$$= -\frac{a^3\left(2Ad\left(2c^2-15cd+76d^2\right)-B\left(2c^3-12c^2d+41c\right.\right.}{120d^3f}$$

$$= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60cd+26\right.\right.$$

Mathematica [A] time = 2.39877, size = 355, normalized size = 0.77

$$-\frac{a^3\cos(e+fx)\left(60\left(A\left(40c^2+60cd+26d^2\right)+B\left(30c^2+52cd+23d^2\right)\right)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(-16\left(A\left(5\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d +
 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(18
40*A*c^2 + 1680*B*c^2 + 3360*A*c*d + 3112*B*c*d + 1556*A*d^2 + 1468*B*d^2 -
 16*(A*(5*c^2 + 30*c*d + 22*d^2) + B*(15*c^2 + 44*c*d + 26*d^2))*Cos[2*(e +
 f*x)] + 12*d*(2*B*c + A*d + 3*B*d)*Cos[4*(e + f*x)] + 720*A*c^2*Sin[e + f*
x] + 990*B*c^2*Sin[e + f*x] + 1980*A*c*d*Sin[e + f*x] + 2100*B*c*d*Sin[e +
f*x] + 1050*A*d^2*Sin[e + f*x] + 1085*B*d^2*Sin[e + f*x] - 30*B*c^2*Sin[3*(
e + f*x)] - 60*A*c*d*Sin[3*(e + f*x)] - 180*B*c*d*Sin[3*(e + f*x)] - 90*A*d
^2*Sin[3*(e + f*x)] - 140*B*d^2*Sin[3*(e + f*x)] + 5*B*d^2*Sin[5*(e + f*x)]
)))/(480*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.075, size = 725, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-1/3*A*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*a^3*c*d*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*A*a^3*d^2*(8/3+sin(f*x+e
)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^3*c*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e
)^2)*cos(f*x+e)+B*a^3*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x
+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*A*a^3*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-2*A*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*A*a^3*d^2*(-1/4*(sin
(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-B*a^3*c^2*(2+sin(f*x+e)
^2)*cos(f*x+e)+6*B*a^3*c*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3
/8*f*x+3/8*e)-3/5*B*a^3*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-
3*A*a^3*c^2*cos(f*x+e)+6*A*a^3*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-A*a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^2*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-2*B*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*d^2*(-
1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+A*a^3*c^2*(f*x+
e)-2*A*a^3*c*d*cos(f*x+e)+A*a^3*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2
*e)-B*a^3*c^2*cos(f*x+e)+2*B*a^3*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/
2*e)-1/3*B*a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Maxima [A] time = 1.02578, size = 950, normalized size = 2.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e -
 sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e)^
3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*si
n(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2
+ 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e + si
```

```
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(
f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c*d + 18
0*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d + 480*(
2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5 - 10*cos(f
*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x +
e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*
A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 192*(3*cos(f*x
 + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 320*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e +
 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*a^3*c^2*cos(f*x +
 e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x + e))/f
```

Fricas [A] time = 2.48562, size = 713, normalized size = 1.54

$$-\frac{48\left(2\,Ba^3cd+(A+3\,B)a^3d^2\right)\cos\left(fx+e\right)^5-80\left((A+3\,B)a^3c^2+2\,(3\,A+5\,B)a^3cd+(5\,A+7\,B)a^3d^2\right)\cos\left(fx+e\right)^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B)
*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 - 15
*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d^2)
*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(f*x
+ e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a^3*c*
d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^2 + 4*
(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/
f
```

Sympy [A] time = 11.3352, size = 1804, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x)
**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*
*3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/(3*
f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 + 3*A*
a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*sin(e + f*x)*
*2 + 3*A*a**3*c*d*x*cos(e + f*x)**4/4 + 3*A*a**3*c*d*x*cos(e + f*x)**2 - 5*
A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e + f*x)**
2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*A*a*
*3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/f - 2*A*a
**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*sin(e + f*x)**2/2 + 9*A
*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/2 - A*a**3*d
**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*A*a**
3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e + f*x)*cos(e +
f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*d**2
*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f + 3*B*a**3*c**2*x
*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*
B*a**3*c**2*x*sin(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e + f*x)**4/8 + 3*B*a
**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(
8*f) - 3*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f)
 - 2*B*a**3*c**2*cos(e + f*x)**3/f - B*a**3*c**2*cos(e + f*x)/f + 9*B*a**3*
c*d*x*sin(e + f*x)**4/4 + 9*B*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2
+ B*a**3*c*d*x*sin(e + f*x)**2 + 9*B*a**3*c*d*x*cos(e + f*x)**4/4 + B*a**3*
c*d*x*cos(e + f*x)**2 - 2*B*a**3*c*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*
a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 8*B*a**3*c*d*sin(e + f*x)**2*
cos(e + f*x)**3/(3*f) - 6*B*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a
**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*d*sin(e + f*x)*cos(e
+ f*x)/f - 16*B*a**3*c*d*cos(e + f*x)**5/(15*f) - 4*B*a**3*c*d*cos(e + f*x)
**3/f + 5*B*a**3*d**2*x*sin(e + f*x)**6/16 + 15*B*a**3*d**2*x*sin(e + f*x)*
*4*cos(e + f*x)**2/16 + 9*B*a**3*d**2*x*sin(e + f*x)**4/8 + 15*B*a**3*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*d**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 5*B*a**3*d**2*x*cos(e + f*x)**6/16 + 9*B*a**3*d**2*x*cos(e
+ f*x)**4/8 - 11*B*a**3*d**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*B*a**3
*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)**3*cos(e
+ f*x)**3/(6*f) - 15*B*a**3*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a
**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - B*a**3*d**2*sin(e + f*x)**2*co
s(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*B*a**3
*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a**3*d**2*cos(e + f*x)**5/(5
*f) - 2*B*a**3*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c
+ d*sin(e))**2*(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.32552, size = 513, normalized size = 1.11

$$-\frac{Ba^3d^2\sin\left(6\,fx+6\,e\right)}{192\,f}+\frac{1}{16}\left(40\,Aa^3c^2+30\,Ba^3c^2+60\,Aa^3cd+52\,Ba^3cd+26\,Aa^3d^2+23\,Ba^3d^2\right)x-\frac{\left(2\,Ba^3cd+Aa^3d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 + 6
0*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a^3
*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 + 12
*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)*cos
(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46*B*a^
3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c^2 + 4
*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4*e)/f -
 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d + 64*A*a
^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f
```

3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=201

$$\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos^3(e+fx)}{60f}-\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{5f}-\frac{3a^3(20Ac+15Ad+1}{}$$

```
[Out] (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*x)/8 - (a^3*(20*A*c + 15*B*c + 15*
A*d + 13*B*d)*Cos[e + f*x])/(5*f) + (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d
)*Cos[e + f*x]^3)/(60*f) - (3*a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e
 + f*x]*Sin[e + f*x])/(40*f) - ((5*B*c + 5*A*d - B*d)*Cos[e + f*x]*(a + a*S
in[e + f*x])^3)/(20*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(5*a*f)
```

Rubi [A] time = 0.334971, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2968, 3023, 2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos^3(e+fx)}{60f}-\frac{a^3(20Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{5f}-\frac{3a^3(20Ac+15Ad+1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*x)/8 - (a^3*(20*A*c + 15*B*c + 15*
A*d + 13*B*d)*Cos[e + f*x])/(5*f) + (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d
)*Cos[e + f*x]^3)/(60*f) - (3*a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e
 + f*x]*Sin[e + f*x])/(40*f) - ((5*B*c + 5*A*d - B*d)*Cos[e + f*x]*(a + a*S
in[e + f*x])^3)/(20*f) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(5*a*f)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^3\left(Ac+(Bc+Ad)\sin(e+fx)+\right.$$
$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af} + \frac{\int (a+a\sin(e$$
$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} -$$
$$= -\frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} -$$
$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{(5Bc+5Ad-$$
$$= \frac{1}{20}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{3a^3(20Ac+1$$
$$= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{a^3(20Ac+15B$$

Mathematica [A] time = 1.06431, size = 156, normalized size = 0.78

$$\frac{\cos(e+fx)\left(-\frac{1}{4}a^4(5Ad+5Bc-Bd)(\sin(e+fx)+1)^3 - \frac{a^4(20Ac+15Ad+15Bc+13Bd)\left(30\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\left(2\sin^2(e+fx)+9\sin(e+fx)\right.\right.}{24\sqrt{\cos^2(e+fx)}}\right.}{5af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] (Cos[e + f*x]*(-(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3)/4 - B*d*(a
 + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*ArcSin[
Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(22 + 9*Sin[e + f*x]
 + 2*Sin[e + f*x]^2)))/(24*Sqrt[Cos[e + f*x]^2])))/(5*a*f)
```

Maple [B] time = 0.059, size = 414, normalized size = 2.1

$$\frac{1}{f}\left(-\frac{Aa^3c\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3} + Aa^3d\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*A*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a^3*d*(-1/4*(sin(f*x+e)^3+3
/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*si
n(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^3*d*(8/3+sin(f*x+e)^4+4/3*sin(f
*x+e)^2)*cos(f*x+e)+3*A*a^3*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*
a^3*d*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a
^3*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*A*a^3*
c*cos(f*x+e)+3*A*a^3*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*B*a^3*c
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*d*(2+sin(f*x+e)^2)*cos(f*
x+e)+A*a^3*c*(f*x+e)-A*a^3*d*cos(f*x+e)-B*a^3*c*cos(f*x+e)+B*a^3*d*(-1/2*si
n(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [B] time = 0.998088, size = 537, normalized size = 2.67

$$160\left(\cos\left(fx+e\right)^{3}-3\,\cos\left(fx+e\right)\right)Aa^{3}c+360\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^{3}c+480\left(fx+e\right)Aa^{3}c+480\left(\cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] 1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*A*a^3*c + 480*(f*x + e)*A*a^3*c + 480*(cos(f*x + e)^3 - 3*
cos(f*x + e))*B*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x
+ 2*e))*B*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 480*(cos(f
*x + e)^3 - 3*cos(f*x + e))*A*a^3*d + 15*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*A*a^3*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*
d - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d + 4
80*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d + 45*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d + 120*(2*f*x + 2*e - sin(2*f*x + 2*e
))*B*a^3*d - 1440*A*a^3*c*cos(f*x + e) - 480*B*a^3*c*cos(f*x + e) - 480*A*a
^3*d*cos(f*x + e))/f
```

Fricas [A] time = 2.18467, size = 437, normalized size = 2.17

$$-\frac{24\,Ba^3d\cos\left(fx+e\right)^5-40\left((A+3\,B)a^3c+(3\,A+5\,B)a^3d\right)\cos\left(fx+e\right)^3-15\left(5\,(4\,A+3\,B)a^3c+(15\,A+13\,B)a^3d\right)f}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a^3*d*cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*d
)*cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 480
*((A + B)*a^3*c + (A + B)*a^3*d)*cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*B)*
a^3*d)*cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d)*cos(f*x
 + e))*sin(f*x + e))/f
```

Sympy [A] time = 5.69284, size = 960, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*c*x*sin(e + f*x)**2/2 + 3*A*a**3*c*x*cos(e + f*x)**2/2
+ A*a**3*c*x - A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*sin(e +
 f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c*cos(e + f*x)**3/(3*f) - 3*A*a**3*c*co
s(e + f*x)/f + 3*A*a**3*d*x*sin(e + f*x)**4/8 + 3*A*a**3*d*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*A*a**3*d*x*sin(e + f*x)**2/2 + 3*A*a**3*d*x*cos(e +
 f*x)**4/8 + 3*A*a**3*d*x*cos(e + f*x)**2/2 - 5*A*a**3*d*sin(e + f*x)**3*co
s(e + f*x)/(8*f) - 3*A*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*d*s
in(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*d*sin(e + f*x)*cos(e + f*x)/(2
*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B*a**3*c*x
*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a
**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B*a**3*c*x*c
os(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c*cos(e + f*
x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x)**4/8 + 9*B*a*
*3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e + f*x)**2/2 + 9
*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2/2 - B*a**3*d*sin
```

```
(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a**3*d*sin(e + f
*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a
**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*d*cos(e + f*x)**5/(15*f) -
 2*B*a**3*d*cos(e + f*x)**3/f, Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*
(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.24446, size = 293, normalized size = 1.46

$$-\frac{Ba^3d\cos\left(5\,fx+5\,e\right)}{80\,f}+\frac{1}{8}\left(20\,Aa^3c+15\,Ba^3c+15\,Aa^3d+13\,Ba^3d\right)x+\frac{\left(4\,Aa^3c+12\,Ba^3c+12\,Aa^3d+17\,Ba^3d\right)\cos\left(3\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/80*B*a^3*d*cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3*
d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*d)
*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a^3*
d)*cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d)*sin(4*f*x + 4*e)/f
 - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d)*sin(2*f*x + 2*e)/f
```

3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=127

$$-\frac{5a^3(4A+3B)\cos(e+fx)}{6f}-\frac{5a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{24f}+\frac{5}{8}a^3x(4A+3B)-\frac{a(4A+3B)\cos(e+fx)(a\sin}{12f}$$

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 - (5*a^3*(4*A + 3*B)*Cos[e + f*x])/(6*f) - (5*a^3*(
4*A + 3*B)*Cos[e + f*x]*Sin[e + f*x])/(24*f) - (a*(4*A + 3*B)*Cos[e + f*x]*
(a + a*Sin[e + f*x])^2)/(12*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4
*f)
```

Rubi [A] time = 0.100562, antiderivative size = 117, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(4A+3B)\cos^3(e+fx)}{12f}-\frac{a^3(4A+3B)\cos(e+fx)}{f}-\frac{3a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{8f}+\frac{5}{8}a^3x(4A+3B)-\frac{B\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*x)/8 - (a^3*(4*A + 3*B)*Cos[e + f*x])/f + (a^3*(4*A + 3*
B)*Cos[e + f*x]^3)/(12*f) - (3*a^3*(4*A + 3*B)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(4*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int (a+a\sin(e+fx))^3$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}(4A+3B)\int \left(a^3+3a^3\sin(e+fx\right.$$

$$= \frac{1}{4}a^3(4A+3B)x-\frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}+\frac{1}{4}\left(a^3(4A+3B)\right)\int$$

$$= \frac{1}{4}a^3(4A+3B)x-\frac{3a^3(4A+3B)\cos(e+fx)}{4f}-\frac{3a^3(4A+3B)\cos(e+fx)}{8f}$$

$$= \frac{5}{8}a^3(4A+3B)x-\frac{a^3(4A+3B)\cos(e+fx)}{f}+\frac{a^3(4A+3B)\cos^3(e+fx)}{12f}$$

Mathematica [A] time = 0.48649, size = 120, normalized size = 0.94

$$-\frac{a^3\cos(e+fx)\left(30(4A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}\left(8(A+3B)\sin^2(e+fx)+9(4A+5B)\sin(e+fx)+\right.\right.}{24f\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] +
 Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*(A + 3*
B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(24*f*Sqrt[Cos[e + f*x]^2])
```

Maple [A] time = 0.046, size = 178, normalized size = 1.4

$$\frac{1}{f}\left(-\frac{a^3A\left(2+\left(\sin\left(fx+e\right)\right)^2\right)\cos\left(fx+e\right)}{3}+Ba^3\left(-\frac{\cos\left(fx+e\right)}{4}\left(\left(\sin\left(fx+e\right)\right)^3+\frac{3\sin\left(fx+e\right)}{2}\right)+\frac{3fx}{8}+\frac{3e}{8}\right)+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

```
[Out] 1/f*(-1/3*a^3*A*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*A*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-B*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*cos(f*x+e)+3*B*a^3*(
-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*(f*x+e)-B*a^3*cos(f*x+e))
```

Maxima [A] time = 0.966436, size = 231, normalized size = 1.82

$$\frac{32\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aa^3+72\left(2fx+2e-\sin\left(2fx+2e\right)\right)Aa^3+96\left(fx+e\right)Aa^3+96\left(\cos\left(fx+e\right)^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3 + 72*(2*f*x + 2*e - sin(2*
f*x + 2*e))*A*a^3 + 96*(f*x + e)*A*a^3 + 96*(cos(f*x + e)^3 - 3*cos(f*x + e
))*B*a^3 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3
+ 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3 - 288*A*a^3*cos(f*x + e) - 96*B
*a^3*cos(f*x + e))/f
```

Fricas [A] time = 1.94411, size = 231, normalized size = 1.82

$$\frac{8\,(A+3\,B)a^3\cos\left(fx+e\right)^3+15\,(4\,A+3\,B)a^3fx-96\,(A+B)a^3\cos\left(fx+e\right)+3\left(2\,Ba^3\cos\left(fx+e\right)^3-(12\,A+17\,B)a^3\right.}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)*
a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e)^3 - (12*A + 17*B)*a^3*cos(f*x +
e))*sin(f*x + e))/f
```

Sympy [A] time = 1.96336, size = 371, normalized size = 2.92

$$\begin{cases}\frac{3Aa^3x\sin^2\left(e+fx\right)}{2}+\frac{3Aa^3x\cos^2\left(e+fx\right)}{2}+Aa^3x-\frac{Aa^3\sin^2\left(e+fx\right)\cos\left(e+fx\right)}{f}-\frac{3Aa^3\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{2Aa^3\cos^3\left(e+fx\right)}{3f}-\frac{3Aa^3\cos\left(e+fx\right)}{f} \\ x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^3\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A*
a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/f +
 3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*a**3*
x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**
3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f*x)**3/f
 - B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3, T
rue))
```

Giac [A] time = 1.28961, size = 157, normalized size = 1.24

$$\frac{Ba^3\sin\left(4\,fx+4\,e\right)}{32\,f}+\frac{5}{8}\left(4\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos\left(3\,fx+3\,e\right)}{12\,f}-\frac{\left(15\,Aa^3+13\,Ba^3\right)\cos\left(fx+e\right)}{4\,f}-\frac{\left(3\,Aa^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3
*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1/4
*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=246

$$\frac{a^3\left(Ad(2c-5d)-B\left(2c^2-5cd+5d^2\right)\right)\cos(e+fx)}{2d^3f}+\frac{2a^3(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f\sqrt{c^2-d^2}}+\frac{a^3x\left(Ad\left(2c^2-6cd\right.\right.}{}$$

```
[Out] (a^3*(A*d*(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*
x)/(2*d^4) + (2*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(d^4*Sqrt[c^2 - d^2]*f) + (a^3*(A*(2*c - 5*d)*d - B*(2*c^2
 - 5*c*d + 5*d^2))*Cos[e + f*x])/(2*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^2)/(3*d*f) + ((3*B*c - 3*A*d - 5*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[
e + f*x]))/(6*d^2*f)
```

Rubi [A] time = 0.895251, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2976, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{a^3\left(Ad(2c-5d)-B\left(2c^2-5cd+5d^2\right)\right)\cos(e+fx)}{2d^3f}+\frac{2a^3(c-d)^3(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f\sqrt{c^2-d^2}}+\frac{a^3x\left(Ad\left(2c^2-6cd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```


```
[Out] (a^3*(A*d*(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*
x)/(2*d^4) + (2*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(d^4*Sqrt[c^2 - d^2]*f) + (a^3*(A*(2*c - 5*d)*d - B*(2*c^2
 - 5*c*d + 5*d^2))*Cos[e + f*x])/(2*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^2)/(3*d*f) + ((3*B*c - 3*A*d - 5*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[
e + f*x]))/(6*d^2*f)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{\int \frac{(a+a\sin(e+fx))^2(a(2Bc+3Ad)-a(3Bc-3A}{c+d\sin(e+fx)}}{3d}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a\right.}{6d^2f}$$

$$= -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} + \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a\right.}{6d^2f}$$

$$= \frac{a^3\left(A(2c-5d)d-B\left(2c^2-5cd+5d^2\right)\right)\cos(e+fx)}{2d^3f} - \frac{aB\cos(e+fx)(a-}{3d}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{a^3\left(A(2c-\right.}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{a^3\left(A(2c-\right.}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{a^3\left(A(2c-\right.}{}$$

$$= \frac{a^3\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\right)\right)x}{2d^4} + \frac{2a^3(c-d)^3}{}$$

Mathematica [A] time = 0.964217, size = 233, normalized size = 0.95

$$\frac{a^3(\sin(e+fx)+1)^3\left(6(e+fx)\left(Ad\left(2c^2-6cd+7d^2\right)+B\left(6c^2d-2c^3-7cd^2+5d^3\right)\right)-3d\left(4Ad(3d-c)+B\left(4c^2-12cd\right.\right.\right.}{12d^4f\left(\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
```

```
),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*c
^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d +
c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c + 3*
d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)] - 3
*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)]))/(12*d^4*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.165, size = 1357, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^3/d^4/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*B*c^4-4/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)
^2*c^2+2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4*c-2/f*
a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c^2+6/f*a^3/d^2/(
1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c+4/f*a^3/d^2/(1+tan(1/2*f
*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^2*c+12/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^
2)^3*B*tan(1/2*f*x+1/2*e)^2*c+1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/
2*f*x+1/2*e)*B*c-1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^
5*B*c-6/f*a^3/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*B*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f
*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/f*a^3/d^3/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^3+6/f*a^3/d
^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*A*c^2-6/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c
^2-d^2)^(1/2))*A*c+3/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^
5*B-12/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^2-16/f*a^3/d
/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2-2/f*a^3/d^4*arctan(tan(1
/2*f*x+1/2*e))*B*c^3-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2
*e)^4-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4-1/f*a^3/d
/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A-3/f*a^3/d/(1+tan(1/2*f*x+1
/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B+2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*A*c-
2/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c^2+6/f*a^3/d^2/(1+tan(1/2*f*x+1/2
*e)^2)^3*B*c+2/f*a^3/d^3*arctan(tan(1/2*f*x+1/2*e))*A*c^2-6/f*a^3/d^2*arcta
n(tan(1/2*f*x+1/2*e))*A*c+6/f*a^3/d^3*arctan(tan(1/2*f*x+1/2*e))*B*c^2-7/f*
```

```
a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*B*c+1/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3
*tan(1/2*f*x+1/2*e)^5*A+5/f*a^3/d*arctan(tan(1/2*f*x+1/2*e))*B+2/f*a^3/(c^2
-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-6/f*
a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A-22/3/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*
B+7/f*a^3/d*arctan(tan(1/2*f*x+1/2*e))*A
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.56341, size = 1393, normalized size = 5.66

$$\left[\frac{2\,Ba^3d^3\cos\left(fx+e\right)^3-3\left(2\,Ba^3c^3-2\,(A+3\,B)a^3c^2d+(6\,A+7\,B)a^3cd^2-(7\,A+5\,B)a^3d^3\right)fx+3\left(Ba^3cd^2-(A+3\,B)a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d +
 (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3
*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) + 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d
 + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d
^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*
x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^
2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d - (A +
 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f), 1/6*(2*B*a^3*
d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a
^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*c
```

```
os(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a
^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sq
rt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 6*(B*a^3*c^2*d - (A + 3*B)*a^
3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28872, size = 833, normalized size = 3.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B*
a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*a^
3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*d^3
 - B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4) + 2
*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 -
 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^4 -
6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 +
18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 +
 12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^2
- 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^2
 + 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*x + 1/2*e) +
 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e) + 6*B*
a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^3*d^2)/((tan(1
/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=283

$$-\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{2d^3f(c+d)}+\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)\sqrt{c^2-d^2}}-$$

```
[Out] -(a^3*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*d^4) + (2*a^3*
(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)*Sqrt[c^2 - d^2]*f) - (a^3*(4*A*c*
d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(2*d^3*(c + d)*f) + ((2*A*d -
B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d^2*(c + d)*f) + (a*
(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Sin[e
+ f*x]))
```

Rubi [A] time = 0.941206, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2975, 2976, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{2d^3f(c+d)}+\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)\sqrt{c^2-d^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -(a^3*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*d^4) + (2*a^3*
(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*ArcTan[(d + c*Tan[(e
+ f*x)/2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)*Sqrt[c^2 - d^2]*f) - (a^3*(4*A*c*
d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(2*d^3*(c + d)*f) + ((2*A*d -
B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d^2*(c + d)*f) + (a*
(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Sin[e
+ f*x]))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
```

```
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^2(-a(B(2c-d)}{c+d}}{d}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(}{d(c+d)}$$

$$= \frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)f} + \frac{a(Bc-Ad)\cos(}{d(c+d)}$$

$$= -\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{2d^3(c+d)f} + \frac{(2Ad-B(3c+d))\cos}{2}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} - \frac{a^3\left(4Acd-B\left(6c^2-3c\right.\right.}{2d^3(c}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} - \frac{a^3\left(4Acd-B\left(6c^2-3c\right.\right.}{2d^3(c}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} - \frac{a^3\left(4Acd-B\left(6c^2-3c\right.\right.}{2d^3(c}$$

$$= -\frac{a^3\left(2A(2c-3d)d-B\left(6c^2-12cd+7d^2\right)\right)x}{2d^4} + \frac{2a^3(c-d)^2\left(Ad(2c+3\right.}{}$$

Mathematica [A] time = 1.48677, size = 244, normalized size = 0.86

$$\frac{a^3(\sin(e+fx)+1)^3\left(2(e+fx)\left(2Ad(3d-2c)+B\left(6c^2-12cd+7d^2\right)\right)-\frac{8(c-d)^2\left(B\left(3c^2+3cd-d^2\right)-Ad(2c+3d)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right.}{(c+d)\sqrt{c^2-d^2}}\right.}{4d^4f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d^
2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2))
```

```
*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]
) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d)*Co
s[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)]))/(4*d^
4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.189, size = 1534, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 4/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c+2/f*a
^3/d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^3-4/f*a^
3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^2+2/f*a^3
/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c-2/f*a^3/d^2/
(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/
2))*A*c^2-8/f*a^3/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e
)+2*d)/(c^2-d^2)^(1/2))*A*c-6/f*a^3/d^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^4*B+2/f*a^3/d^2/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c^2*tan(1/2*f*x+1/2*e)*B-4/f*a^3
/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/
2*e)*B+4/f*a^3/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)
+2*d)/(c^2-d^2)^(1/2))*A*c^3-10/f*a^3/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/f*a^3/d/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/2*e)*A-2/f*a^3*d/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/2*f*x+1/2*e)*A
+6/f*a^3/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*B*c^3+6/f*a^3/d^4*arctan(tan(1/2*f*x+1/2*e))*B*c^2+8/f*a^3
/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2
)^(1/2))*B*c^2-12/f*a^3/d^3*arctan(tan(1/2*f*x+1/2*e))*B*c+6/f*a^3/(c+d)/(c
^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+2/
f*a^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*B+4/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
*tan(1/2*f*x+1/2*e)*A+2/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B-6/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*ta
n(1/2*f*x+1/2*e)^2-1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2
*e)+4/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^2*B*c-4/f*a^3/d^3*arctan(tan(1/2*f
*x+1/2*e))*A*c+1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^
3-2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2+4/f*a^3/d^3
/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2*c-2/f*a^3/d^2/(c*tan(1/2
```

```
*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c^2-2/f*a^3/(c*tan(1/2*f*x+
1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e
)^2)^2*A-6/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B+6/f*a^3/d^2*arctan(tan(1/
2*f*x+1/2*e))*A+7/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.86849, size = 2242, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*
B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*B*
a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3*
B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*si
n(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*
d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (
2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*c^3*d -
2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3*d^4)*f*
x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)*cos(f*x
 + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f),
1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*B
)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + 2*(3*B
*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3
```

```
*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*s
in(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d
)/(c + d))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2
*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*
c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3
*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)
*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d
^5)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31483, size = 794, normalized size = 2.81

$$-\frac{\frac{4\left(3Ba^3c^4-2Aa^3c^3d-3Ba^3c^3d+Aa^3c^2d^2-4Ba^3c^2d^2+4Aa^3cd^3+5Ba^3cd^3-3Aa^3d^4-Ba^3d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(cd^4+d^5\right)\sqrt{c^2-d^2}}-\frac{4\left(Ba^3c^3d\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B*
a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(pi*
floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/
sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(1/2*
f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(1/2*f*
x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*d - 2*B
*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d^3 + c*d
```

```
^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (6*B*a^3*c
^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x + e)/d^4
- 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*
A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)^2 - B*a^3*d
*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((tan(1/2*f*x +
1/2*e)^2 + 1)^2*d^3))/f
```

3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=305

$$-\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(4c^2d+2c^3+cd^2-2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)^2\sqrt{c^2-d^2}}-\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-\right.\right.}{2d^2f(c+d}$$

```
[Out] -((a^3*(3*B*c - A*d - 3*B*d)*x)/d^4) - (a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7
*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/
2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (a^3*(3*B*c*(2*c
+ 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x])/(2*d^3*(c + d)^2*f) + (a*(B*c - A*d
)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^
2) - ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e + f*x]*(a^3 + a^3*S
in[e + f*x]))/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.93441, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2975, 2968, 3023, 2735, 2660, 618, 204}

$$-\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(4c^2d+2c^3+cd^2-2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4f(c+d)^2\sqrt{c^2-d^2}}-\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-\right.\right.}{2d^2f(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```


```
[Out] -((a^3*(3*B*c - A*d - 3*B*d)*x)/d^4) - (a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7
*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/
2])/Sqrt[c^2 - d^2]])/(d^4*(c + d)^2*Sqrt[c^2 - d^2]*f) - (a^3*(3*B*c*(2*c
+ 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x])/(2*d^3*(c + d)^2*f) + (a*(B*c - A*d
)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^
2) - ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e + f*x]*(a^3 + a^3*S
in[e + f*x]))/(2*d^2*(c + d)^2*f*(c + d*Sin[e + f*x]))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))^2(-2a(B(c-d)-2}{(c+d\sin}}{2d(}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\left(Ad(c+4d)-B\left(3c^2+4c\right.\right.}{2d^2(c+}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{\left(Ad(c+4d)-B\left(3c^2+4c\right.\right.}{2d^2(c+}$$

$$= -\frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(}{2d(c+d)f(c+d\,s}$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} +$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} +$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} +$$

$$= -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c\right.\right.}{d^4(c+d)^2\sqrt{c}}$$

Mathematica [B] time = 3.18217, size = 830, normalized size = 2.72

$$\frac{a^3(\sin(e+fx)+1)^3\left(\frac{4(c-d)\left(3B\left(2c^3+4dc^2+d^2c-2d^3\right)-Ad\left(2c^2+6dc+7d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{-12Bec^5-12Bfxc^5+4Adec^4-12Bdec^4+4Adfxc^4-1}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*B
*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^
2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e + 8*
A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4*e +
 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x - 12*B*
c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B*c^2*d
^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x - d*(2*A
*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^2*d^2 +
4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B*d)*(e +
f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Cos[3*(e + f
*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*c^3*d^2*e*S
in[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[e + f*x] + 2
4*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c*d^4*e*Sin[e
+ f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e + f*x] - 24*B*
c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 24*B*c^2*d^3*f*x
*Sin[e + f*x] + 8*A*c*d^4*f*x*Sin[e + f*x] + 24*B*c*d^4*f*x*Sin[e + f*x] -
9*B*c^3*d^2*Sin[2*(e + f*x)] + 3*A*c^2*d^3*Sin[2*(e + f*x)] - 9*B*c^2*d^3*S
in[2*(e + f*x)] + 3*A*c*d^4*Sin[2*(e + f*x)] + 4*B*c*d^4*Sin[2*(e + f*x)] -
 6*A*d^5*Sin[2*(e + f*x)] - 2*B*d^5*Sin[2*(e + f*x)])/(c + d*Sin[e + f*x])^
2))/(4*d^4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Maple [B] time = 0.217, size = 2906, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -4/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d
^2)*tan(1/2*f*x+1/2*e)^3*A-16/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-4/f*a^3*d/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B
-1/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2
)*c*tan(1/2*f*x+1/2*e)^2*A+2/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*
x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c^3+4/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c^2-4/f*a^3/d^3/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^4*B-2/f*a^3/d^2/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3+7/f*a^
```

3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2+22/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-4/f*a^3/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2/f*a^3/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^3-2/f*a^3*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A-13/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^3/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-1/f*a^3/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c-9/f*a^3/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-9/f*a^3/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-10/f*a^3*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A+6/f*a^3/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+6/f*a^3/d^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^4*B-3/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*B+2/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*A-2/f*a^3*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A-4/f*a^3/d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^4*tan(1/2*f*x+1/2*e)^2*B+4/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*A-5/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-2/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B-1/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-2/f*a^3*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*B+7/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+1/f*a^3/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*A-2/f*a^3*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3*A-3/f*a^3/d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^3*B+14/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*B-5/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c-1/f*a^3*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A-6/f*a^3/d^4*B*arctan(tan(1/2*f*x+1/2*e))*c-1/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c+7/f*a^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+6/f*a^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+5/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*A+6/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*

```
f*x+1/2*e)^3*B-5/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/
(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B+11/f*a^3/(c*tan(1/2*f*x+1/2*e)^2+2
*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+7/f*a^3*d
/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/
2*f*x+1/2*e)^2*A-2/f*a^3/d^3*B/(1+tan(1/2*f*x+1/2*e)^2)+2/f*a^3/d^3*A*arcta
n(tan(1/2*f*x+1/2*e))+6/f*a^3/d^3*B*arctan(tan(1/2*f*x+1/2*e))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.72469, size = 3568, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 -
 (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 +
 B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3
*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x
 - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*
A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a
^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*
a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2
*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e))*sqrt(-(c - d)/(c
 + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 +
 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(
-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) -
 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(
```

```
A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*B*a^3*c^4*d -
(A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x
+ (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3
*A + B)*a^3*d^5)*cos(f*x + e))*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*c
os(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 +
 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B*a^3*c^3*d^2 - (A -
 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x
+ e)^2 + 2*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 + B*a^3*d^5)*cos(f*x + e)^3 - 2*(
3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*
A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^
3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A -
B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2
*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B
*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B
)*a^3*c*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) +
d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - (6*B*a^3*c^4*d - 2*(A -
3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B
)*a^3*d^5)*cos(f*x + e) - (4*(3*B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A
+ 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3
*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e)
)*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 +
2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*
d^7 + d^8)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34504, size = 1331, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] ((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*c
^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(
c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((tan(1/
2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A*a^3*
c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 5
*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e
)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/2*f*x +
1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(1/2*f*x +
 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^2*tan(1/2*
f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^3*d^3*tan(1
/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*A*a^3*c^2*d^
4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 10*A*a
^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*
A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e) - 7*
A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) -
 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2
*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2*d^4*tan(1/2*f*x +
 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^6 - 2*A*a^3*c^5*d
+ 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 + 5*A*a^3*c^3*d^3 + B*a
^3*c^3*d^3 + A*a^3*c^2*d^4)/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*(c*tan(1/2*f*x
 + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - (3*B*a^3*c - A*a^3*d - 3*B
*a^3*d)*(f*x + e)/d^4)/f
```

3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=220

$$\frac{2d\left(3A\left(c^2-3cd+d^2\right)-B\left(7c^2-9cd+4d^2\right)\right)\cos(e+fx)}{3af}+\frac{x\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(-6c^2d+2c^3+9cd^2-3d^3\right)\right)}{2a}+$$

```
[Out] ((3*A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/(
2*a) + (2*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f
*x])/(3*a*f) + (d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f
*x])/(6*a*f) + ((3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a*f)
- ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.361171, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2977, 2753, 2734}

$$\frac{2d\left(3A\left(c^2-3cd+d^2\right)-B\left(7c^2-9cd+4d^2\right)\right)\cos(e+fx)}{3af}+\frac{x\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(-6c^2d+2c^3+9cd^2-3d^3\right)\right)}{2a}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```


```
[Out] ((3*A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/(
2*a) + (2*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f
*x])/(3*a*f) + (d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f
*x])/(6*a*f) + ((3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a*f)
- ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x]))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))} + \frac{\int (c+d\sin(e+fx))^2(a(B(\cdots}{\cdots}$$

$$= \frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(A-B)\cos(e+fx)(c+\cdots}{f(a+a\sin(e+\cdots}$$

$$= \frac{\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(2c^3-6c^2d+9cd^2-3d^3\right)\right)x}{2a} + \frac{2d\left(3A\left(c^2-\cdots\right.\right.}{\cdots}$$

Mathematica [B] time = 1.30916, size = 788, normalized size = 3.58

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3\cos\left(\frac{1}{2}(e+fx)\right)\left(4Ad\left(6c^2(e+fx)-3cd(2e+2fx+1)+d^2(3e+3fx+1)\right)+B\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*(
1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d*(
```

```
1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x)))*C
os[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos[(3*
(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f*x))/
2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A*c^3*S
in[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f*x)/2]
+ 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180*B*c*d^2
*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e + f*x)/2] +
 24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72*B*c^2*d*e
*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*e*Sin[(e +
f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x)/2] + 24*B*
c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 72*B*c^2*d*f*x
*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c*d^2*f*x*Sin[(
e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*Sin[(e + f*x)/2]
 - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e + f*x))/2] + 27*B
*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2] - 18*B*d^3*Sin[(
3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^3*Sin[(5*(e + f*x)
)/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e + f*x))/2]))/(24*a*f*
(1 + Sin[e + f*x]))
```

Maple [B] time = 0.102, size = 1110, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c^3+2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d^3+2/a/
f/(tan(1/2*f*x+1/2*e)+1)*B*c^3-2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*d^3+2/a/f/(1+
tan(1/2*f*x+1/2*e)^2)^3*A*d^3-10/3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*d^3+3/a
/f*arctan(tan(1/2*f*x+1/2*e))*A*d^3+2/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c^3-
3/a/f*arctan(tan(1/2*f*x+1/2*e))*B*d^3-3/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan
(1/2*f*x+1/2*e)*B*c*d^2-12/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2
*e)^2*c*d^2-12/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2*c^2*d+
12/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2*c*d^2+3/a/f/(1+tan
(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*
e)^2)^3*A*tan(1/2*f*x+1/2*e)^4*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan
(1/2*f*x+1/2*e)^4*c^2*d+6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*
e)^4*c*d^2+9/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c*d^2+6/a/f/(tan(1/2*f*x+1/2*
e)+1)*A*c^2*d-6/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c*d^2-6/a/f/(tan(1/2*f*x+1/2*e
)+1)*B*c^2*d+6/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c*d^2+1/a/f/(1+tan(1/2*f*x+1/2*
e)^2)^3*tan(1/2*f*x+1/2*e)^5*A*d^3-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2
```

```
*f*x+1/2*e)*A*d^3-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B*d
^3+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4*d^3-2/a/f/(1+tan
(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*d^3+4/a/f/(1+tan(1/2*f*x+1/2*e)
^2)^3*A*tan(1/2*f*x+1/2*e)^2*d^3-8/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2
*f*x+1/2*e)^2*d^3+1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B*d^3
-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*A*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*
B*c^2*d+6/a/f/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c*d^2+6/a/f*arctan(tan(1/2*f*x+1
/2*e))*A*c^2*d-6/a/f*arctan(tan(1/2*f*x+1/2*e))*A*c*d^2-6/a/f*arctan(tan(1/
2*f*x+1/2*e))*B*c^2*d
```

Maxima [B] time = 1.5744, size = 1517, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -1/3*(B*d^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 9*B*c*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*d^3*((sin(f*x + e)/(cos(f*x +
e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x +
e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e)
+ 1))/a) + 18*B*c^2*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arct
an(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 18*A*c*d^2*((sin(f*x + e)/(cos(f*x
```

```
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 6*B*
c^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1))) - 18*A*c^2*d*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 6*A*c^3/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1)))/f
```

Fricas [B] time = 2.45702, size = 1062, normalized size = 4.83

$$\frac{2\,Bd^3\cos\left(fx+e\right)^4-6\,(A-B)c^3+18\,(A-B)c^2d-18\,(A-B)cd^2+6\,(A-B)d^3+\left(9\,Bcd^2+(3\,A-B)d^3\right)\cos\left(fx+e\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B)
*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(2*
B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(3*B
*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)*c^3
 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^3 + 6
*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x + e) + (
2*B*d^3*cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A - B)*c*d^
2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*
(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^2 - 3*(6*B*c^2*
d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32047, size = 647, normalized size = 2.94

$$\frac{\frac{3\left(2Bc^3+6Ac^2d-6Bc^2d-6Acd^2+9Bcd^2+3Ad^3-3Bd^3\right)(fx+e)}{a}-\frac{12\left(Ac^3-Bc^3-3Ac^2d+3Bc^2d+3Acd^2-3Bcd^2-Ad^3+Bd^3\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(9Bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3 -
 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c*d
^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*c*d
^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*tan(1/
2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(1/2*f
*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f*x + 1
/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x + 1/2*e)^
2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 +
 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c*
d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3*tan(1/2*f
*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 10*B*d^3)/((
tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=143

$$\frac{x\left(2Ad(2c-d)+B\left(2c^2-4cd+3d^2\right)\right)}{2a}+\frac{2d(A(c-d)-B(2c-d))\cos(e+fx)}{af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

```
[Out] ((2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*x)/(2*a) + (2*(A*(c - d) - B
*(2*c - d))*d*Cos[e + f*x])/(a*f) + ((2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e + f
*x])/(2*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[
e + f*x]))
```

Rubi [A] time = 0.205708, antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2977, 2734}

$$\frac{x\left(d^2(-(2A-3B))+4Acd+2Bc(c-2d)\right)}{2a}+\frac{2d(A(c-d)-B(2c-d))\cos(e+fx)}{af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/(2*a) + (2*(A*(c - d) - B
*(2*c - d))*d*Cos[e + f*x])/(a*f) + ((2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e + f
*x])/(2*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[
e + f*x]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))} + \frac{\int (c+d\sin(e+fx))(a(B(c}{}$$

$$= \frac{\left(2Bc(c-2d)+4Acd-(2A-3B)d^2\right)x}{2a} + \frac{2(A(c-d)-B(2c-d))d\cos(}{af}$$

Mathematica [A] time = 0.459728, size = 200, normalized size = 1.4

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(e+fx)\left(2Ad(2c-d)+B\left(2c^2-4cd+3d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)*Sin[2*(e + f*x)]))/(4*a*f*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.085, size = 524, normalized size = 3.7

$$\frac{Bd^2}{af}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2\right)^{-2} - 2\,\frac{A\left(\tan\left(1/2\,fx+e/2\right)\right)^2 d^2}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2} - 4\,\frac{B\left(\tan\left(1/2\,fx+e/2\right)\right)^2 cd}{af\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3*d^2-2/a/f/(1+tan(1/
2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2*d^2-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)
^2*B*tan(1/2*f*x+1/2*e)^2*c*d+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*
x+1/2*e)^2*d^2-1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)*d^2-2/
a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A*d^2-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*c*d
+2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*d^2+4/a/f*arctan(tan(1/2*f*x+1/2*e))*A*
c*d-2/a/f*arctan(tan(1/2*f*x+1/2*e))*A*d^2+2/a/f*arctan(tan(1/2*f*x+1/2*e))
*B*c^2-4/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c*d+3/a/f*arctan(tan(1/2*f*x+1/2*
e))*B*d^2-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c^2+4/a/f/(tan(1/2*f*x+1/2*e)+1)*A
*c*d-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d^2+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c^2-
4/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c*d+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*d^2
```

Maxima [B] time = 1.4929, size = 818, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] (B*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4*B*c*d*((sin(f*x + e)/(cos(f*x
 + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(
cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*
d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(c
os(f*x + e) + 1))/a) + 2*B*c^2*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 4*A*c*d*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - 2*A*c^
2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] time = 2.30239, size = 684, normalized size = 4.78

$$Bd^2\cos\left(fx+e\right)^3-2\,(A-B)c^2+4\,(A-B)cd-2\,(A-B)d^2+\left(2\,Bc^2+4\,(A-B)cd-(2\,A-3\,B)d^2\right)fx-2\left(2\,Bcd+(A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] 1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 +
 (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d^2
)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 - (2*
B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos(f*x
 + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4*(A -
 B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x + e))*si
n(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 12.1062, size = 5583, normalized size = 39.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((4*A*c**2*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c**2*tan(e/2 + f*x/2)**3/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c**2*tan
(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a
*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)
**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**4/(2*
a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2
)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*
d*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/
2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) + 8*A*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x
```

```
/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f
*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/
2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
 + 4*A*c*d*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a
*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
 + 2*a*f) - 8*A*c*d*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*
tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 16*A*c*d*tan(e/2 + f*x/2)**3/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*A*c*d*tan(
e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f
*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
 2*a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a
*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)
**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2 + f*x/2)**4/(2
*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/
2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d
**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) - 4*A*d**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 +
f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*t
an(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 2*A*d**2*f*x*tan(e/2
 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*ta
n(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*
a*f) - 2*A*d**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 4*A*d**2*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x
/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*d**2*tan(e/2 + f*x/2)**3/(2*
a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2
)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*d*
*2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) - 4*A*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f
*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*ta
n(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 +
 f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*
tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x*tan(e/
2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*
f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2)
+ 2*a*f) + 4*B*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*B*c**2*f*x*tan(e/2 + f*x/2)**2/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
```

/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*
c**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/
2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) + 2*B*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(
e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2
*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c**2*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c**2*tan(e/2
 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f
*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
 2*a*f) - 4*B*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(
e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2
*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan
(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 +
4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x*t
an(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 +
 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*
x/2) + 2*a*f) - 8*B*c*d*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5
+ 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f
*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c*d*f*x*tan(e/2 + f*x/2)**
2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 +
f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4
*B*c*d*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*
x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan
(e/2 + f*x/2) + 2*a*f) - 4*B*c*d*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*c*d*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/
2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a
*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*c*d*tan(e/2
+ f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*
tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +
2*a*f) - 8*B*c*d*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*B*c*d/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*
f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)*
*2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2*f*x*tan(e/2 + f*x/2)**5/(2*
a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2
)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d*
*2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x
/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(
e/2 + f*x/2) + 2*a*f) + 6*B*d**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f
*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*ta
n(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d**2*f*x*tan(e/2
+ f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*
tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) +

```
2*a*f) + 3*B*d**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*t
an(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2
+ 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 3*B*d**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5
 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 +
f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*B*d**2*tan(e/2 + f*x/2)**5/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*B
*d**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/
2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) - 2*B*d**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)
**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*B*d**2*tan(e/2 + f*x/2)/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B
*d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/
2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
, Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2/(a*sin(e) + a), True))
```

Giac [A] time = 1.27307, size = 300, normalized size = 2.1

$$\frac{\frac{\left(2\,Bc^2+4\,Acd-4\,Bcd-2\,Ad^2+3\,Bd^2\right)(fx+e)}{a} - \frac{4\left(Ac^2-Bc^2-2\,Acd+2\,Bcd+Ad^2-Bd^2\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{2\left(Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + \cdots\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\cdots\right.\right.}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*c
^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e) +
1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 - 2*
A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*tan(1
/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*a))/f
```

3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=67

$$-\frac{(A-B)(c-d)\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{x(Ad+B(c-d))}{a}-\frac{Bd\cos(e+fx)}{af}$$

```
[Out] ((B*(c - d) + A*d)*x)/a - (B*d*Cos[e + f*x])/(a*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(a*f*(1 + Sin[e + f*x]))
```

Rubi [A] time = 0.20323, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3023, 2735, 2648}

$$-\frac{(A-B)(c-d)\cos(e+fx)}{af(\sin(e+fx)+1)}+\frac{x(Ad+B(c-d))}{a}-\frac{Bd\cos(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((B*(c - d) + A*d)*x)/a - (B*d*Cos[e + f*x])/(a*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(a*f*(1 + Sin[e + f*x]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx &= \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{a+a\sin(e+fx)}\,dx\\ &= -\frac{Bd\cos(e+fx)}{af}+\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{a}\\ &= \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}+((A-B)(c-d))\int \frac{1}{a+a\sin(e+fx}\\ &= \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}-\frac{(A-B)(c-d)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.468425, size = 126, normalized size = 1.88

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)((e+fx)(Ad+B(c-d))-Bd\cos(e+fx))+\sin\left(\frac{1}{2}(e+fx)\right)(2Ac -\right.}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),
x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d)
*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*(-
2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])
)
```

Maple [B] time = 0.065, size = 179, normalized size = 2.7

$$-2\,{\frac {Bd}{af \left(1+ \left(\tan \left(1/2\,fx+e/2 \right) \right) ^{2} \right) }}+2\,{\frac {A\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) d}{af}}+2\,{\frac {B\arctan \left(\tan \left(1/2\,fx+e/2 \right) \right) c}{af}}-2\,{\frac {B\arctan}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] -2/a/f*B*d/(1+tan(1/2*f*x+1/2*e)^2)+2/a/f*A*arctan(tan(1/2*f*x+1/2*e))*d+2/
a/f*B*arctan(tan(1/2*f*x+1/2*e))*c-2/a/f*B*arctan(tan(1/2*f*x+1/2*e))*d-2/a
/f/(tan(1/2*f*x+1/2*e)+1)*A*c+2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d+2/a/f/(tan(1
/2*f*x+1/2*e)+1)*B*c-2/a/f/(tan(1/2*f*x+1/2*e)+1)*B*d
```

Maxima [B] time = 1.47055, size = 346, normalized size = 5.16

$$-\frac{2\left(Bd\left(\frac{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+2}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{a\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}}+\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}\right)-Bc\left(\frac{\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a}+\frac{1}{a+\frac{a\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)-Ad\left(\frac{\arctan\left(\frac{s}{c}\right.}{a}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -2*(B*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a +
 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*d*(arctan(sin(f*x + e)/(cos
(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + A*c/(a + a
*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [B] time = 1.9737, size = 354, normalized size = 5.28

$$-\frac{Bd\cos\left(fx+e\right)^{2}-(Bc+(A-B)d)fx+(A-B)c-(A-B)d-\left((Bc+(A-B)d)fx-(A-B)c+(A-2\,B)d\right)\cos\left(fx\right.}{af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B*
c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A - B
)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*cos
(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 5.03552, size = 1244, normalized size = 18.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/
2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*tan(e/2 + f*x/2)**
3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2
 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e
/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 +
 a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 +
f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
 a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x
/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*
x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*ta
n(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
 B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**
2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*ta
n(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2
+ f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/
2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f
*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x/(a*f*tan(e/2 +
```

```
f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*d*
tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*
f*tan(e/2 + f*x/2) + a*f) - 2*B*d/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
d*sin(e))/(a*sin(e) + a), True))
```

Giac [B] time = 1.23198, size = 216, normalized size = 3.22

$$\frac{\frac{(Bc+Ad-Bd)(fx+e)}{a}-\frac{2\left(Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Ac-Bc-Ad+2\,Bd\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] ((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1/
2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^2
+ B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/2*e
)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=35

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

```
[Out] (B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rubi [A] time = 0.0485771, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2735, 2648}

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] (B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx &= \frac{Bx}{a}-(-A+B)\int \frac{1}{a+a\sin(e+fx)}\,dx\\ &= \frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}\end{aligned}$$

Mathematica [B] time = 0.155121, size = 79, normalized size = 2.26

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)(2A+B(e+fx-2))+B(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*A
 + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

Maple [A] time = 0.042, size = 65, normalized size = 1.9

$$2\,\frac{B\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{af}-2\,\frac{A}{af\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+2\,\frac{B}{af\left(\tan\left(1/2\,fx+e/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] 2/a/f*B*arctan(tan(1/2*f*x+1/2*e))-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(ta
n(1/2*f*x+1/2*e)+1)*B
```

Maxima [B] time = 1.43761, size = 105, normalized size = 3.

$$\frac{2\left(B\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)-\frac{A}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(co
s(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Fricas [A] time = 1.83652, size = 166, normalized size = 4.74

$$\frac{Bfx + \left(Bfx - A + B\right)\cos\left(fx + e\right) + \left(Bfx + A - B\right)\sin\left(fx + e\right) - A + B}{af\cos\left(fx + e\right) + af\sin\left(fx + e\right) + af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] (B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A +
B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [A] time = 2.03555, size = 109, normalized size = 3.11

$$\begin{cases} -\frac{2A}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{Bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{Bfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} + \frac{2B}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for}\ f \neq 0 \\ \frac{x(A+B\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f*
tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*tan
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True))
```

Giac [A] time = 1.22262, size = 54, normalized size = 1.54

$$\frac{\frac{(fx+e)B}{a} - \frac{2(A-B)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] ((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=101

$$\frac{2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

```
[Out] (2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)
*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x]
))
```

Rubi [A] time = 0.170218, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] (2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)
*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x]
))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{\int \frac{a(Bc-Ad)}{c+d\sin(e+fx)}\,dx}{a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}+\frac{(2(Bc-Ad))\,\mathrm{Subst}\left(\int \frac{1}{c+2dx+cx^2}\,dx, x, \tan\right.}{a(c-d)f}$$

$$= -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}-\frac{(4(Bc-Ad))\,\mathrm{Subst}\left(\int \frac{1}{-4\left(c^2-d^2\right)-x^2}\,dx, x, 2d\right.}{a(c-d)f}$$

$$= \frac{2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}f}-\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

Mathematica [A] time = 0.325058, size = 148, normalized size = 1.47

$$\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\sqrt{c^2-d^2}\sin\left(\frac{1}{2}(e+fx)\right)+(Bc-Ad)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\right.}{af(c-d)\sqrt{c^2-d^2}(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e +
f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Sin[e
 + f*x]))
```

Maple [A] time = 0.114, size = 176, normalized size = 1.7

$$-2\,\frac{Ad}{af\,(c-d)\,\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right)+2\,\frac{Bc}{af\,(c-d)\,\sqrt{c^2-d^2}}\arctan\left(1/2\,\frac{2\,c\tan\left(1/2\,fx+e/2\right)+2\,d}{\sqrt{c^2-d^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/a/f/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*A*d+2/a/f/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*
e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/a/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(c-
d)/(tan(1/2*f*x+1/2*e)+1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01083, size = 1297, normalized size = 12.84

$$\left[-\frac{2(A-B)c^2-2(A-B)d^2+\left(Bc-Ad+(Bc-Ad)\cos\left(fx+e\right)+(Bc-Ad)\sin\left(fx+e\right)\right)\sqrt{-c^2+d^2}\log\left(\frac{\left(2c^2-d^2\right)\cos\left(fx\right.}{}\right.}{2\left(\left(ac^3-ac^2d-acd^2+ad^3\right)f\cos\left(fx+e\right)+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x + e
) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x +
 e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d
*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) -
 c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)*c^2
- (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x
 + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2
*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d + (B*c -
 A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*s
in(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 - (A - B)*d
^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^
2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)
*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24632, size = 153, normalized size = 1.51

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(Bc-Ad)}{(ac-ad)\sqrt{c^2-d^2}}-\frac{A-B}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e
) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A - B
)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=181

$$-\frac{2\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{(A-B}{f(c-d)(a\sin(e+f}$$

```
[Out] (-2*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(3/2)*f) + (d*(B*(2*c + d) - A*(c +
 2*d))*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])) - ((A - B
)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.349443, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{2\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{3/2}}+\frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{af(c-d)^2(c+d)(c+d\sin(e+fx))}-\frac{(A-B}{f(c-d)(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/S
qrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(3/2)*f) + (d*(B*(2*c + d) - A*(c +
 2*d))*Cos[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*Sin[e + f*x])) - ((A - B
)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
```

<u>Rule 2978</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))} - \frac{\int \frac{a(2Ad-B(c+d))-a(A-B)d\sin}{(c+d\sin(e+fx))^2}}{a^2(c-d)}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\,\mathrm{s}}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\,\mathrm{s}}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\,\mathrm{s}}$$

$$= \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\,\mathrm{s}}$$

$$= -\frac{2\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)^2(c+d)\sqrt{c^2-d^2}\,f} + \frac{d(B(2c+d)}{a(c-d)^2(c}$$

Mathematica [A] time = 1.23584, size = 209, normalized size = 1.15

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{2\left(B\left(c^2+cd+d^2\right)-Ad(2c+d)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + \frac{d(Bc-Ad)\cos(e+fx)(}{(c+d)(}\right.}{af(c-d)^2(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-(
A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d^2]
) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c
+ d)*(c + d*Sin[e + f*x]))))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.141, size = 615, normalized size = 3.4

$$-2\frac{d^3\tan\left(1/2\,fx+e/2\right)A}{af\,(c-d)^2\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\,\tan\left(1/2\,fx+e/2\right)d+c\right)(c+d)\,c}+2\frac{d^2\tan\left(1/2\,fx\right.}{af\,(c-d)^2\left(c\left(\tan\left(1/2\,fx+e/2\right)\right)^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/a/f/(c-d)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)*d^3/(c+d)/
c*tan(1/2*f*x+1/2*e)*A+2/a/f/(c-d)^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)*d^2/(c+d)*tan(1/2*f*x+1/2*e)*B-2/a/f/(c-d)^2/(c*tan(1/2*f*x+1/2
*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)*d^2/(c+d)*A+2/a/f/(c-d)^2/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c+d)*B*c-4/a/f/(c-d)^2/(c+d)/(c^2-d^2)^
(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c*d-2/a/f/
(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-
d^2)^(1/2))*A*d^2+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c*d+2/a/f/(c-
d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2
)^(1/2))*B*d^2-2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(c-d)^2/(tan(1/
2*f*x+1/2*e)+1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.45626, size = 3247, normalized size = 17.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] [1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3*
d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (
B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A
 - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A -
 B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A
 - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x
 + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((A -
B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*x + e
) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d + (
2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x +
 e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f
*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4
- a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f -
 ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos
(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), (
(A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B)*c^3*d + (2*A - B
)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (B*c^3 - 2*(A
 - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A - B)*c*d^2
- (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A - B)*c*d^2)*c
os(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 +
(B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(
c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (
(A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*
x + e) - ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d
+ (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*
x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6
)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d
^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*
f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*
cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30671, size = 983, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -((B*a*c^5 - 2*A*a*c^4*d + A*a*c^3*d^2 - B*a*c^3*d^2 + 3*A*a*c^2*d^3 - B*a*
c^2*d^3 - A*a*c*d^4 - A*a*d^5 + B*a*d^5)*sqrt(-c^2 + d^2)*log(abs((d + sqrt
(-c^2 + d^2))*tan(1/2*f*x + 1/2*e) + c))/(a^2*c^8 - 2*a^2*c^7*d - 2*a^2*c^6
*d^2 + 6*a^2*c^5*d^3 - 6*a^2*c^3*d^5 + 2*a^2*c^2*d^6 + 2*a^2*c*d^7 - a^2*d^
8) - (B*a*c^5 - 2*A*a*c^4*d + A*a*c^3*d^2 - B*a*c^3*d^2 + 3*A*a*c^2*d^3 - B
*a*c^2*d^3 - A*a*c*d^4 - A*a*d^5 + B*a*d^5)*sqrt(-c^2 + d^2)*log(abs(-(d -
sqrt(-c^2 + d^2))*tan(1/2*f*x + 1/2*e) - c))/(a^2*c^8 - 2*a^2*c^7*d - 2*a^2
*c^6*d^2 + 6*a^2*c^5*d^3 - 6*a^2*c^3*d^5 + 2*a^2*c^2*d^6 + 2*a^2*c*d^7 - a^
2*d^8) + 2*(A*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A
*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*ta
n(1/2*f*x + 1/2*e)^2 + A*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x
 + 1/2*e) - 3*B*c^2*d*tan(1/2*f*x + 1/2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e)
 - 3*B*c*d^2*tan(1/2*f*x + 1/2*e) + A*d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*
c^3 + A*c^2*d - 2*B*c^2*d + A*c*d^2)/((a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^
3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x +
 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=283

$$-\frac{\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(4c^2d+2c^3+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d\left(2Ac^2+9Acd+4Ad^2-5Bc^2-6Bc\right.}{2af(c-d)^3(c+d)^2(c+d\,\mathrm{si}}$$

```
[Out] -(((3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(5/2
)*f)) - (d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*S
in[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*A*c^2 - 5*B*c^2 + 9*A*c*d - 6*
B*c*d + 4*A*d^2 - 4*B*d^2)*Cos[e + f*x])/(2*a*(c - d)^3*(c + d)^2*f*(c + d*
Sin[e + f*x]))
```

Rubi [A] time = 0.550317, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(4c^2d+2c^3+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{af(c-d)\left(c^2-d^2\right)^{5/2}}-\frac{d\left(2Ac^2+9Acd+4Ad^2-5Bc^2-6Bc\right.}{2af(c-d)^3(c+d)^2(c+d\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -(((3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*Ar
cTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^(5/2
)*f)) - (d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*a*(c - d)^2*(c
+ d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*S
in[e + f*x])*(c + d*Sin[e + f*x])^2) - (d*(2*A*c^2 - 5*B*c^2 + 9*A*c*d - 6*
B*c*d + 4*A*d^2 - 4*B*d^2)*Cos[e + f*x])/(2*a*(c - d)^3*(c + d)^2*f*(c + d*
Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{\int \frac{a(3Ad-B(c+2d))-2a(A-B)}{(c+d\sin(e+fx))^3}}{a^2(c-d)}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+}{(c-d)f(a+a\sin(e+fx))(c-}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+}{(c-d)f(a+a\sin(e+fx))(c-}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+}{(c-d)f(a+a\sin(e+fx))(c-}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+}{(c-d)f(a+a\sin(e+fx))(c-}$$

$$= -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} - \frac{(A-B)\cos(e+}{(c-d)f(a+a\sin(e+fx))(c-}$$

$$= -\frac{\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e\right.}{\sqrt{c^2-d^2}}\right.}{a(c-d)^3(c+d)^2\sqrt{c^2-d^2}\,f}$$

Mathematica [A] time = 1.38614, size = 313, normalized size = 1.11

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{d\left(B\left(3c^2+2cd+2d^2\right)-Ad(5c+2d)\right)\cos(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))}+\frac{2\left(B\left(4c^2d+2c^3+7cd^2+2d^3\right)-3Ad\right.}{}\right.}{2af(c-d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-3
*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[
(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) + (d*(
```

```
-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*a*(c - d)^3*
f*(1 + Sin[e + f*x]))
```

Maple [B] time = 0.165, size = 2482, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A-2/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)
^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*A
-4/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/(c^2
+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-6/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2
)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2*d-7/
a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3*c/(c^2+
2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A+2/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2*d^5/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A+5/a/f
/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2*c^2/(c^2+2
*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B+2/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*ta
n(1/2*f*x+1/2*e)*d+c)^2*d^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B-6/a/f/
(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^2+2*c*d+
d^2)*c^2*tan(1/2*f*x+1/2*e)^2*A-11/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*ta
n(1/2*f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A-3/a/f/(c
-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*
d)/(c^2-d^2)^(1/2))*A*d^3-6/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2*d^2/(c^2+2*c*d+d^2)*A*c^2+1/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2
*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2*c*d+d^2)*B*c+4/a/f/(c-d)^3/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/
2*f*x+1/2*e)^2*B+2/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(
2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+2/a/f/(c-d)^3/(c^2+2*c*d
+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/
2))*B*d^3+2/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2
*d^2/(c^2+2*c*d+d^2)*B*c^2-6/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+4/a/f/(c-d)^3/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1
/2*f*x+1/2*e)*B-2/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)^2*d^4/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-2/a/f/(c-d)^3/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2*c*d+d^2)*A*c+4/a/f/(c-d
)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d/(c^2+2*c*d+d^2)*B
```

```
*c^3+1/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^4/
(c^2+2*c*d+d^2)*A+4/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2*d+7/a/f/(c-d)^3/(c^2+2*
c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^
(1/2))*B*c*d^2-6/a/f/(c-d)^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*
c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c*d^2+4/a/f/(c-d)^3/(c*tan(1/2
*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x
+1/2*e)^2*B+2/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)
^2*d^6/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A+2/a/f/(c-d)^3/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*
x+1/2*e)^2*B+9/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2*d^3/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B+2/a/f/(c-d)^3/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1
/2*e)^2*B-17/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^
2*d^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+2/a/f/(c-d)^3/(c*tan(1/2*f*x+1
/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^5/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)*A+11/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^2*
c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+6/a/f/(c-d)^3/(c*tan(1/2*f*x+1/2*e
)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*d^3*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+
2/a/f/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.19584, size = 7112, normalized size = 25.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(4*(A - B)*c^6 - 12*(A - B)*c^4*d^2 + 12*(A - B)*c^2*d^4 - 4*(A - B)*d
^6 - 2*((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3
*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + 2*(4*(A - 2*B)*c^5*d +
 4*(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A
 - B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 - (2*B*c^5 - 2*(3*A - 4*B)*c^
4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 -
 (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4
 - (3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 1
8*(A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 +
 (2*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2
*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*
A - 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11
*B)*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A -
 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B
)*c^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
 e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*(A
- B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (7
*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(f*x + e) - 2*
(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*d^6 - ((
2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*(3*A - 2
*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d + (14*A - 13
*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A - 7*B)*c*d^
5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2*d^7 - a*c*
d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
)), 1/2*(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*
d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*
(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + (4*(A - 2*B)*c^5*d + 4*
(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A -
B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 + (2*B*c^5 - 2*(3*A - 4*B)*c^4*d
 - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3
*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4 -
(3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 18*(
A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + (2
```

```
*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2*d^
3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*A -
 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)
*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*
B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B)*c
^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x + e
))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x +
e))) + (2*(A - B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)
*c^3*d^3 - (7*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(
f*x + e) - (2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A -
B)*d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 -
 3*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d +
 (14*A - 13*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A
- 7*B)*c*d^5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c
^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 +
a*d^9)*f*cos(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^
4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x +
e)^2 - (a*c^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^3*d^6 - 2*a*c^2
*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4
*a*c^6*d^3 + 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^
8 + a*d^9)*f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^
3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^
7*d^2 - 3*a*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 +
 a*c*d^8)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6
*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33618, size = 1017, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*B*c^3 - 6*A*c^2*d + 4*B*c^2*d - 6*A*c*d^2 + 7*B*c*d^2 - 3*A*d^3 + 2*B*d
^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a
*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) - 2*(A - B)/((a*c^3 - 3*a*c^2*d + 3*a*c*d^
2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)) + (5*B*c^4*d^2*tan(1/2*f*x + 1/2*e)^
3 - 7*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*c^3*d^3*tan(1/2*f*x + 1/2*e)^3
 - 2*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^5*tan(1/2*f*x + 1/2*e)^3 +
4*B*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B
*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*
c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 11*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 4*B*
c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 4*A*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d
^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^6*tan(1/2*f*x + 1/2*e)^2 + 11*B*c^4*d^2*t
an(1/2*f*x + 1/2*e) - 17*A*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*B*c^3*d^3*tan(1
/2*f*x + 1/2*e) - 6*A*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^4*tan(1/2*f*
x + 1/2*e) + 2*A*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^5*d - 6*A*c^4*d^2 + 2*B
*c^4*d^2 - 2*A*c^3*d^3 + B*c^3*d^3 + A*c^2*d^4)/((a*c^7 - a*c^6*d - 2*a*c^5
*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 - a*c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d
*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=228

$$\frac{2d\left(A\left(c^2+6cd-5d^2\right)+B\left(2c^2-15cd+8d^2\right)\right)\cos(e+fx)}{3a^2f}+\frac{dx\left(2Ad(3c-2d)+B\left(6c^2-12cd+7d^2\right)\right)}{2a^2}+\frac{d^2(2A(c+6d$$

```
[Out] (d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*a^2) + (2*d*(A*(c
^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d + 8*d^2))*Cos[e + f*x])/(3*a^2*f) +
 (d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f)
 - ((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a
^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3
*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.523275, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2977, 2734}

$$\frac{2d\left(A\left(c^2+6cd-5d^2\right)+B\left(2c^2-15cd+8d^2\right)\right)\cos(e+fx)}{3a^2f}+\frac{dx\left(2Ad(3c-2d)+B\left(6c^2-12cd+7d^2\right)\right)}{2a^2}+\frac{d^2(2A(c+6d$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```



```
[Out] (d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*a^2) + (2*d*(A*(c
^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d + 8*d^2))*Cos[e + f*x])/(3*a^2*f) +
 (d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f)
 - ((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a
^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3
*f*(a + a*Sin[e + f*x])^2)
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx))^2(a(Ac+2Bc+3A}{a+a\sin}}{3a}$$

$$= -\frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} - \frac{(A-B)\cos}{3}$$

$$= \frac{d\left(2A(3c-2d)d+B\left(6c^2-12cd+7d^2\right)\right)x}{2a^2} + \frac{2d\left(A\left(c^2+6cd-5d^2\right)+\right.}{}$$

Mathematica [B] time = 3.51596, size = 547, normalized size = 2.4

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3\cos\left(\frac{1}{2}(e+fx)\right)\left(8Ad\left(6c^2+3cd(3e+3fx-4)+d^2(-6e-6fx+5)\right)+B\left(24c^2d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(8*A*d*(6*c^2 + d^2*(5 - 6*e - 6*
f*x) + 3*c*d*(-4 + 3*e + 3*f*x)) + B*(16*c^3 + 24*c^2*d*(-4 + 3*e + 3*f*x)
- 24*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x)))*Cos[(e + f*x)/
2] - (4*A*(4*c^3 + 24*c^2*d + d^3*(41 - 12*e - 12*f*x) + 6*c*d^2*(-10 + 3*e
 + 3*f*x)) + B*(32*c^3 + 24*c^2*d*(-10 + 3*e + 3*f*x) - 12*c*d^2*(-41 + 12*
e + 12*f*x) + d^3*(-239 + 84*e + 84*f*x)))*Cos[(3*(e + f*x))/2] + 3*(d^2*(1
2*B*c + 4*A*d - 5*B*d)*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] +
2*(8*A*c^3 + 8*B*c^3 + 24*A*c^2*d - 72*B*c^2*d - 72*A*c*d^2 + 108*B*c*d^2 +
 36*A*d^3 - 50*B*d^3 + 48*B*c^2*d*e + 48*A*c*d^2*e - 96*B*c*d^2*e - 32*A*d^
3*e + 56*B*d^3*e + 48*B*c^2*d*f*x + 48*A*c*d^2*f*x - 96*B*c*d^2*f*x - 32*A*
```

```
d^3*f*x + 56*B*d^3*f*x + d*(8*A*d*(3*c*(e + f*x) - 2*d*(1 + e + f*x)) + B*(
24*c^2*(e + f*x) - 48*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f*x)))*Cos[e
+ f*x] + 2*d^2*(-6*B*c - 2*A*d + 3*B*d)*Cos[2*(e + f*x)] + B*d^3*Cos[3*(e +
 f*x)])*Sin[(e + f*x)/2])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.105, size = 946, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*d^3+4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3
*B*c^3+7/f/a^2*arctan(tan(1/2*f*x+1/2*e))*B*d^3-2/f/a^2/(tan(1/2*f*x+1/2*e)
+1)*A*c^3-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B*d^3-2/f/a^2/(1+tan(1/2*f*x+1
/2*e)^2)^2*A*d^3+4/f/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*d^3-4/f/a^2/(tan(1/2*
f*x+1/2*e)+1)*A*d^3+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)*B*d^3+2/f/a^2/(tan(1/2*f
*x+1/2*e)+1)^2*A*c^3-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A*d^3-2/f/a^2/(tan(1/
2*f*x+1/2*e)+1)^2*B*c^3+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B*d^3-4/3/f/a^2/(t
an(1/2*f*x+1/2*e)+1)^3*A*c^3-6/f/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f
*x+1/2*e)^2*c*d^2+4/f/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2
*d^3-1/f/a^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)*d^3-6/f/a^2/(1
+tan(1/2*f*x+1/2*e)^2)^2*B*c*d^2+6/f/a^2*arctan(tan(1/2*f*x+1/2*e))*A*c*d^2
+6/f/a^2*d*arctan(tan(1/2*f*x+1/2*e))*B*c^2+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)*
A*c*d^2+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)*B*c^2*d-12/f/a^2/(tan(1/2*f*x+1/2*e)
+1)*B*c*d^2-6/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2*d+6/f/a^2/(tan(1/2*f*x+1
/2*e)+1)^2*A*c*d^2+4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B*c*d^2+1/f/a^2/(1+tan(
1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3*d^3-2/f/a^2/(1+tan(1/2*f*x+1/2*e
)^2)^2*A*tan(1/2*f*x+1/2*e)^2*d^3-12/f/a^2*arctan(tan(1/2*f*x+1/2*e))*B*c*d
^2-4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c*d^2-4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^
3*B*c^2*d+6/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B*c^2*d-6/f/a^2/(tan(1/2*f*x+1/2
*e)+1)^2*B*c*d^2+4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2*d-4/f/a^2*arctan(ta
n(1/2*f*x+1/2*e))*A*d^3
```

Maxima [B] time = 1.5814, size = 1866, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] 1/3*(B*d^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
+ 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*
x + e) + 1))/a^2) - 12*B*c*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(
f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) - 4*A*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)
 + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a^2) + 6*B*c^2*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 6*A*c*d^2*((9*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^
2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^2) - 2*A*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3) - 2*B*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 6*A*c^2*d*(3*sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) +
 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3))/f
```

Fricas [B] time = 2.20721, size = 1334, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/6*(3*B*d^3*cos(f*x + e)^4 - 2*(A - B)*c^3 + 6*(A - B)*c^2*d - 6*(A - B)*
c*d^2 + 2*(A - B)*d^3 + 6*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^3 + 6*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - (2*(A + 2*B)*c^3 + 6*(2
*A - 5*B)*c^2*d - 6*(5*A - 11*B)*c*d^2 + (22*A - 31*B)*d^3 + 3*(6*B*c^2*d +
 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e)^2 - (2*(2*A + B)*c^
3 + 6*(A - 4*B)*c^2*d - 6*(4*A - 13*B)*c*d^2 + 2*(13*A - 19*B)*d^3 - 3*(6*B
*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e) + (3*B*d^3*
cos(f*x + e)^3 + 2*(A - B)*c^3 - 6*(A - B)*c^2*d + 6*(A - B)*c*d^2 - 2*(A -
 B)*d^3 + 6*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - 3*(6*B*
c*d^2 + (2*A - 3*B)*d^3)*cos(f*x + e)^2 - (2*(A + 2*B)*c^3 + 6*(2*A - 5*B)*
c^2*d - 6*(5*A - 14*B)*c*d^2 + 4*(7*A - 10*B)*d^3 - 3*(6*B*c^2*d + 6*(A - 2
*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*
x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*si
n(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.322, size = 667, normalized size = 2.93

$$\frac{\frac{3\left(6\,Bc^2d+6\,Acd^2-12\,Bcd^2-4\,Ad^3+7\,Bd^3\right)\left(fx+e\right)}{a^2}+\frac{6\left(Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bd^3\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/6*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/a
^2 + 6*(B*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 2
*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*tan(1/2*f*x + 1/2*e)^2 - B*d^3*tan(
1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((tan(1/2*f*x + 1/2*e)^2
+ 1)^2*a^2) - 4*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*tan(1/2*f*x + 1
/2*e)^2 - 9*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e
)^2 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*A
*c^3*tan(1/2*f*x + 1/2*e) + 3*B*c^3*tan(1/2*f*x + 1/2*e) + 9*A*c^2*d*tan(1/
2*f*x + 1/2*e) - 27*B*c^2*d*tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*tan(1/2*f*x +
 1/2*e) + 45*B*c*d^2*tan(1/2*f*x + 1/2*e) + 15*A*d^3*tan(1/2*f*x + 1/2*e) -
 21*B*d^3*tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 12*B*c^2*d -
 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(tan(1/2*f*x + 1/2*e) +
 1)^3))/f
```

3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=132

$$-\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{dx(Ad+2B(c-d))}{a^2}+\frac{d^2(A-4B)\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)(c+}{3f(a\sin(e+fx}$$

```
[Out] (d*(2*B*(c - d) + A*d)*x)/a^2 + ((A - 4*B)*d^2*Cos[e + f*x])/(3*a^2*f) - ((
c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rubi [A] time = 0.509724, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2968, 3023, 2735, 2648}

$$-\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{dx(Ad+2B(c-d))}{a^2}+\frac{d^2(A-4B)\cos(e+fx)}{3a^2f}-\frac{(A-B)\cos(e+fx)(c+}{3f(a\sin(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*B*(c - d) + A*d)*x)/a^2 + ((A - 4*B)*d^2*Cos[e + f*x])/(3*a^2*f) - ((
c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*
x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f*(a + a*Sin[e + f*
x])^2)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{(c+d\sin(e+fx))(a(2B(c-d)+A(c\cdots}{a+a\sin(e\cdots}}{3a^2\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int \frac{ac(2B(c-d)+A(c+2d))+(-a(A-4\cdots}{\cdots}}{\cdots}$$

$$= \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2} + \frac{\int\cdots}{\cdots}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)(c\cdots}{3f(a+a\sin(\cdots}$$

$$= \frac{d(2B(c-d)+Ad)x}{a^2} + \frac{(A-4B)d^2\cos(e+fx)}{3a^2f} - \frac{(c-d)(2B(c-3d)+A\cdots}{3f\left(a^2+a^2\cdots\right.}$$

Mathematica [B] time = 1.63466, size = 338, normalized size = 2.56

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(6\cos\left(\frac{1}{2}(e+fx)\right)\left(Ad(4c+d(3e+3fx-4))+B\left(2c^2+2cd(3e+3fx-4)+d^2(-6e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x))
 + B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e + f
*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x))
+ 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] + 3*B
*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d - 6*A
*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*A*d^2*
f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1 + e +
f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12*a^2*f*(
1 + Sin[e + f*x])^2)
```

Maple [B] time = 0.092, size = 489, normalized size = 3.7

$$-2\,\frac{Bd^2}{a^2f\left(1+\left(\tan\left(1/2\,fx+e/2\right)\right)^2\right)}+2\,\frac{A\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)d^2}{a^2f}+4\,\frac{Bd\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)c}{a^2f}-4\,\frac{B\,\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

```
[Out] -2/f/a^2*B*d^2/(1+tan(1/2*f*x+1/2*e)^2)+2/f/a^2*A*arctan(tan(1/2*f*x+1/2*e)
)*d^2+4/f/a^2*d*B*arctan(tan(1/2*f*x+1/2*e))*c-4/f/a^2*B*arctan(tan(1/2*f*x
+1/2*e))*d^2-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)*A*c^2+2/f/a^2/(tan(1/2*f*x+1/2*
e)+1)*A*d^2+4/f/a^2/(tan(1/2*f*x+1/2*e)+1)*B*c*d-4/f/a^2/(tan(1/2*f*x+1/2*e
)+1)*B*d^2+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2-4/f/a^2/(tan(1/2*f*x+1/2*
e)+1)^2*A*c*d+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A*d^2-2/f/a^2/(tan(1/2*f*x+1
/2*e)+1)^2*B*c^2+4/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B*c*d-2/f/a^2/(tan(1/2*f*
x+1/2*e)+1)^2*B*d^2-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2+8/3/f/a^2/(tan
(1/2*f*x+1/2*e)+1)^3*A*c*d-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*d^2+4/3/f/a
^2/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2-8/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B*c*d+
```

```
4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B*d^2
```

Maxima [B] time = 1.51886, size = 1122, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] -2/3*(2*B*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4
*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*
B*c*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*ar
ctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*d^2*((9*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e
) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*c*d*(3*sin(f*x + e)/(cos(f*x +
e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Fricas [B] time = 2.15878, size = 863, normalized size = 6.54

$$-\frac{3\,Bd^2\cos\left(fx+e\right)^3-(A-B)c^2+2\,(A-B)cd-(A-B)d^2+6\left(2\,Bcd+(A-2\,B)d^2\right)fx-\left((A+2\,B)c^2+2\,(2\,A-5\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5*A
 - 11*B)*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e)^2 - ((2*A + B)
*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x
)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d - (A
 - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A - 5*B
)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e))*s
in(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*
cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 23.2995, size = 5358, normalized size = 40.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e/2
+ f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c**2/(3*a**2*f*tan(e/2 + f*
x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12
*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*A*
c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 +
f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d*tan(e/2 + f*x/2)**2/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 12*A*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d/(3*a**2*f*tan
```

(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2
)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f
) + 3*A*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) + 12*A*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a*
*2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*d**2*f*x*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 3*A*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*A*d**2*tan(e/2
+ f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/
2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**
3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
14*A*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*A*d**2*tan(e/2 + f*x/2)/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 8*A*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2
+ f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2
+ 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/2 + f*x/2)**3/(3*
a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e
/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
+ 3*a**2*f) - 2*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c**2*tan(e/
2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 2*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B*c*d*f*x*tan(e/2 +
f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(
e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)

```
**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 + 24*B*c*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*
f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
 f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*B*c*d*f*x*tan(e/2 +
 f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2
 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
6*B*c*d*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*B*c*d*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2
 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
36*B*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e
/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)
**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 28*B*c*d*tan(e/2 + f*x/2)**2/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 36*B*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 16*B*c*d/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f*x*
tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a
**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/
2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
 3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**
5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f
*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*d**2*f*x/(3*a**2*f*tan(e/2 + f*x/2
)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*B*d**
2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*
x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9
*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*d**2*tan(e/2 + f*x/2)**3/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
```

```
*a**2*f) - 44*B*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*
a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 48*B*d**2*tan(e/2
 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12
*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(
e/2 + f*x/2) + 3*a**2*f) - 20*B*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*s
in(e))*(c + d*sin(e))**2/(a*sin(e) + a)**2, True))
```

Giac [B] time = 1.23681, size = 374, normalized size = 2.83

$$\frac{\frac{3\left(2\,Bcd+Ad^2-2\,Bd^2\right)(fx+e)}{a^2}-\frac{6\,Bd^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^2}-\frac{2\left(3\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 1/3*(3*(2*B*c*d + A*d^2 - 2*B*d^2)*(f*x + e)/a^2 - 6*B*d^2/((tan(1/2*f*x +
1/2*e)^2 + 1)*a^2) - 2*(3*A*c^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c*d*tan(1/2*f*
x + 1/2*e)^2 - 3*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*d^2*tan(1/2*f*x + 1/2*e
)^2 + 3*A*c^2*tan(1/2*f*x + 1/2*e) + 3*B*c^2*tan(1/2*f*x + 1/2*e) + 6*A*c*d
*tan(1/2*f*x + 1/2*e) - 18*B*c*d*tan(1/2*f*x + 1/2*e) - 9*A*d^2*tan(1/2*f*x
 + 1/2*e) + 15*B*d^2*tan(1/2*f*x + 1/2*e) + 2*A*c^2 + B*c^2 + 2*A*c*d - 8*B
*c*d - 4*A*d^2 + 7*B*d^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] (B*d*x)/a^2 - ((A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((A - B)*(c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.210815, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3019, 2735, 2648}

$$-\frac{(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{3a^2f(\sin(e+fx)+1)}+\frac{Bdx}{a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (B*d*x)/a^2 - ((A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(3*a^2*f*(1 + Si
n[e + f*x])) - ((A - B)*(c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^2}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2B(c-d)+A(c+2d))-3aBd\sin(e+fx)}{a+a\sin(e+fx)}\,dx}{3a^2}$$

$$= \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(Ac+2Bc+2Ad-5Bd)\int \frac{1}{a+a\sin(e}}{3a}$$

$$= \frac{Bdx}{a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{(Ac+2Bc+2Ad-5Bd)\cos(e+f}{3f\left(a^2+a^2\sin(e+fx)\right)}$$

Mathematica [B] time = 0.341272, size = 180, normalized size = 2.12

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+2(Ac+2Ad+2Bc-5Bd)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\right.\right.\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c +
2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3
*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 + Sin[
e + f*x])^2)
```

Maple [B] time = 0.077, size = 252, normalized size = 3.

$$2\,\frac{Bd\arctan\left(\tan\left(1/2\,fx+e/2\right)\right)}{a^2f}-2\,\frac{Ac}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+2\,\frac{Bd}{a^2f\left(\tan\left(1/2\,fx+e/2\right)+1\right)}+2\,\frac{Ac}{a^2f\left(\tan\left(1/2\,fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*B*d*arctan(tan(1/2*f*x+1/2*e))-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)*A*c+2
/f/a^2/(tan(1/2*f*x+1/2*e)+1)*B*d+2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*A*c-2/f/
a^2/(tan(1/2*f*x+1/2*e)+1)^2*A*d-2/f/a^2/(tan(1/2*f*x+1/2*e)+1)^2*B*c+2/f/a
^2/(tan(1/2*f*x+1/2*e)+1)^2*B*d-4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c+4/3/
f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*A*d+4/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B*c-4
/3/f/a^2/(tan(1/2*f*x+1/2*e)+1)^3*B*d
```

Maxima [B] time = 1.46278, size = 613, normalized size = 7.21

$$\frac{2\left(Bd\left(\frac{\frac{9\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+4}{a^2+\frac{3a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}+\frac{3\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)}{a^2}\right)-\frac{Ac\left(\frac{3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+2\right)}{a^2+\frac{3a^2\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3a^2\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{a^2\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}}-\frac{\cdots}{a^2+\frac{3a^2\sin\left(fx\right.}{\cos\left(fx+e\right.}}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(B*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3
*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*c*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f
*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - B*c*(3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*d*(3
*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(c
```

```
os(f*x + e) + 1)^3))/f
```

Fricas [B] time = 1.88666, size = 491, normalized size = 5.78

$$-\frac{6\,Bdfx - \left(3\,Bdfx + (A+2\,B)c + (2\,A-5\,B)d\right)\cos\left(fx+e\right)^2 - (A-B)c + (A-B)d + \left(3\,Bdfx - (2\,A+B)c - (A-4}{3\left(a^2f\cos\left(fx+e\right)^2 - a^2f\cos\left(fx+e\right) - 2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2
- (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x +
 e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2*A
- 5*B)*d)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
 + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 10.6344, size = 986, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*A*c*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(3*a
**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) - 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**
3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
2*A*d/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2
+ f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*
a**2*f) - 2*B*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**
2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(
3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(
e/2 + f*x/2) + 3*a**2*f) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2
```

```
+ f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*
a**2*f) + 9*B*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/
(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 2*B*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f
*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**
2*f) + 12*B*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan
(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B*d/(3*a**2*f*
tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x
/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a)
**2, True))
```

Giac [A] time = 1.27019, size = 190, normalized size = 2.24

$$\frac{\frac{3(fx+e)Bd}{a^2} - \frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 3\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 3\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 9\,Bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 2\,Ac + Bc +}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(3*(f*x + e)*B*d/a^2 - 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 3*B*d*tan(1/2*
f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) +
3*A*d*tan(1/2*f*x + 1/2*e) - 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c + A*d
 - 4*B*d)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

```
[Out] -((A - B)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e + f
*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rubi [A] time = 0.0517211, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2750, 2648}

$$-\frac{(A+2B)\cos(e+fx)}{3f\left(a^2\sin(e+fx)+a^2\right)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e + f
*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} + \frac{(A+2B)\int \frac{1}{a+a\sin(e+fx)}\,dx}{3a}$$
$$= -\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2} - \frac{(A+2B)\cos(e+fx)}{3f\left(a^2+a^2\sin(e+fx)\right)}$$

Mathematica [A] time = 0.0523491, size = 43, normalized size = 0.66

$$-\frac{\cos(e+fx)((A+2B)\sin(e+fx)+2A+B)}{3a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(3*a^2*f*(1 + Sin[e + f*
x])^2)
```

Maple [A] time = 0.056, size = 70, normalized size = 1.1

$$2\,\frac{1}{a^2f}\left(-1/2\,\frac{-2\,A+2\,B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-\frac{A}{\tan\left(1/2\,fx+e/2\right)+1}-1/3\,\frac{2\,A-2\,B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2*(-1/2*(-2*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2-A/(tan(1/2*f*x+1/2*e)+1)-
1/3*(2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] time = 0.977802, size = 289, normalized size = 4.45

$$-\frac{2\left(\frac{A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{B\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*(
3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3))/f
```

Fricas [A] time = 1.8238, size = 288, normalized size = 4.43

$$\frac{(A+2B)\cos\left(fx+e\right)^2+(2A+B)\cos\left(fx+e\right)+\left((A+2B)\cos\left(fx+e\right)-A+B\right)\sin\left(fx+e\right)+A-B}{3\left(a^2f\cos\left(fx+e\right)^2-a^2f\cos\left(fx+e\right)-2a^2f-\left(a^2f\cos\left(fx+e\right)+2a^2f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*x
 + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
+ e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [A] time = 4.62994, size = 309, normalized size = 4.75

$$\begin{cases}\frac{2A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}-\frac{2A}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f}+\frac{}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right.} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((2*A*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A/(3*a**2*
f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f
```

```
*x/2) + 3*a**2*f) + 2*B*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 +
 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B
*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x
/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e)
)/(a*sin(e) + a)**2, True))
```

Giac [A] time = 1.23517, size = 92, normalized size = 1.42

$$-\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,A+B\right)}{3\,a^{2}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*f
*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=152

$$-\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

```
[Out] (-2*d*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c
 - d)^2*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/(3*
a^2*(c - d)^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(
a + a*Sin[e + f*x])^2)
```

Rubi [A] time = 0.419258, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 12, 2660, 618, 204}

$$-\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^2\sqrt{c^2-d^2}}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2f(c-d)^2(\sin(e+fx)+1)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*d*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c
 - d)^2*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/(3*
a^2*(c - d)^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(
a + a*Sin[e + f*x])^2)
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} - \frac{\int \frac{-a(2Bc+A(c-3d))-a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}dx}{3a^2(c-d)}$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} +$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} -$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} -$$

$$= -\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2} +$$

$$= -\frac{2d(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}f} - \frac{(A(c-4d)+B(2c+d))\cos(e+}{3a^2(c-d)^2f(1+\sin(e+fx)}$$

Mathematica [A] time = 0.638223, size = 229, normalized size = 1.51

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{6d(Ad-Bc)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 2(A-B)(c-d)\sin\left(\frac{1}{2}(e+\right.\right.}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d) +
 B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6
*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]))/(3*a^2*(c - d)^2*f*(1 +
Sin[e + f*x])^2)
```

Maple [B] time = 0.131, size = 327, normalized size = 2.2

$$2\frac{Ad^2}{a^2f(c-d)^2\sqrt{c^2-d^2}}\arctan\left(1/2\frac{2c\tan(1/2fx+e/2)+2d}{\sqrt{c^2-d^2}}\right)-2\frac{Bcd}{a^2f(c-d)^2\sqrt{c^2-d^2}}\arctan\left(1/2\frac{2c\tan(1/2fx+e/2)+2d}{\sqrt{c^2-d^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2*d^2/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))*A-2/f/a^2*d/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1
)^2*A-2/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)/(tan(1/2*f*x
+1/2*e)+1)^3*A+4/3/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3*B-2/f/a^2/(c-d)^2/(
tan(1/2*f*x+1/2*e)+1)*A*c+4/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*A*d-2/f/a^
2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*B*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42788, size = 2755, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 + 2
*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*
```

```
x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d -
 A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin
(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sq
rt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*
((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos
(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 -
((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x
 + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos
(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e
) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2
*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d +
 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 - (A - B)*c^2*d
- (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B
)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A
*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 +
(B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(
f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^3 - (5*A - 2*B
)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos(f*x + e) - ((A - B)*c^3 -
(A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^
2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^
4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^
2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^
4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*si
n(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2664, size = 350, normalized size = 2.3

$$-\frac{2\left(\frac{3\left(Bcd-Ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^2-2a^2cd+a^2d^2\right)\sqrt{c^2-d^2}}+\frac{3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}{\left(a^2c^2-2a^2cd+a^2d^2\right)\left(\right.}\right.}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((
c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x +
1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*
c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*x + 1
/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(tan(
1/2*f*x + 1/2*e) + 1)^3))/f
```

3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=275

$$\frac{2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}$$

```
[Out] (2*d*(A*d*(3*c + 2*d) - B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(A*(
c^2 - 6*c*d - 10*d^2) + B*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/(3*a^2*(c
- d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos
[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - ((
A - B)*Cos[e + f*x])/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x
]))
```

Rubi [A] time = 0.672215, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^3(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+}{3a^2f(c-d)^3(c+d)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d*(A*d*(3*c + 2*d) - B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^3*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(A*(
c^2 - 6*c*d - 10*d^2) + B*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/(3*a^2*(c
- d)^3*(c + d)*f*(c + d*Sin[e + f*x])) - ((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos
[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])*(c + d*Sin[e + f*x])) - ((
A - B)*Cos[e + f*x])/(3*(c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x
]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))} - \frac{\int \frac{-a(A(c-4d)+B(2c+d))}{(a+a\sin(e+fx))}}{3a^2(}$$

$$= -\frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-}{3(c-d)f(a+a\sin}$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(A}{3a^2(c-}$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(A}{3a^2(c-}$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(A}{3a^2(c-}$$

$$= -\frac{d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(A}{3a^2(c-}$$

$$= \frac{2d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}f} - \frac{d\left(A\left(\right.\right.}{}$$

Mathematica [A] time = 2.88501, size = 313, normalized size = 1.14

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{6d\left(B\left(2c^2+2cd+d^2\right)-Ad(3c+2d)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + \frac{3d^2(Ad-Bc)}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d) +
 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
(6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)
*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] +
```

```
Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)^3*f*(1
 + Sin[e + f*x])^2)
```

Maple [B] time = 0.16, size = 770, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^2/(c-d)^3*d^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)*A-2/f/a^2/(c-d)^3*d^3/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1
/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B+2/f/a^2/(c-d)^3*d^3/(c*tan(1/
2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2/f/a^2/(c-d)^3*d^2/(c*tan
(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c+6/f/a^2/(c-d)^3*d^2/(
c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2
))*A*c+4/f/a^2/(c-d)^3*d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-4/f/a^2/(c-d)^3*d/(c+d)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-4/f/a^2/(c-d)^
3*d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*B*c-2/f/a^2/(c-d)^3*d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)
+1)^2*A-2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)^2/(tan(1
/2*f*x+1/2*e)+1)^3*A+4/3/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B-2/f/a^2/(
c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c+6/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*
d-4/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*B*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.02852, size = 6543, normalized size = 23.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*d
^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - 2*((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c
^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*co
s(f*x + e)^3 + 2*((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (
A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B
*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2
*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*B*c^3*d
 - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^
2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*
(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x +
 e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^
4)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 -
 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e)
 - 2*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 +
 (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (
11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)
^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*
c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e))*sin(f*x + e
))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5
- 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^
2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(
f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 -
 a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5
- a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^
2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c
^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^
7)*f)*sin(f*x + e)), 1/3*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 +
```

```
 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*
A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*
B)*d^5)*cos(f*x + e)^3 + ((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3
*d^2 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2
- 3*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*
d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2
*B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f
*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B
)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*
d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*c
os(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A
 - B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x +
 e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^5 - (5*A - 8*B)*c^4
*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A
 - 4*B)*d^5)*cos(f*x + e) - ((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^
2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*
(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A -
 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)
*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*co
s(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2
*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7
 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*
d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 +
3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(
f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c
^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d
^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*c
os(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^
2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^
7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d
^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2982, size = 574, normalized size = 2.09

$$-\frac{2\left(\frac{3\left(2Bc^2d-3Acd^2+2Bcd^2-2Ad^3+Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(Bcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Bc^2d^2-\cdots\right.}{\left(a^2c^5-2a^2c^4d+2a^2c^2d^3-a^2cd^4\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2d\tan\left(\cdots\right.\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] -2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2))
+ 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*d^2
- A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*
f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x + 1/2*
e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*
tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*f*x + 1/
2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/((a^2*c^3
- 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=386

$$\frac{d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(12c^2d+6c^3+13cd^2+4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(A\left(-16c^2d+2c^3-59cd^2-32d\right.\right.}{6a^2f(c-d)}$$

```
[Out] (d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3)
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^
2*Sqrt[c^2 - d^2]*f) - (d*(A*(2*c^2 - 16*c*d - 21*d^2) + B*(4*c^2 + 19*c*d
+ 12*d^2))*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2)
 - ((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(a
 + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) - (d*(A*(2*c^3 - 16*c^2*d - 59
*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f*x])/
(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.961331, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(12c^2d+6c^3+13cd^2+4d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^2f(c-d)^4(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(A\left(-16c^2d+2c^3-59cd^2-32d\right.\right.}{6a^2f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] (d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3)
)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^
2*Sqrt[c^2 - d^2]*f) - (d*(A*(2*c^2 - 16*c*d - 21*d^2) + B*(4*c^2 + 19*c*d
+ 12*d^2))*Cos[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*Sin[e + f*x])^2)
 - ((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/(3*a^2*(c - d)^2*f*(1 + Sin
[e + f*x])*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(3*(c - d)*f*(a
 + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) - (d*(A*(2*c^3 - 16*c^2*d - 59
*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f*x])/
(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(A(c-5d)+2B(c+d))\cdots}{(a+a\sin(e+fx))(\cdots}}{3a^2(\cdots}$$

$$= -\frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} - \frac{(A-\cdots}{3(c-d)f(a+a\sin\cdots}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c\cdots}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c\cdots}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c\cdots}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c\cdots}$$

$$= -\frac{d\left(A\left(2c^2-16cd-21d^2\right)+B\left(4c^2+19cd+12d^2\right)\right)\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} - \frac{\cdots}{3a^2(c\cdots}$$

$$= \frac{d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\tan^{-1}\left(\frac{d+c\,\mathrm{t}\cdots}{\cdots}\right.}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}f}$$

Mathematica [B] time = 6.35709, size = 1522, normalized size = 3.94

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] -((d*(6*B*c^3 - 12*A*c^2*d + 12*B*c^2*d - 16*A*c*d^2 + 13*B*c*d^2 - 7*A*d^3
 + 4*B*d^3)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/
2]))/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c - d)^4*(
c + d)^2*Sqrt[c^2 - d^2]*f*(a + a*Sin[e + f*x])^2)) + ((Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])*(48*B*c^5*Cos[(e + f*x)/2] - 96*A*c^4*d*Cos[(e + f*x)/2]
```

```
+ 240*B*c^4*d*Cos[(e + f*x)/2] - 524*A*c^3*d^2*Cos[(e + f*x)/2] + 536*B*c^3
*d^2*Cos[(e + f*x)/2] - 776*A*c^2*d^3*Cos[(e + f*x)/2] + 701*B*c^2*d^3*Cos[
(e + f*x)/2] - 487*A*c*d^4*Cos[(e + f*x)/2] + 400*B*c*d^4*Cos[(e + f*x)/2]
- 112*A*d^5*Cos[(e + f*x)/2] + 70*B*d^5*Cos[(e + f*x)/2] - 16*A*c^5*Cos[(3*
(e + f*x))/2] - 32*B*c^5*Cos[(3*(e + f*x))/2] + 80*A*c^4*d*Cos[(3*(e + f*x)
)/2] - 224*B*c^4*d*Cos[(3*(e + f*x))/2] + 536*A*c^3*d^2*Cos[(3*(e + f*x))/2
] - 728*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 1028*A*c^2*d^3*Cos[(3*(e + f*x))/2
] - 893*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 695*A*c*d^4*Cos[(3*(e + f*x))/2] -
 482*B*c*d^4*Cos[(3*(e + f*x))/2] + 134*A*d^5*Cos[(3*(e + f*x))/2] - 98*B*d
^5*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 12*A*c^2*d^3*
Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 15*A*c*d^4*Cos[(
5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] + 6*A*d^5*Cos[(5*(e + f*x
))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 8
*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 59*B*
c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*(e + f*x))/2] + 76*B*c*d^4
*Cos[(7*(e + f*x))/2] - 52*A*d^5*Cos[(7*(e + f*x))/2] + 34*B*d^5*Cos[(7*(e
+ f*x))/2] + 48*A*c^5*Sin[(e + f*x)/2] + 48*B*c^5*Sin[(e + f*x)/2] - 224*A*
c^4*d*Sin[(e + f*x)/2] + 416*B*c^4*d*Sin[(e + f*x)/2] - 872*A*c^3*d^2*Sin[(
e + f*x)/2] + 992*B*c^3*d^2*Sin[(e + f*x)/2] - 1144*A*c^2*d^3*Sin[(e + f*x)
/2] + 967*B*c^2*d^3*Sin[(e + f*x)/2] - 685*A*c*d^4*Sin[(e + f*x)/2] + 496*B
*c*d^4*Sin[(e + f*x)/2] - 168*A*d^5*Sin[(e + f*x)/2] + 126*B*d^5*Sin[(e + f
*x)/2] + 48*B*c^4*d*Sin[(3*(e + f*x))/2] - 132*A*c^3*d^2*Sin[(3*(e + f*x))/
2] + 96*B*c^3*d^2*Sin[(3*(e + f*x))/2] - 204*A*c^2*d^3*Sin[(3*(e + f*x))/2]
 + 207*B*c^2*d^3*Sin[(3*(e + f*x))/2] - 165*A*c*d^4*Sin[(3*(e + f*x))/2] +
174*B*c*d^4*Sin[(3*(e + f*x))/2] - 66*A*d^5*Sin[(3*(e + f*x))/2] + 42*B*d^5
*Sin[(3*(e + f*x))/2] - 16*A*c^4*d*Sin[(5*(e + f*x))/2] - 32*B*c^4*d*Sin[(5
*(e + f*x))/2] + 116*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 224*B*c^3*d^2*Sin[(5*
(e + f*x))/2] + 412*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 409*B*c^2*d^3*Sin[(5*(
e + f*x))/2] + 403*A*c*d^4*Sin[(5*(e + f*x))/2] - 286*B*c*d^4*Sin[(5*(e + f
*x))/2] + 114*A*d^5*Sin[(5*(e + f*x))/2] - 78*B*d^5*Sin[(5*(e + f*x))/2] +
15*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 21*A*c*d^4*Sin[(7*(e + f*x))/2] + 12*B*
c*d^4*Sin[(7*(e + f*x))/2] - 12*A*d^5*Sin[(7*(e + f*x))/2] + 6*B*d^5*Sin[(7
*(e + f*x))/2]))/(48*(c - d)^4*(c + d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Si
n[e + f*x])^2)
```

Maple [B] time = 0.172, size = 2641, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -6/f/a^2*d/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+16/f/a^2*d^3/(c-d)^4/(c^2+2*c*d+d^2)
/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A
*c+12/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2/f/a^2*d^7/(c-d)^4/(c*tan(1/2*
f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/
2*e)^2*A-6/f/a^2*d^2/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B-4/f/a^2*d^3/(c-d)^4/(c*tan
(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f
*x+1/2*e)^2*B-2/f/a^2*d^6/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*B+23/f/a^2*d^4/(c-d)^4/(c
*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2
*f*x+1/2*e)*A-4/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2
*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B+8/f/a^2*d^3/(c-d)^4/(c*
tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/
2*f*x+1/2*e)^2*A-2/f/a^2*d^6/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+
1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-7/f/a^2*d^3/(c-d)^4/
(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan
(1/2*f*x+1/2*e)^3*B+4/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*A+8/f/a^2*d^6/(c-d)
^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*ta
n(1/2*f*x+1/2*e)^2*A-12/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*a
rctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-13/f/a^2*d^3/
(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+
2*d)/(c^2-d^2)^(1/2))*B*c-1/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A-1/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c+12/f/a^2*d^5/(c-d
)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan
(1/2*f*x+1/2*e)*A-4/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x
+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+7/f/a^2*d^4/(c-d)^4/(c^
2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d
^2)^(1/2))*A-6/f/a^2*d^2/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*
e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3-4/f/a^2*d^3/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2
+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2-4/f/a^2*d^4/(c-d)^4/(c^2
+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*B-8/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e
)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*B+8/f/a^2*d^3/(c-d)^4/(c*tan(
1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c^2+4/f/a^2*
d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+
d^2)*A*c+15/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*A-6/f/a^2/(c-d)^4/(tan(1/2*f*x+
1/2*e)+1)*B*d-2/f/a^2/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*A*c+8/f/a^2/(c-d)^4/(t
an(1/2*f*x+1/2*e)+1)*A*d+4/f/a^2*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-4/3/f/a^2/(c-d
)^3/(tan(1/2*f*x+1/2*e)+1)^3*A-17/f/a^2*d^3/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2
```

```
+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+4/3/f
/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*B+2/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)
+1)^2*A-12/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+9/f/a^2*d^4/(c-d)^4/(c*tan(1/2
*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2
*e)^3*A-13/f/a^2*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d
+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B-2/f/a^2*d^6/(c-d)^4/(c*tan(1
/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1
/2*e)*A-2/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.14617, size = 10961, normalized size = 28.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/12*(4*(A - B)*c^7 - 4*(A - B)*c^6*d - 12*(A - B)*c^5*d^2 + 12*(A - B)*c
^4*d^3 + 12*(A - B)*c^3*d^4 - 12*(A - B)*c^2*d^5 - 4*(A - B)*c*d^6 + 4*(A -
 B)*d^7 - 2*(2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^
3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*co
s(f*x + e)^4 - 2*(4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*
c^4*d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c
*d^6 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + 2*(2*(A + 2*B)*c^7 - 6*(2*A - 3*
B)*c^6*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*
c^3*d^4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7
)*cos(f*x + e)^2 + 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c
```

```
^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6
 + (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^
6)*cos(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)
*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^
5*d - 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4
 - (76*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 1
2*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(
10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2
*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A -
 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A
 - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A
- 3*B)*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*
d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3
*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*co
s(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)
^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*co
s(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^
2 - d^2)) + 4*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 19*B)*c^5*d^2
 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A - 21*B)*c^2*d
^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - 2*(2*(A - B)*
c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6*(A - B)*c
^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2*(A + 2*B)
*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A + 17*B)*c^
2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 - (4*(A + 2
*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(3*A + B)*c
^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A - 8*B)*d^7)
*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A - 18*B)*c
^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21*A - 22*B)
*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))*sin(f*x +
 e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^4 - (2*a^2
*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 1
2*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e)^3
- (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4
+ 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2*
c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c
^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) + 2*(a^2
*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a
^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5
- 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^3
+ 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^
5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f
*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d
^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2
 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*sin(f*x +
```

e)), -1/6*(2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)
*c^4*d^3 + 6*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A -
B)*d^7 - (2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*
d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(
f*x + e)^4 - (4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*c^4*
d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c*d^6
+ (43*A - 28*B)*d^7)*cos(f*x + e)^3 + (2*(A + 2*B)*c^7 - 6*(2*A - 3*B)*c^6
d - 12(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*c^3*d^
4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7)*cos(
f*x + e)^2 - 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3
- 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 + (6*
B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*cos
(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)*c^2*d
^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - (6*B*c^5*d -
12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4 - (76
*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*cos(f*x + e)^2 + (6*B*c^5*d - 12*(A -
2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A -
7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e) + (12*B*c^5*d - 24*(A - 2*B)*c^
4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*
c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*
B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A - 3*B)
*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*d^6)*c
os(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 -
3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*cos(f*x
+ e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2
- d^2)*cos(f*x + e))) + 2*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 1
9*B)*c^5*d^2 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A -
21*B)*c^2*d^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*cos(f*x + e) - (
2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6
*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2
*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A
+ 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^3 -
(4*(A + 2*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(
3*A + B)*c^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A -
8*B)*d^7)*cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A
- 18*B)*c^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21
*A - 22*B)*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*cos(f*x + e))
*sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^
5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^
4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c
^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*cos(f
*x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^
2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^
8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2
+ 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e

```
) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c
^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^
2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*cos(f
*x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*
a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a
^2*d^10)*f*cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10
*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*cos(f*x + e) - 2*(a^2*c^10 - 5*a
^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)
*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39803, size = 1274, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3
- 7*A*d^4 + 4*B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*t
an(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^
4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(c^2 - d^2
)) + 3*(7*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*tan(1/2*f*x + 1/2*
e)^3 + 4*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*tan(1/2*f*x + 1/2*e
)^3 + 2*A*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*tan(1/2*f*x + 1/2*e)^2
 - 8*A*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^2
- 4*A*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^2
- 15*A*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*tan(1/2*f*x + 1/2*e)^2
```

```
- 8*A*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*A
*d^7*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*tan(1/2*f*x + 1/2*e) - 23*A*c^3*
d^4*tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*tan(1/2*f*x + 1/2*e) - 12*A*c^2*d^5
*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*A*c*d^6*tan(1/
2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4*A*c^3*d^4 + B*
c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3
- a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*
d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 12*A*d*t
an(1/2*f*x + 1/2*e)^2 + 9*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x +
1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 21*A*d*tan(1/2*f*x + 1/2*e) + 15*B*d*
tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 11*A*d + 8*B*d)/((a^2*c^4 - 4*a^2*c^3*
d + 6*a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=225

$$-\frac{(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}+\frac{d^2x(A}{}$$

```
[Out] (d^2*(3*B*(c - d) + A*d)*x)/a^3 + (d^2*(3*B*(c - 9*d) + A*(2*c + 7*d))*Cos[
e + f*x])/(15*a^3*f) - ((c - d)*(3*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*
c*d + 15*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((3*B*(c - 3
*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a*f*(a + a*Si
n[e + f*x])^2) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(5*f*(a + a*
Sin[e + f*x])^3)
```

Rubi [A] time = 0.809307, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2968, 3023, 2735, 2648}

$$-\frac{(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}+\frac{d^2x(A}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*(3*B*(c - d) + A*d)*x)/a^3 + (d^2*(3*B*(c - 9*d) + A*(2*c + 7*d))*Cos[
e + f*x])/(15*a^3*f) - ((c - d)*(3*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*
c*d + 15*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((3*B*(c - 3
*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(15*a*f*(a + a*Si
n[e + f*x])^2) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(5*f*(a + a*
Sin[e + f*x])^3)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
```

```
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{(c+d\sin(e+fx))^2(a(2Ac+3Bc+3Ad}{(a+a\sin(e+}}{5a^2}$$

$$= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\,\mathrm{co}}{5}$$

$$= -\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\,\mathrm{co}}{5}$$

$$= \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(3B(c-3d)+2A(c+2d))\,\mathrm{co}}{15af(a+a\,\mathrm{s}}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(3B(c}{}$$

$$= \frac{d^2(3B(c-d)+Ad)x}{a^3} + \frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f} - \frac{(c-c}{}$$

Mathematica [A] time = 6.05691, size = 366, normalized size = 1.63

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(c-d)\left(A\left(2c^2+11cd+32d^2\right)+3B\left(c^2+8cd-24d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2
*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*(2*c^
2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
)/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.106, size = 936, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -6/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2*d+6/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*
B*c*d^2+12/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2*d-8/f/a^3/(tan(1/2*f*x+1/2*
e)+1)^3*A*c*d^2-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2*d+4/f/a^3/(tan(1/2*f
*x+1/2*e)+1)^3*B*c*d^2-12/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*A*c^2*d+12/f/a^3/(
tan(1/2*f*x+1/2*e)+1)^4*A*c*d^2+12/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B*c^2*d-1
2/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B*c*d^2+24/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^
5*A*c^2*d-24/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*A*c*d^2-24/5/f/a^3/(tan(1/2*f
*x+1/2*e)+1)^5*B*c^2*d+24/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B*c*d^2+6/f/a^3*
d^2*B*arctan(tan(1/2*f*x+1/2*e))*c+6/f/a^3/(tan(1/2*f*x+1/2*e)+1)*B*c*d^2+2
/f/a^3*d^3*A*arctan(tan(1/2*f*x+1/2*e))-6/f/a^3*d^3*B*arctan(tan(1/2*f*x+1/
2*e))+8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B*c^3-8/5/f/a^3/(tan(1/2*f*x+1/2*e
)+1)^5*B*d^3-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)*A*c^3+2/f/a^3/(tan(1/2*f*x+1/2*
e)+1)*A*d^3-6/f/a^3/(tan(1/2*f*x+1/2*e)+1)*B*d^3+4/f/a^3/(tan(1/2*f*x+1/2*e
)+1)^2*A*c^3-2/f/a^3*d^3*B/(1+tan(1/2*f*x+1/2*e)^2)+4/3/f/a^3/(tan(1/2*f*x+
1/2*e)+1)^3*A*d^3+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*B*c^3+4/f/a^3/(tan(1/2*f
*x+1/2*e)+1)^4*A*c^3-4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*A*d^3-4/f/a^3/(tan(1/
2*f*x+1/2*e)+1)^4*B*c^3+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B*d^3-8/5/f/a^3/(t
an(1/2*f*x+1/2*e)+1)^5*A*c^3+8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*A*d^3-16/3/
f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*A*c^3+2/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*A*d^3
-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*B*c^3-4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*B*
d^3
```

Maxima [B] time = 1.64417, size = 2271, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] -2/15*(3*B*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*
```

```
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1
1*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*sin(f*x + e)/(cos(f*x + e) +
1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(
f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a^3) - A*d^3*((95*sin(f*x + e)/(cos(f*x + e) + 1)
+ 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x
 + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*
a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^3) + A*c^3*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*B*c^2*d*(5*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^
3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*c*d^2*(5
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
 + 3*B*c^3*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*
x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10
*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 9*A*c^2*d*(5*sin(f*x
 + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
+ 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 2.31478, size = 1511, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(15*B*d^3*cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A - B
)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A -
 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos
(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x - (2*(2*A + 3*B)*c^3 + 3*(
6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^2 + (
A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((3*A + 2*B)*c^3 + 3*(2*A + 3*B)*c^2*
d + 9*(A - 6*B)*c*d^2 - 9*(2*A - 7*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*
f*x)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A - B)*c^
2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x
- ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 - 2*(16*A -
 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((2*A +
 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A - 62*B)*d^3
- 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(a^3*f*co
s(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a
^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31477, size = 805, normalized size = 3.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] -1/15*(30*B*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3
- 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^2*
tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*tan(1/2
*f*x + 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*tan(1/2*f*x +
1/2*e)^3 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*tan(1/2*f*x + 1/
2*e)^3 - 75*A*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*tan(1/2*f*x + 1/2*e)^3
 + 40*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*tan(1/2*f*x + 1/2*e)^2 + 45*A
*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*A*c*
d^2*tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*A*d^3
*tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^3*tan(1
/2*f*x + 1/2*e) + 15*B*c^3*tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*tan(1/2*f*x +
1/2*e) + 30*B*c^2*d*tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*tan(1/2*f*x + 1/2*e)
- 285*B*c*d^2*tan(1/2*f*x + 1/2*e) - 95*A*d^3*tan(1/2*f*x + 1/2*e) + 240*B*
d^3*tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*B*c^2*d + 6*A*
c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5
))/f
```

3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{\left(2A\left(c^2+3cd+2d^2\right)+B\left(3c^2+14cd-29d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bd^2x}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d}{}$$

```
[Out] (B*d^2*x)/a^3 - ((c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 + 14*c*d - 29*d^2) + 2*A*(c^2 + 3*c*
d + 2*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((A - B)*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rubi [A] time = 0.461075, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2977, 2968, 3019, 2735, 2648}

$$-\frac{\left(2A\left(c^2+3cd+2d^2\right)+B\left(3c^2+14cd-29d^2\right)\right)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}+\frac{Bd^2x}{a^3}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}-\frac{(c-d}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (B*d^2*x)/a^3 - ((c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(15*a*
f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 + 14*c*d - 29*d^2) + 2*A*(c^2 + 3*c*
d + 2*d^2))*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x])) - ((A - B)*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^2)/(5*f*(a + a*Sin[e + f*x])^3)
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \frac{\int \frac{(c+d\sin(e+fx))(a(B(3c-2d)+2A}{(a+a\sin(e+f}}{5a^2}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3} + \int \frac{ac(B(3c-2d)+2A(c+d))+(5aBcd+}{}$$

$$= -\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e+fx)(c-}{5f(a+a\sin(e}$$

$$= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(A-B)\cos(e+}{5f(a+}$$

$$= \frac{Bd^2x}{a^3} - \frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{\left(B\left(3c^2+14cd-\right.\right.}{}$$

Mathematica [B] time = 0.896707, size = 514, normalized size = 3.13

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(30\cos\left(\frac{1}{2}(e+fx)\right)\left(2Ad(c+d)+B\left(c^2+4cd+d^2(5e+5fx-9)\right)\right)-5\cos\left(\frac{3}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d
+ d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2)
+ B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] - 15
*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*A*c^
2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x)/2]
+ 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*Sin[(e
 + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f*x)/2]
+ 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 90*B*d^2*
Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*f*x*Sin[(
3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(e + f*x))/
2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/2] - 14*A*d
^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*d^2*e*Sin[(5
*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f*(1 + Sin[e +
 f*x])^3)
```

Maple [B] time = 0.098, size = 617, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*A*c*d-16/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*
B*c*d-4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*A*c*d-8/f/a^3/(tan(1/2*f*x+1/2*e)+1)
^4*A*c*d+8/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B*c*d-2/f/a^3/(tan(1/2*f*x+1/2*e)
+1)*A*c^2+2/f/a^3/(tan(1/2*f*x+1/2*e)+1)*B*d^2+4/f/a^3/(tan(1/2*f*x+1/2*e)+
1)^2*A*c^2-2/f/a^3/(tan(1/2*f*x+1/2*e)+1)^2*B*c^2+2/f/a^3/(tan(1/2*f*x+1/2*
e)+1)^2*B*d^2+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*A*c^2+4/f/a^3/(tan(1/2*f*x+1
/2*e)+1)^4*A*d^2-4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^4*B*c^2+16/5/f/a^3/(tan(1/2
*f*x+1/2*e)+1)^5*A*c*d-16/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B*c*d-8/5/f/a^3/
(tan(1/2*f*x+1/2*e)+1)^5*A*c^2-8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*A*d^2+8/5
/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B*c^2+8/5/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5*B*
d^2-16/3/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2-8/3/f/a^3/(tan(1/2*f*x+1/2*e)
+1)^3*A*d^2+4/f/a^3/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2+4/3/f/a^3/(tan(1/2*f*x+1
/2*e)+1)^3*B*d^2+2/f/a^3*B*d^2*arctan(tan(1/2*f*x+1/2*e))-4/f/a^3/(tan(1/2*
f*x+1/2*e)+1)^4*B*d^2
```

Maxima [B] time = 1.59682, size = 1528, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] 2/15*(B*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3)
 - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
```

```
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) - 4*B*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*d^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*c^2*(5*sin(f*x + e)/(co
s(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5) - 6*A*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))
/f
```

Fricas [B] time = 2.18067, size = 1002, normalized size = 6.11

$$-\frac{60\,Bd^2fx-\left(15\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+7\,B)cd-(7\,A-32\,B)d^2\right)\cos\left(fx+e\right)^3-3\,(A-B)c^2+6\,(A-B)cd-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] -1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d -
 (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B)*d
^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*c*d
- 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A - B)
*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B)*c*d
+ (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*
(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(
```

```
f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3
*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21794, size = 516, normalized size = 3.15

$$\frac{15\left(fx+e\right)Bd^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 75\,Bd^2\tan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^
2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1
/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e
)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 2
0*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^
2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*tan(1/2*f
*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x + 1/2*e)
 - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4*B*c*d +
2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -((A - B)*(c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A*c + 3*
B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A*
c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.226532, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2968, 3019, 2750, 2648}

$$-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] -((A - B)*(c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A*c + 3*
B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A*
c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^3}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc+3Ad-3Bd)-5aBd\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} + \cdots$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \cdots$$

Mathematica [A] time = 0.671652, size = 176, normalized size = 1.39

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(15(Ad+B(c+2d))\cos\left(\frac{1}{2}(e+fx)\right)-5(2Ac+3Ad+3Bc+4Bd)\cos\left(\frac{3}{2}(e+fx)\right)-\cdots\right.}{30a^3f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*x
)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*A*
```

```
c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x] +
 (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(30*a
^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.085, size = 151, normalized size = 1.2

$$2\,\frac{1}{fa^3}\left(-1/4\,\frac{-8\,Ac+8\,Ad+8\,Bc-8\,Bd}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^4}-1/2\,\frac{-4\,Ac+2\,Ad+2\,Bc}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^2}-1/5\,\frac{4\,Ac-4\,Ad-4\,Bc+4\,Bd}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}-\frac{Ac}{\tan\left(1/2\,fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A*c+8*A*d+8*B*c-8*B*d)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-4*A
*c+2*A*d+2*B*c)/(tan(1/2*f*x+1/2*e)+1)^2-1/5*(4*A*c-4*A*d-4*B*c+4*B*d)/(tan
(1/2*f*x+1/2*e)+1)^5-A*c/(tan(1/2*f*x+1/2*e)+1)-1/3*(8*A*c-6*A*d-6*B*c+4*B*
d)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Maxima [B] time = 1.04049, size = 990, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*
a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e)
 + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1)
 + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
```

```
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*
a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*s
in(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [B] time = 1.82323, size = 641, normalized size = 5.05

$$-\frac{((2A+3B)c+(3A+7B)d)\cos\left(fx+e\right)^{3}-(2(2A+3B)c+(6A-B)d)\cos\left(fx+e\right)^{2}-3(A-B)c+3(A-B)d-3(}{15\left(a^{3}f\cos\left(fx+e\right)^{3}+3a^{3}f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c +
(6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*c
+ (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x +
e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*cos(f*
x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^
3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) -
 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 23.6428, size = 1819, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
```

```
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(
e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)/(15*a**3*f*t
an(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f
*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15
*a**3*f) - 6*A*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2
)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
*3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*
f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*d*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*B*d*tan(e/2 + f*x/2)/(15*a**3*f
*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 4*B*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
d*sin(e))/(a*sin(e) + a)**3, True))
```

Giac [A] time = 1.20122, size = 301, normalized size = 2.37

$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,Ac\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B
*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/2*
f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/2*e
)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*B*c*
tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*x + 1
/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2A+3B)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

```
[Out] -((A - B)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A + 3*B)*Cos[e +
 f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A + 3*B)*Cos[e + f*x])/(15*f*(
a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.0750472, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2750, 2650, 2648}

$$-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2A+3B)\cos(e+fx)}{15af(a\sin(e+fx)+a)^2}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((A - B)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A + 3*B)*Cos[e +
 f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A + 3*B)*Cos[e + f*x])/(15*f*(
a^3 + a^3*Sin[e + f*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{(2A+3B)\int \frac{1}{(a+a\sin(e+fx))^2}\,dx}{5a}$$
$$= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}+\frac{(2A+3B)\int \frac{1}{a+a\sin(e+fx)}\,dx}{15a^2}$$
$$= -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2}-\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

Mathematica [A] time = 0.0784365, size = 63, normalized size = 0.62

$$-\frac{\cos(e+fx)\left((2A+3B)\sin^2(e+fx)+(6A+9B)\sin(e+fx)+7A+3B\right)}{15a^3f(\sin(e+fx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Sin[e +
f*x]^2))/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Maple [A] time = 0.064, size = 114, normalized size = 1.1

$$2\,\frac{1}{fa^3}\left(-1/4\,\frac{-8\,A+8\,B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^4}-1/5\,\frac{4\,A-4\,B}{\left(\tan\left(1/2\,fx+e/2\right)+1\right)^5}-\frac{A}{\tan\left(1/2\,fx+e/2\right)+1}-1/3\,\frac{8\,A-6\,B}{\left(\tan\left(1/2\,fx+e/2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/4*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*B)/(tan(1/2*f*
x+1/2*e)+1)^5-A/(tan(1/2*f*x+1/2*e)+1)-1/3*(8*A-6*B)/(tan(1/2*f*x+1/2*e)+1)
^3-1/2*(-4*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] time = 1.01367, size = 522, normalized size = 5.12

$$-\frac{2\left(\frac{A\left(\frac{20\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{40\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{30\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{15\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+7\right)}{a^3+\frac{5a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{5a^3\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4}+\frac{a^3\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}+\frac{3B\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{5\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{5\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}\right.}{a^3+\frac{5a^3\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10a^3\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+\frac{10a^3\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{5a^3\sin\left(fx+\right.}{\left(\cos\left(fx+e\right)+\right.}}\right.}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5) + 3*B*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3
 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

Fricas [A] time = 1.77057, size = 466, normalized size = 4.57

$$-\frac{(2A+3B)\cos\left(fx+e\right)^3-2(2A+3B)\cos\left(fx+e\right)^2-3(3A+2B)\cos\left(fx+e\right)-\left((2A+3B)\cos\left(fx+e\right)^2+3(2\right.}{15\left(a^3f\cos\left(fx+e\right)^3+3a^3f\cos\left(fx+e\right)^2-2a^3f\cos\left(fx+e\right)-4a^3f+\left(a^3f\cos\left(fx+e\right)^2-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A +
 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x +
```

```
e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*
a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [A] time = 11.3529, size = 899, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((6*A*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3
*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*A*tan(e/2 + f*
x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 15
0*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) - 10*A*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f
*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 8*A/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a
**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(
e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f
*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
 30*B*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/
2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2
)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)/(15*
a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x
/2) + 15*a**3*f) - 6*B/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 +
 f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**
2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))/(
a*sin(e) + a)**3, True))
```

Giac [A] time = 1.29214, size = 176, normalized size = 1.73

$$-\frac{2\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+30\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+40\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,B\tan\left(\frac{1}{2}\,fx\right.\right.}{15\,a^{3}f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*tan
(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/2*
e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B)/(
a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=229

$$\frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac-7Ad+3Bc}{15af(c-d)^2(a\,\mathrm{s}}$$

```
[Out] (2*d^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(
c - d)^3*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Si
n[e + f*x])^3) - ((2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/(15*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 - 16*c*d - 2*d^2) + A*(2*c^2 -
9*c*d + 22*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```

Rubi [A] time = 0.723971, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^3\sqrt{c^2-d^2}}-\frac{\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{15f(c-d)^3\left(a^3\sin(e+fx)+a^3\right)}-\frac{(2Ac-7Ad+3Bc}{15af(c-d)^2(a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```


```
[Out] (2*d^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(
c - d)^3*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Si
n[e + f*x])^3) - ((2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/(15*a*(c -
d)^2*f*(a + a*Sin[e + f*x])^2) - ((B*(3*c^2 - 16*c*d - 2*d^2) + A*(2*c^2 -
9*c*d + 22*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x]))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{\int \frac{-a(2Ac+3Bc-5Ad)-2a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx}{5a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2}$$

$$= \frac{2d^2(Bc-Ad)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3}$$

Mathematica [B] time = 1.24329, size = 502, normalized size = 2.19

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{60d^2(Ad-Bc)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + 20Ac^2\sin\left(\frac{1}{2}(e+fx)\right) -\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c*
d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2]
- 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c*d*
Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*(e +
```

```
f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2] + 15*
B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e + f*x)
/2] + 145*A*d^2*Sin[(e + f*x)/2] - 20*B*d^2*Sin[(e + f*x)/2] - (60*d^2*(-(B
*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/Sqrt[c^2 - d^2] - 15*B*c*d*Sin[(3*(e + f*x))/2] +
 15*A*d^2*Sin[(3*(e + f*x))/2] - 2*A*c^2*Sin[(5*(e + f*x))/2] - 3*B*c^2*Sin
[(5*(e + f*x))/2] + 9*A*c*d*Sin[(5*(e + f*x))/2] + 16*B*c*d*Sin[(5*(e + f*x
))/2] - 22*A*d^2*Sin[(5*(e + f*x))/2] + 2*B*d^2*Sin[(5*(e + f*x))/2]))/(30*
a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.138, size = 606, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

```
[Out] -2/f/a^3*d^3/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2))*A+2/f/a^3*d^2/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+4/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e
)+1)^4*A-4/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)/(tan(1/2*
f*x+1/2*e)+1)^5*A+8/5/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5*B+4/f/a^3/(c-d)^
2/(tan(1/2*f*x+1/2*e)+1)^2*A*c-6/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*A*d
-2/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B*c+4/f/a^3/(c-d)^2/(tan(1/2*f*x+
1/2*e)+1)^2*B*d-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c+20/3/f/a^3/
(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*A*d+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)
^3*B*c-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^3/(tan
(1/2*f*x+1/2*e)+1)*A*c^2+6/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c*d-6/f/a
^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*d^2+2/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)
+1)*B*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63274, size = 4929, normalized size = 21.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4
- 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16
*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (18*A
 + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A - 4*B)
*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x
+ e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x +
 e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2
- A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x +
 e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d + 5*(3*A - B)
*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) - 2*(3*(A - B
)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^
4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A
 - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B)*c^3*d + 5*(3*
A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos(f*x + e))*sin(f
*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*
d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
+ 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*
a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*
x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3
 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a
^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3
*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)
*f)*sin(f*x + e)), 1/15*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3
- 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d
^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + (2*(2*A + 3*B)
*c^4 - (18*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 -
(29*A - 4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d
^3)*cos(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^
```

```
3)*cos(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 +
 2*(B*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c
*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^4 - (
11*A + 9*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*
d^4)*cos(f*x + e) - (3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*
(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 +
 (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4
 - (9*A + 11*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A -
2*B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d
^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 -
 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos
(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*
a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*
d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d +
2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2
*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3
*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28868, size = 780, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] 2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arcta
n((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d +
```

```
3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
- 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*
d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan
(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan(1/2*f
*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*f*x + 1
/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2
 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 185
*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*
tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*tan(1/2*f*x
 + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f*x + 1/2*e)
- 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d - 11*B*c*d +
 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(
1/2*f*x + 1/2*e) + 1)^5))/f
```

3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=381

$$-\frac{2d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(-12c^2d+2c^3+43cd^2+72d^3\right)+B\left(-23c^2d+\right.\right.}{15a^3f(c-d)^4(c+d)(c+d\,\mathrm{si}}$$

```
[Out] (-2*d^2*(A*d*(4*c + 3*d) - B*(3*c^2 + 3*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(
B*(3*c^3 - 23*c^2*d - 63*c*d^2 - 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 +
 72*d^3))*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e +
f*x])) - ((2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(
a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) - ((B*(3*c^2 - 23*c*d - 15*d^2)
 + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Si
n[e + f*x])*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.08106, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{2d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^4(c+d)\sqrt{c^2-d^2}}-\frac{d\left(A\left(-12c^2d+2c^3+43cd^2+72d^3\right)+B\left(-23c^2d+\right.\right.}{15a^3f(c-d)^4(c+d)(c+d\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (-2*d^2*(A*d*(4*c + 3*d) - B*(3*c^2 + 3*c*d + d^2))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f) - (d*(
B*(3*c^3 - 23*c^2*d - 63*c*d^2 - 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 +
 72*d^3))*Cos[e + f*x])/(15*a^3*(c - d)^4*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((A - B)*Cos[e + f*x])/(5*(c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e +
f*x])) - ((2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(
a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) - ((B*(3*c^2 - 23*c*d - 15*d^2)
 + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Si
n[e + f*x])*(c + d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{\int \frac{-a(2A(c-3d)+B(3c+d}{(a+a\sin(e+fx))}}{5a^2(}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{(2Ac+3Bc}{15a(c-d)^2f(a+a}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))} - \frac{(2Ac+3Bc}{15a(c-d)^2f(a+a}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+72d^3\right)\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+72d^3\right)\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+72d^3\right)\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{d\left(B\left(3c^3-23c^2d-63cd^2-22d^3\right)+A\left(2c^3-12c^2d+43cd^2+72d^3\right)\right.}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}$$

$$= -\frac{2d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\tan^{-1}\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f} - \frac{d\,(}{}$$

Mathematica [B] time = 6.37469, size = 1253, normalized size = 3.29

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^2),x]
```

```
[Out] (2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*x
)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a + a*
Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos[(e
```

```
+ f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2] + 54
0*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430*A*c*d
^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(e + f*x
)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] - 60*B*c^
4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*c^3*d*Cos
[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c^2*d^2*Cos
[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c*d^3*Cos[(3
*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos[(3*(e + f*x
))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*(e + f*x))/2]
 + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x))/2] + 30*B*d
^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6*B*c^3*d*Cos[(7
*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c^2*d^2*Cos[(7*(e
 + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^3*Cos[(7*(e + f*x
))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*(e + f*x))/2] + 80
*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 340*A*c^3*d*Sin[(e +
f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d^2*Sin[(e + f*x)/2] - 1
520*B*c^2*d^2*Sin[(e + f*x)/2] + 2140*A*c*d^3*Sin[(e + f*x)/2] - 1435*B*c*d
^3*Sin[(e + f*x)/2] + 975*A*d^4*Sin[(e + f*x)/2] - 340*B*d^4*Sin[(e + f*x)/
2] - 90*B*c^3*d*Sin[(3*(e + f*x))/2] + 120*A*c^2*d^2*Sin[(3*(e + f*x))/2] -
 390*B*c^2*d^2*Sin[(3*(e + f*x))/2] + 540*A*c*d^3*Sin[(3*(e + f*x))/2] - 31
5*B*c*d^3*Sin[(3*(e + f*x))/2] + 285*A*d^4*Sin[(3*(e + f*x))/2] - 150*B*d^4
*Sin[(3*(e + f*x))/2] - 8*A*c^4*Sin[(5*(e + f*x))/2] - 12*B*c^4*Sin[(5*(e +
 f*x))/2] + 28*A*c^3*d*Sin[(5*(e + f*x))/2] + 62*B*c^3*d*Sin[(5*(e + f*x))/
2] - 52*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 362*B*c^2*d^2*Sin[(5*(e + f*x))/2]
 - 568*A*c*d^3*Sin[(5*(e + f*x))/2] + 553*B*c*d^3*Sin[(5*(e + f*x))/2] - 55
5*A*d^4*Sin[(5*(e + f*x))/2] + 190*B*d^4*Sin[(5*(e + f*x))/2] - 15*B*c*d^3*
Sin[(7*(e + f*x))/2] + 15*A*d^4*Sin[(7*(e + f*x))/2]))/(120*(c - d)^4*(c +
d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]))
```

Maple [B] time = 0.162, size = 1049, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a^3*d^3/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
*B*c+2/f/a^3*d^4/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+
1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2/f/a^3*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+
2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B-6/f/a^3*d^4/(c-d)^4/(c
+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
```

```
)*A-2/f/a^3*d^4/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(
c+d)*A+8/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*A*c*d-4/f/a^3/(c-d)^2/(tan(1/
2*f*x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5*A+8/5/f/a^3/
(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5*B+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^4
*A-16/3/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*A*c+8/f/a^3/(c-d)^3/(tan(1/2
*f*x+1/2*e)+1)^3*A*d+4/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*B*c-20/3/f/a^
3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+
1)*A*c^2-12/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)*A*d^2+6/f/a^3/(c-d)^4/(tan
(1/2*f*x+1/2*e)+1)*B*d^2-2/f/a^3*d^5/(c-d)^4/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/2*f*x+1/2*e)*A-8/f/a^3*d^3/(c-d)^4/(c+d)/
(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*
c+6/f/a^3*d^2/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2
*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+6/f/a^3*d^3/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)*a
rctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+4/f/a^3/(c-d)^3
/(tan(1/2*f*x+1/2*e)+1)^2*A*c-8/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*A*d-
2/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2*B*c+6/f/a^3/(c-d)^3/(tan(1/2*f*x+1
/2*e)+1)^2*B*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.05564, size = 9643, normalized size = 25.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/30*(6*(A - B)*c^6 - 12*(A - B)*c^5*d - 6*(A - B)*c^4*d^2 + 24*(A - B)*c
^3*d^3 - 6*(A - B)*c^2*d^4 - 12*(A - B)*c*d^5 + 6*(A - B)*d^6 - 2*((2*A + 3
```

*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*
d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - 2*((2*A +
 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*B)*c^
3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^
6)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A -
7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A -
 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(
2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 -
 (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A -
9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*
c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos
(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 -
(3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*
A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A
 - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*
B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)
*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e))*sin(f*x + e))*s
qrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^
2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2)
)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c
^6 - (11*A + 9*B)*c^5*d + (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 +
 (47*A + 28*B)*c^2*d^4 - 71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e)
 - 2*(3*(A - B)*c^6 - 6*(A - B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*
d^3 - 3*(A - B)*c^2*d^4 - 6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^
5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 -
(43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^
6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 +
5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x
+ e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 +
2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*
A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3
*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*
d^8)*f*cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^
3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)
*f*cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*
d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^
3*d^8)*f*cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*
c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x
+ e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3
*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d
^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^
7 - a^3*d^8)*f*cos(f*x + e)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 +
10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2
 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5
 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a

^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 +
2*a^3*c*d^7 - a^3*d^8)*f)*sin(f*x + e)), -1/15*(3*(A - B)*c^6 - 6*(A - B)*c
^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 - 6*(A -
B)*c*d^5 + 3*(A - B)*d^6 - ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41
*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 1
1*B)*d^6)*cos(f*x + e)^4 - ((2*A + 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A -
18*B)*c^4*d^2 + (147*A - 152*B)*c^3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A -
166*B)*c*d^5 - 3*(57*A - 17*B)*d^6)*cos(f*x + e)^3 + (2*(2*A + 3*B)*c^6 -
(19*A + 16*B)*c^5*d + 11*(2*A - 7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*
(32*A + 23*B)*c^2*d^4 - (109*A - 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x
+ e)^2 + 15*(12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 -
4*(3*A - B)*d^5 + (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x
+ e)^4 - (3*B*c^3*d^2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A
- B)*d^5)*cos(f*x + e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18
*B)*c*d^4 - 5*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B
)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e) + (12*B*c^3*d^2
- 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 - (3*B*c^2
*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^3 - (3*B*c^3*d^2 - 4
*(A - 3*B)*c^2*d^3 - 5*(3*A - 2*B)*c*d^4 - 3*(3*A - B)*d^5)*cos(f*x + e)^2
+ 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 - (3*A - B)*d^
5)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)
/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^6 - (11*A + 9*B)*c^5*d
+ (12*A - 47*B)*c^4*d^2 + 2*(41*A - 31*B)*c^3*d^3 + (47*A + 28*B)*c^2*d^4 -
71*(A - B)*c*d^5 - (62*A - 17*B)*d^6)*cos(f*x + e) - (3*(A - B)*c^6 - 6*(A
- B)*c^5*d - 3*(A - B)*c^4*d^2 + 12*(A - B)*c^3*d^3 - 3*(A - B)*c^2*d^4 -
6*(A - B)*c*d^5 + 3*(A - B)*d^6 + ((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^
2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(3
6*A - 11*B)*d^6)*cos(f*x + e)^3 - ((2*A + 3*B)*c^6 - (8*A + 17*B)*c^5*d + (
17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 + 5*(16*A + 7*B)*c^2*d^4 - (
98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^
6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 + 2*(39*A - 29*B)*c^3*d^3 +
3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*A - 2*B)*d^6)*cos(f*x + e)
)*sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 -
5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^4 - (a
^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3
*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*cos(f*x + e)^3 - (3*a^3
*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*
a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*cos(f*x + e)^2 +
2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 +
2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*
c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a
^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c
^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e
)^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^
3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*

```
d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c
*d^7 - a^3*d^8)*f*cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 +
 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*
sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34181, size = 1042, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] 2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor(
1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3
*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) - A*d
^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*
a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1
/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4
 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*B
*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*ta
n(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*d*tan(1/2*
f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*tan(1/2*f*x +
 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)
^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x + 1/2*e)^2 +
420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*
c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 110*A*c*d*tan(1/
```

```
2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*tan(1/2*f*x + 1/
2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 34*A*c*d - 16*B
*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3
*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=508

$$-\frac{d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(8c^2d+4c^3+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(A\left(142c^2d^2-30c^3d+4c^4+\right.\right.}{}$$

```
[Out] -((d^2*(A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2
*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^5*(c
+ d)^2*Sqrt[c^2 - d^2]*f)) - (d*(3*B*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3)
 + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3))*Cos[e + f*x])/(30*a^3*(c - d
)^4*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f
*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2) - ((2*A*c + 3*B*c - 11*A*d
+ 6*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[
e + f*x])^2) - ((3*B*(c^2 - 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))
*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2) - (d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4) + A*(4
*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.44656, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$-\frac{d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(8c^2d+4c^3+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3f(c-d)^5(c+d)^2\sqrt{c^2-d^2}}-\frac{d\left(A\left(142c^2d^2-30c^3d+4c^4+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] -((d^2*(A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2
*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a^3*(c - d)^5*(c
+ d)^2*Sqrt[c^2 - d^2]*f)) - (d*(3*B*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3)
 + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3))*Cos[e + f*x])/(30*a^3*(c - d
)^4*(c + d)*f*(c + d*Sin[e + f*x])^2) - ((A - B)*Cos[e + f*x])/(5*(c - d)*f
*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2) - ((2*A*c + 3*B*c - 11*A*d
+ 6*B*d)*Cos[e + f*x])/(15*a*(c - d)^2*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[
```

```
e + f*x])^2) - ((3*B*(c^2 - 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))
*Cos[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2) - (d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4) + A*(4
*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/(30*a^3
*(c - d)^5*(c + d)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{\int \frac{-a(2Ac+3Bc-7Ad+2B}{(a+a\sin(e+fx))^2}}{5a^2(}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-}{15a(c-d)^2f(a+a\,\mathrm{s}}$$

$$= -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2} - \frac{(2Ac+3Bc-}{15a(c-d)^2f(a+a\,\mathrm{s}}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd^2+195d^3\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd^2+195d^3\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd^2+195d^3\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd^2+195d^3\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d\left(3B\left(2c^3-20c^2d-57cd^2-30d^3\right)+A\left(4c^3-30c^2d+146cd^2+195d^3\right.\right.}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$

$$= -\frac{d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\tan^{-1}\left(\frac{d}{-}\right.}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}f}$$

Mathematica [A] time = 4.57133, size = 548, normalized size = 1.08

$$\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{15d^3\left(B\left(7c^2+6cd+2d^2\right)-3Ad(3c+2d)\right)\cos(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{(c+d)^2(c+d\sin(e+fx))}+4\left(A\left(2c^2-19cd+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - d)
*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2 - 19*
c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 +
(30*d^2*(-(A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2
 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d)*d^3*(B*
c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)*(c
+ d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 + 6*c*d + 2*d
^2))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*(c +
d*Sin[e + f*x]))))/(30*a^3*(c - d)^5*f*(1 + Sin[e + f*x])^3)
```

Maple [B] time = 0.193, size = 2918, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 12/f/a^3/(c-d)^5/(tan(1/2*f*x+1/2*e)+1)*B*d^2-20/f/a^3/(c-d)^5/(tan(1/2*f*x
+1/2*e)+1)*A*d^2-10/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2*A*d-2/f/a^3/(c-d
)^4/(tan(1/2*f*x+1/2*e)+1)^2*B*c+8/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2*B
*d-16/3/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3*A*c+28/3/f/a^3/(c-d)^4/(tan(
1/2*f*x+1/2*e)+1)^3*A*d+4/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3*B*c-8/f/a^
3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^5/(tan(1/2*f*x+1/2*e)+
```

1)*A*c^2+4/f/a^3/(c-d)^4/(tan(1/2*f*x+1/2*e)+1)^2*A*c+6/f/a^3*d^4/(c-d)^5/(
c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(
1/2*f*x+1/2*e)^2*B+17/f/a^3*d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f
*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^2*B+2/f/a^3*d^7/(c-d)
^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*ta
n(1/2*f*x+1/2*e)^2*B-29/f/a^3*d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2
*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+23/f/a^3*d^4/(c-d
)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)
*tan(1/2*f*x+1/2*e)*B+12/f/a^3*d^2/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*
arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+2/f/a^3*d^7/
(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^
2)*tan(1/2*f*x+1/2*e)^3*A+9/f/a^3*d^4/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan
(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*B+18/f/a^3*
d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*
d+d^2)*tan(1/2*f*x+1/2*e)*B-11/f/a^3*d^5/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*
tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-20/f/a^3
*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-30/f/a^3*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-
d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+24/
f/a^3*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f
*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+6/f/a^3*d^4/(c-d)^5/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2+10/f/a^3/(c-d)^5/(
tan(1/2*f*x+1/2*e)+1)*A*c*d+1/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A+6/f/a^3*d^5/(c-d)^5/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)^3*B-10/f/a^3*d^4/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c
)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*A-6/f/a^3*d^5/(c-d)^5/(c*tan(1
/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1
/2*e)^2*A+2/f/a^3*d^7/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*
d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+21/f/a^3*d^4/(c-d)^5/(c^2+2*c
*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(
1/2))*B*c-12/f/a^3*d^7/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-19/f/a^3*d^6/(c-d)^5/(c*ta
n(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+
1/2*e)^2*A+8/f/a^3*d^3/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)
*d+c)^2/(c^2+2*c*d+d^2)*B*c^3-13/f/a^3*d^5/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2
)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+6/f/a^3*
d^5/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2
*e)+2*d)/(c^2-d^2)^(1/2))*B+4/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*t
an(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+1/f/a^3*d^5/(
c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*
B*c+12/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^
2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^2*B-18/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*
x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A
-6/f/a^3*d^6/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c

```
^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3*A-10/f/a^3*d^4/(c-d)^5/(c*tan(1/2*f*x+1/
2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c^2-6/f/a^3*d^5/(c-d)^
5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c-4
/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)^3/(tan(1/2*f*x+1/
2*e)+1)^5*A+8/5/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^5*B+4/f/a^3/(c-d)^3/(t
an(1/2*f*x+1/2*e)+1)^4*A+2/f/a^3*d^8/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A+8/f/a^3*d^
3/(c-d)^5/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^
2)*c^3*tan(1/2*f*x+1/2*e)^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.11804, size = 16342, normalized size = 32.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] [-1/60*(12*(A - B)*c^8 - 24*(A - B)*c^7*d - 24*(A - B)*c^6*d^2 + 72*(A - B)
*c^5*d^3 - 72*(A - B)*c^3*d^5 + 24*(A - B)*c^2*d^6 + 24*(A - B)*c*d^7 - 12*
(A - B)*d^8 + 2*(2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 1
21*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35
*A - 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*cos(f*x + e)^5 - 2*(4*(2*A + 3*B)*c
^7*d - 4*(13*A + 27*B)*c^6*d^2 + 18*(12*A - 37*B)*c^5*d^3 + 6*(181*A - 171*
B)*c^4*d^4 + 3*(328*A - 33*B)*c^3*d^5 - 9*(69*A - 104*B)*c^2*d^6 - (1208*A
- 753*B)*c*d^7 - (413*A - 198*B)*d^8)*cos(f*x + e)^4 - 2*(2*(2*A + 3*B)*c^8
 - 6*(A + 4*B)*c^7*d - 20*(A + 21*B)*c^6*d^2 + 6*(128*A - 293*B)*c^5*d^3 +
3*(892*A - 827*B)*c^4*d^4 + 3*(769*A - 49*B)*c^3*d^5 - (1573*A - 2373*B)*c^
```

2*d^6 - 3*(1023*A - 643*B)*c*d^7 - (1087*A - 522*B)*d^8)*cos(f*x + e)^3 + 4
(2(2*A + 3*B)*c^8 - 5*(4*A + 3*B)*c^7*d + (19*A - 174*B)*c^6*d^2 + 15*(22
*A - 35*B)*c^5*d^3 + 3*(233*A - 173*B)*c^4*d^4 + 15*(23*A + 10*B)*c^3*d^5 -
(526*A - 591*B)*c^2*d^6 - 5*(131*A - 78*B)*c*d^7 - 4*(49*A - 24*B)*d^8)*co
s(f*x + e)^2 - 15*(48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)
*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B
)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*
A - 6*B)*d^7)*cos(f*x + e)^5 + (24*B*c^4*d^3 - 4*(10*A - 21*B)*c^3*d^4 - 6*
(20*A - 19*B)*c^2*d^5 - (116*A - 75*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x
+ e)^4 - (12*B*c^5*d^2 - 4*(5*A - 18*B)*c^4*d^3 - (110*A - 153*B)*c^3*d^4 -
(193*A - 162*B)*c^2*d^5 - (142*A - 87*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f
*x + e)^3 - (36*B*c^5*d^2 - 12*(5*A - 16*B)*c^4*d^3 - (290*A - 387*B)*c^3*d
^4 - (479*A - 396*B)*c^2*d^5 - (340*A - 207*B)*c*d^6 - 7*(13*A - 6*B)*d^7)*
cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c
^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*
cos(f*x + e) + (48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^
3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d
^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A -
6*B)*d^7)*cos(f*x + e)^4 - 2*(12*B*c^4*d^3 - 4*(5*A - 9*B)*c^3*d^4 - 5*(10
*A - 9*B)*c^2*d^5 - (43*A - 27*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^3
- (12*B*c^5*d^2 - 4*(5*A - 24*B)*c^4*d^3 - 75*(2*A - 3*B)*c^3*d^4 - (293*A
- 252*B)*c^2*d^5 - 3*(76*A - 47*B)*c*d^6 - 5*(13*A - 6*B)*d^7)*cos(f*x + e)
^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(
31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)
)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d
*sin(f*x + e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e)
)*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2))
+ 12*((3*A + 2*B)*c^8 - (11*A + 9*B)*c^7*d + (9*A - 109*B)*c^6*d^2 + (213*A
- 353*B)*c^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (237*A + 103*B)*c^3*d^5 - (35
9*A - 399*B)*c^2*d^6 - (439*A - 259*B)*c*d^7 - (128*A - 63*B)*d^8)*cos(f*x
+ e) - 2*(6*(A - B)*c^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B
)*c^5*d^3 - 36*(A - B)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*
(A - B)*d^8 + (2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121
*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A
- 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*cos(f*x + e)^4 + (4*(2*A + 3*B)*c^7*d
- 6*(8*A + 17*B)*c^6*d^2 + 6*(31*A - 121*B)*c^5*d^3 + 3*(408*A - 463*B)*c^
4*d^4 + 3*(513*A - 143*B)*c^3*d^5 - 3*(153*A - 383*B)*c^2*d^6 - (1733*A - 1
143*B)*c*d^7 - 3*(239*A - 114*B)*d^8)*cos(f*x + e)^3 - 2*((2*A + 3*B)*c^8 -
(7*A + 18*B)*c^7*d + (14*A - 159*B)*c^6*d^2 + 3*(97*A - 172*B)*c^5*d^3 + 6
*(121*A - 91*B)*c^4*d^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^
6 - (668*A - 393*B)*c*d^7 - 5*(37*A - 18*B)*d^8)*cos(f*x + e)^2 - 6*((2*A +
3*B)*c^8 - (9*A + 11*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c
^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)
*c^2*d^6 - 9*(49*A - 29*B)*c*d^7 - (127*A - 62*B)*d^8)*cos(f*x + e))*sin(f*
x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a

^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^5 + (2
*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d
^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^
3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^
9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*
a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*
f*cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^
3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*
a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*cos(f*x + e)^2
 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^
4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^
3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*
d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^
3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a
^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6
- 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^4
 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^7*d^4 + 2*a^3*c^
6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c^2*d^9 - 2*
a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*a^3*c^10*d - 13*a^3
*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5 - 58*a^3*c^5*d^6
+ 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^3*c*d^10 + 5*a^3*d
^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^
8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 +
 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*
(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 1
0*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2
*d^9 - a^3*c*d^10 + a^3*d^11)*f)*sin(f*x + e)), -1/30*(6*(A - B)*c^8 - 12*(
A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B)*c^3*d^5
 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A + 3*B)*c
^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37*A - 22*B)
*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(19*A - 9*
B)*d^8)*cos(f*x + e)^5 - (4*(2*A + 3*B)*c^7*d - 4*(13*A + 27*B)*c^6*d^2 + 1
8*(12*A - 37*B)*c^5*d^3 + 6*(181*A - 171*B)*c^4*d^4 + 3*(328*A - 33*B)*c^3*
d^5 - 9*(69*A - 104*B)*c^2*d^6 - (1208*A - 753*B)*c*d^7 - (413*A - 198*B)*d
^8)*cos(f*x + e)^4 - (2*(2*A + 3*B)*c^8 - 6*(A + 4*B)*c^7*d - 20*(A + 21*B)
*c^6*d^2 + 6*(128*A - 293*B)*c^5*d^3 + 3*(892*A - 827*B)*c^4*d^4 + 3*(769*A
 - 49*B)*c^3*d^5 - (1573*A - 2373*B)*c^2*d^6 - 3*(1023*A - 643*B)*c*d^7 - (
1087*A - 522*B)*d^8)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^8 - 5*(4*A + 3*B)*
c^7*d + (19*A - 174*B)*c^6*d^2 + 15*(22*A - 35*B)*c^5*d^3 + 3*(233*A - 173*
B)*c^4*d^4 + 15*(23*A + 10*B)*c^3*d^5 - (526*A - 591*B)*c^2*d^6 - 5*(131*A
 - 78*B)*c*d^7 - 4*(49*A - 24*B)*d^8)*cos(f*x + e)^2 + 15*(48*B*c^5*d^2 - 16
*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5
 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B
)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^5 + (24*B
*c^4*d^3 - 4*(10*A - 21*B)*c^3*d^4 - 6*(20*A - 19*B)*c^2*d^5 - (116*A - 75*

B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x + e)^4 - (12*B*c^5*d^2 - 4*(5*A - 18
*B)*c^4*d^3 - (110*A - 153*B)*c^3*d^4 - (193*A - 162*B)*c^2*d^5 - (142*A -
87*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*cos(f*x + e)^3 - (36*B*c^5*d^2 - 12*(5*A
- 16*B)*c^4*d^3 - (290*A - 387*B)*c^3*d^4 - (479*A - 396*B)*c^2*d^5 - (340*
A - 207*B)*c*d^6 - 7*(13*A - 6*B)*d^7)*cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4
*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (
56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e) + (48*B*c^5*d^2 - 16*(5
*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4
*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c
^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^4 - 2*(12*B*
c^4*d^3 - 4*(5*A - 9*B)*c^3*d^4 - 5*(10*A - 9*B)*c^2*d^5 - (43*A - 27*B)*c*
d^6 - (13*A - 6*B)*d^7)*cos(f*x + e)^3 - (12*B*c^5*d^2 - 4*(5*A - 24*B)*c^4
d^3 - 75(2*A - 3*B)*c^3*d^4 - (293*A - 252*B)*c^2*d^5 - 3*(76*A - 47*B)*c
d^6 - 5(13*A - 6*B)*d^7)*cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B
)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)
*c*d^6 - (13*A - 6*B)*d^7)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arct
an(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 6*((3*A + 2*B)*c
^8 - (11*A + 9*B)*c^7*d + (9*A - 109*B)*c^6*d^2 + (213*A - 353*B)*c^5*d^3 +
5*(95*A - 71*B)*c^4*d^4 + (237*A + 103*B)*c^3*d^5 - (359*A - 399*B)*c^2*d^
6 - (439*A - 259*B)*c*d^7 - (128*A - 63*B)*d^8)*cos(f*x + e) - (6*(A - B)*c
^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B
)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A
+ 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37*
A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(
19*A - 9*B)*d^8)*cos(f*x + e)^4 + (4*(2*A + 3*B)*c^7*d - 6*(8*A + 17*B)*c^6
d^2 + 6(31*A - 121*B)*c^5*d^3 + 3*(408*A - 463*B)*c^4*d^4 + 3*(513*A - 14
3*B)*c^3*d^5 - 3*(153*A - 383*B)*c^2*d^6 - (1733*A - 1143*B)*c*d^7 - 3*(239
*A - 114*B)*d^8)*cos(f*x + e)^3 - 2*((2*A + 3*B)*c^8 - (7*A + 18*B)*c^7*d +
(14*A - 159*B)*c^6*d^2 + 3*(97*A - 172*B)*c^5*d^3 + 6*(121*A - 91*B)*c^4*d
^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^6 - (668*A - 393*B)*c
d^7 - 5(37*A - 18*B)*d^8)*cos(f*x + e)^2 - 6*((2*A + 3*B)*c^8 - (9*A + 11
*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c^5*d^3 + 5*(95*A - 71
*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)*c^2*d^6 - 9*(49*A -
29*B)*c*d^7 - (127*A - 62*B)*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^9*d^2
- 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^
3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c
^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 -
2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11
)*f*cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 +
26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*
c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^3 - (3
*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 -
22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*
c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*
c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 -

```
10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^1
0 + a^3*d^11)*f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5
*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^
4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^
9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a
^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*
a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^
6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11
)*f*cos(f*x + e)^3 - (a^3*c^11 + 3*a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*
d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37
*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^3*c*d^10 + 5*a^3*d^11)*f*cos(f*x + e)^2
 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^
4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^
3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*
d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^
3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a
^3*d^11)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.55749, size = 1717, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] 1/15*(15*(12*B*c^3*d^2 - 20*A*c^2*d^3 + 24*B*c^2*d^3 - 30*A*c*d^4 + 21*B*c*
d^4 - 13*A*d^5 + 6*B*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan
```

```
((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a
^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^
3*d^7)*sqrt(c^2 - d^2)) + 15*(9*B*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 11*A*c^3
*d^5*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*A*c^2*
d^6*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*B*c^5*d^3
*tan(1/2*f*x + 1/2*e)^2 - 10*A*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^4*d^4
*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^3*d^5
*tan(1/2*f*x + 1/2*e)^2 - 19*A*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 12*B*c^2*d^
6*tan(1/2*f*x + 1/2*e)^2 - 12*A*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^7*ta
n(1/2*f*x + 1/2*e)^2 + 2*A*d^8*tan(1/2*f*x + 1/2*e)^2 + 23*B*c^4*d^4*tan(1/
2*f*x + 1/2*e) - 29*A*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*B*c^3*d^5*tan(1/2*f
*x + 1/2*e) - 18*A*c^2*d^6*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^6*tan(1/2*f*x +
 1/2*e) + 2*A*c*d^7*tan(1/2*f*x + 1/2*e) + 8*B*c^5*d^3 - 10*A*c^4*d^4 + 6*B
*c^4*d^4 - 6*A*c^3*d^5 + B*c^3*d^5 + A*c^2*d^6)/((a^3*c^9 - 3*a^3*c^8*d + a
^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4 - a^3*c^4*d^5 + 3*a^3*c^3*d^6 -
a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) -
 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 150
*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 90*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*
tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 195*A*c*d*tan(1/
2*f*x + 1/2*e)^3 - 75*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 525*A*d^2*tan(1/2*f*x
+ 1/2*e)^3 - 300*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*
e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 245*A*c*d*tan(1/2*f*x + 1/2*e)^2 -
 135*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 745*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 420*
B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan
(1/2*f*x + 1/2*e) - 145*A*c*d*tan(1/2*f*x + 1/2*e) - 105*B*c*d*tan(1/2*f*x
+ 1/2*e) + 485*A*d^2*tan(1/2*f*x + 1/2*e) - 270*B*d^2*tan(1/2*f*x + 1/2*e)
+ 7*A*c^2 + 3*B*c^2 - 44*A*c*d - 21*B*c*d + 127*A*d^2 - 72*B*d^2)/((a^3*c^5
 - 5*a^3*c^4*d + 10*a^3*c^3*d^2 - 10*a^3*c^2*d^3 + 5*a^3*c*d^4 - a^3*d^5)*(
tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

3.286 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=256

$$\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}+\frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (4*a*(c + d)*(B*c - 9*A*d - 8*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/
(315*d*f*Sqrt[a + a*Sin[e + f*x]]) + (8*(5*c - d)*(c + d)*(B*c - 9*A*d - 8*
B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*d*(c + d)*(B*c - 9
*A*d - 8*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*a*f) + (2*a*(B*
c - 9*A*d - 8*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Rubi [A] time = 0.459978, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2981, 2770, 2761, 2751, 2646}

$$\frac{4a(c+d)\left(15c^2+10cd+7d^2\right)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}}+\frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```



```
[Out] (4*a*(c + d)*(B*c - 9*A*d - 8*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/
(315*d*f*Sqrt[a + a*Sin[e + f*x]]) + (8*(5*c - d)*(c + d)*(B*c - 9*A*d - 8*
B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(315*f) + (4*d*(c + d)*(B*c - 9
*A*d - 8*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*a*f) + (2*a*(B*
c - 9*A*d - 8*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d*f*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a
+ a*Sin[e + f*x]])
```


Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
```

```
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}} + \frac{(9aAd-B(a$$

$$= \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+}{105af}$$

$$= \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+fx)\sqrt{a+a}}{315f}$$

$$= \frac{4a(c+d)(Bc-9Ad-8Bd)\left(15c^2+10cd+7d^2\right)\cos(}{315df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 1.27302, size = 305, normalized size = 1.19

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-4d\left(27Ad(7c+2d)+B\left(189c^2+162cd+83d^2\right)\right)\cos(2(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2520*A*
c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 4104*B*c*d^
2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^2 + 162*c*
d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*B*c^3*Sin[e
 + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x] + 2016*A*c*
d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e + f*x] + 752
*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3*Sin[3*(e + f*
x)] - 80*B*d^3*Sin[3*(e + f*x)]))/(1260*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]))
```

Maple [A] time = 1.207, size = 242, normalized size = 1.

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a\left(-1+\sin\left(fx+e\right)\right)\left(\left(-45\,Ad^3-135\,Bcd^2-40\,Bd^3\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2+\left(315\,Ac^2d+2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*((-45*A*d^3-135*B*c*d^2-40*B*d^3)*si
n(f*x+e)*cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^3+105*B*c^3+252*B*c^
2*d+351*B*c*d^2+104*B*d^3)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^3+(-189*A*c*d^2-5
4*A*d^3-189*B*c^2*d-162*B*c*d^2-118*B*d^3)*cos(f*x+e)^2+315*A*c^3+630*A*c^2
*d+693*A*c*d^2+198*A*d^3+210*B*c^3+693*B*c^2*d+594*B*c*d^2+211*B*d^3)/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^3, x)
```

Fricas [A] time = 2.16706, size = 1156, normalized size = 4.52

$$-\frac{2\left(35\,Bd^{3}\cos\left(fx+e\right)^{5}-5\left(27\,Bcd^{2}+(9\,A+B)d^{3}\right)\cos\left(fx+e\right)^{4}+105\,(3\,A+B)c^{3}+63\,(5\,A+7\,B)c^{2}d+9\,(49\,A+27\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*d^3*cos(f*x + e)^5 - 5*(27*B*c*d^2 + (9*A + B)*d^3)*cos(f*x +
e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A + 27*B)*c*d^2 + (
81*A + 107*B)*d^3 - (189*B*c^2*d + 27*(7*A + 6*B)*c*d^2 + 2*(27*A + 59*B)*d
^3)*cos(f*x + e)^3 + (105*B*c^3 + 63*(5*A + B)*c^2*d + 9*(7*A + 36*B)*c*d^2
 + 2*(54*A + 13*B)*d^3)*cos(f*x + e)^2 + (105*(3*A + 2*B)*c^3 + 63*(10*A +
```

```
11*B)*c^2*d + 99*(7*A + 6*B)*c*d^2 + (198*A + 211*B)*d^3)*cos(f*x + e) - (3
5*B*d^3*cos(f*x + e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A
 + 27*B)*c*d^2 + (81*A + 107*B)*d^3 + 5*(27*B*c*d^2 + (9*A + 8*B)*d^3)*cos(
f*x + e)^3 - 3*(63*B*c^2*d + 9*(7*A + B)*c*d^2 + (3*A + 26*B)*d^3)*cos(f*x
+ e)^2 - (105*B*c^3 + 63*(5*A + 4*B)*c^2*d + 9*(28*A + 39*B)*c*d^2 + 13*(9*
A + 8*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f
*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.287 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=192

$$\frac{2a\left(15c^2+10cd+7d^2\right)(-7Ad+Bc-6Bd)\cos(e+fx)}{105df\sqrt{a\sin(e+fx)+a}}+\frac{2d(-7Ad+Bc-6Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}+\frac{4(5c$$

```
[Out] (2*a*(B*c - 7*A*d - 6*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(105*d*f
*Sqrt[a + a*Sin[e + f*x]]) + (4*(5*c - d)*(B*c - 7*A*d - 6*B*d)*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(105*f) + (2*d*(B*c - 7*A*d - 6*B*d)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.339343, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2981, 2761, 2751, 2646}

$$\frac{2a\left(15c^2+10cd+7d^2\right)(-7Ad+Bc-6Bd)\cos(e+fx)}{105df\sqrt{a\sin(e+fx)+a}}+\frac{2d(-7Ad+Bc-6Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35af}+\frac{4(5c$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```



```
[Out] (2*a*(B*c - 7*A*d - 6*B*d)*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(105*d*f
*Sqrt[a + a*Sin[e + f*x]]) + (4*(5*c - d)*(B*c - 7*A*d - 6*B*d)*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(105*f) + (2*d*(B*c - 7*A*d - 6*B*d)*Cos[e + f*
x]*(a + a*Sin[e + f*x])^(3/2))/(35*a*f) - (2*a*B*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}} + \frac{(7aAd-B(a}{}$$

$$= \frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/}}{35af}$$

$$= \frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e}}{105f}$$

$$= \frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{105df\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A] time = 0.746042, size = 176, normalized size = 0.92

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\left(56Ad(5c+2d)+B\left(140c^2+224cd+141d^2\right)\right)\sin(e+fx)-\right.}{210f\left(\sin\left(\frac{1}{2}(e+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(420*A*c
^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*d*(14*B*
c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140*c^2 + 22
4*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(210*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 1.049, size = 161, normalized size = 0.8

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a\left(-1+\sin\left(fx+e\right)\right)\left(-15B\left(\cos\left(fx+e\right)\right)^{2}\sin\left(fx+e\right)d^{2}+\left(70Acd+28Ad^{2}+35Bc^{2}+56Bcd+\right.\right.}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*(-15*B*cos(f*x+e)^2*sin(f*x+e)*d^2+(
70*A*c*d+28*A*d^2+35*B*c^2+56*B*c*d+39*B*d^2)*sin(f*x+e)+(-21*A*d^2-42*B*c*
d-18*B*d^2)*cos(f*x+e)^2+105*A*c^2+140*A*c*d+77*A*d^2+70*B*c^2+154*B*c*d+66
*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^2, x)
```

Fricas [A] time = 2.00698, size = 761, normalized size = 3.96

$$2\left(15\,Bd^{2}\cos\left(fx+e\right)^{4}+3\left(14\,Bcd+(7\,A+6\,B)d^{2}\right)\cos\left(fx+e\right)^{3}-35\,(3\,A+B)c^{2}-14\,(5\,A+7\,B)cd-(49\,A+27\,B)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x + e
)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c^2
 + 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)*c^
2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2*cos
(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^2 - 3
*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*B)*c*d
 + (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.288 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=118

$$-\frac{2(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2Bd\cos(e+fx)(a}{5a}$$

```
[Out] (-2*a*(15*A*c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(15*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```

Rubi [A] time = 0.249048, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2968, 3023, 2751, 2646}

$$-\frac{2(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2a(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2Bd\cos(e+fx)(a}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```


```
[Out] (-2*a*(15*A*c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(15*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int \sqrt{a+a\sin(e+fx)}\left(Ac+(Bc+Ad)\sin(e+fx)+Bd\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af} + \frac{2\int\sqrt{a+a\,\mathrm{si}}}{}$$

$$= -\frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} -$$

$$= -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(5Bc+}{}$$

Mathematica [A] time = 0.36511, size = 117, normalized size = 0.99

$$-\frac{\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(2(5Ad+5Bc+4Bd)\sin(e+fx)+30Ac+20Ad+20Bc-3B}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
),x]
```

```
[Out] -((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(30*A*c
+ 20*B*c + 20*A*d + 19*B*d - 3*B*d*Cos[2*(e + f*x)] + 2*(5*B*c + 5*A*d + 4*
B*d)*Sin[e + f*x]))/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.954, size = 102, normalized size = 0.9

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a\left(-1+\sin\left(fx+e\right)\right)\left(3B\left(\sin\left(fx+e\right)\right)^{2}d+5A\sin\left(fx+e\right)d+5B\sin\left(fx+e\right)c+4B\sin\left(fx+\right.\right.}{15f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+
5*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*A*d+10*B*c+8*B*d)/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
), x)
```

Fricas [A] time = 1.99923, size = 436, normalized size = 3.69

$$\frac{2\left(3Bd\cos\left(fx+e\right)^{3}-(5Bc+(5A+B)d)\cos\left(fx+e\right)^{2}-5(3A+B)c-(5A+7B)d-(5(3A+2B)c+(10A+11B\right.}{15\left(f\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*B*d*cos(f*x + e)^3 - (5*B*c + (5*A + B)*d)*cos(f*x + e)^2 - 5*(3*A
+ B)*c - (5*A + 7*B)*d - (5*(3*A + 2*B)*c + (10*A + 11*B)*d)*cos(f*x + e) -
 (3*B*d*cos(f*x + e)^2 - 5*(3*A + B)*c - (5*A + 7*B)*d + (5*B*c + (5*A + 4*
B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)
+ f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.289 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=62

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

```
[Out] (-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rubi [A] time = 0.0576309, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2751, 2646}

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e +
 f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}+\frac{1}{3}(3A+B)\int \sqrt{a+a\sin(e+fx)}\,dx$$
$$= -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

Mathematica [A] time = 0.124571, size = 82, normalized size = 1.32

$$-\frac{2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3A+B\sin(e+fx)+2B)}{3f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*A +
 2*B + B*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.926, size = 58, normalized size = 0.9

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a\left(-1+\sin\left(fx+e\right)\right)\left(B\sin\left(fx+e\right)+3A+2B\right)}{3f\cos\left(fx+e\right)}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*a*(-1+sin(f*x+e))*(B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

Fricas [A] time = 1.88627, size = 225, normalized size = 3.63

$$-\frac{2\left(B\cos\left(fx+e\right)^{2}+\left(3\,A+2\,B\right)\cos\left(fx+e\right)+\left(B\cos\left(fx+e\right)-3\,A-B\right)\sin\left(fx+e\right)+3\,A+B\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(B*cos(f*x + e)^2 + (3*A + 2*B)*cos(f*x + e) + (B*cos(f*x + e) - 3*A -
 B)*sin(f*x + e) + 3*A + B)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*si
n(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.245729, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2981, 2773, 208}

$$\frac{2\sqrt{a}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx &= -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{(-aBc+aAd)\int\frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{ad}\\
&= -\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}+\frac{(2a(Bc-Ad))\operatorname{Subst}\left(\int\frac{1}{ac+ad-dx^2}\,dx,x,\frac{a\cos(e}{\sqrt{a+a\sin}}\right.}{df}\\
&= \frac{2\sqrt{a}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}f}-\frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 8.78459, size = 903, normalized size = 9.03

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{(2-2i)B\sqrt{d}\cos\left(\frac{fx}{2}\right)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)}{f}+\frac{(Ad-Bc)\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\cdots}{\cdots}\right]\cdots}{\cdots}\right)}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x]),x]
```

```
[Out] ((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f +
 ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d +
 (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*
f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt
[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I
)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)
/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sq
rt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*
(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e
]))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((-(
B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] +
(RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt
[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + S
```

```
qrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #
1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f
*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 +
 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*
e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/(Sqr
t[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((2 - 2*I)*B
*Sqrt[d]*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2])/f)*Sqrt[a*(1 + Sin[e + f*x])])
/(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 1.545, size = 139, normalized size = 1.4

$$-2\,\frac{\left(1+\sin\left(fx+e\right)\right)\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}}{d\sqrt{a\left(c+d\right)d}\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}f}\left(A\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}d}{\sqrt{a\left(c+d\right)d}}\right)ad-B\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\right.}}{\sqrt{a\left(c\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -2*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(A*arctanh((-a*(-1+sin(f*x+e))
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a*d-B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a*c+B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2))/d/(a*(c
+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
), x)
```

Fricas [A] time = 9.3318, size = 1539, normalized size = 15.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*
a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3
)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^
2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2
*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*cos(f*x + e) - B*sin(f
*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e)
 + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))
*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) -
 c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f*x + e) - B*si
n(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x +
 e) + d*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

```
[Out] -((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c -
A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
]))
```

Rubi [A] time = 0.261733, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2980, 2773, 208}

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] -((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt
[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c -
A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + -\frac{(-aAd-B(ac+2ad))}{2d(ac+}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(a(Ad+B(c+2d)))\,\mathrm{Su}}{}$$

$$= -\frac{\sqrt{a}(Ad+B(c+2d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a(Bc-A}{d(c+d)f\sqrt{a+a\sin(}}$$

Mathematica [C] time = 8.71981, size = 901, normalized size = 7.15

$$\left(\frac{1}{4}+\frac{i}{4}\right)\sqrt{a(\sin(e+fx)+1)}\left(\frac{(Ad+B(c+2d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{d}\sqrt{c\cdots}}{\cdots}\right]}{\cdots}\right)}{\cdots}\cdots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] + I
*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((
2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)
*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqr
t[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)]
 + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sq
rt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)
*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqr
t[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(Cos[e] + I
*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(Cos[e/2] +
 I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E
^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 +
 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*
```

```
#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*
#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^
((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*
E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e
] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(3/2)*(Cos[e] + I*
(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqrt[d]*(-(B*c) + A*d)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x]))))/
(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [B] time = 1.881, size = 274, normalized size = 2.2

$$-\frac{1+\sin\left(fx+e\right)}{d\left(c+d\right)\left(c+d\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)\mathrm{Artanh}\left(d\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{acd}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(A*d+B*c+2*B*d)+A*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+B*arctanh((a-a*sin(f*x+e))^(1/2)*d
/(a*c*d+a*d^2)^(1/2))*a*c^2+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a*c*d+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d-B*(a-a*sin(f*
x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c)/d/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c
)^2, x)
```

Fricas [B] time = 10.5287, size = 2354, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*cos
(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c*d
 + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*
c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*
cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
+ 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*c
os(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B*c - A*d)*co
s(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 +
d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 +
 d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*
x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d
^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3
*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e)
 - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c - A*d + (B*c -
A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c
*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*
c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)
*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a}(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a(3Ad+B(c+4d))\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{a}{2df(c+d)\sqrt{a}}$$

```
[Out] -(Sqrt[a]*(3*A*d + B*(c + 4*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(3/2)*(c + d)^(5/2)*f) + (a*(B*c
- A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
 f*x])^2) - (a*(3*A*d + B*(c + 4*d))*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.370379, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2980, 2772, 2773, 208}

$$-\frac{\sqrt{a}(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{3/2}f(c+d)^{5/2}}-\frac{a(3Ad+B(c+4d))\cos(e+fx)}{4df(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{a}{2df(c+d)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,
x]
```


```
[Out] -(Sqrt[a]*(3*A*d + B*(c + 4*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(3/2)*(c + d)^(5/2)*f) + (a*(B*c
- A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
 f*x])^2) - (a*(3*A*d + B*(c + 4*d))*Cos[e + f*x])/(4*d*(c + d)^2*f*Sqrt[a
+ a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```


Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
```

```
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + -\frac{(-3aAd-B(ac+4a}{4d(a}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{a(3Ad+B(c}{4d(c+d)^2f\sqrt{a+a\,\mathrm{si}}}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{a(3Ad+B(c}{4d(c+d)^2f\sqrt{a+a\,\mathrm{si}}}$$

$$= -\frac{\sqrt{a}(3Ad+B(c+4d))\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} + \frac{a(Bc-}{2d(c+d)f\sqrt{a+a\,\mathrm{si}}}$$

Mathematica [C] time = 10.064, size = 967, normalized size = 5.04

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\sqrt{a(\sin(e+fx)+1)}\left((3Ad+B(c+4d))\left(\cos\left(\frac{e}{2}\right)+i\sin\left(\frac{e}{2}\right)\right)\left((-1+i)x\cos(e)+(1+i)x\sin(e)+\frac{\text{RootSum}\left[de^{2ie}\#1^4+2ice^{ie}\#1^2-d\&,\frac{-\sqrt{d}\sqrt{c+d}e^{ie}fx\#1^3-2i\sqrt{}\cdots}{\cdots}\right]\cdots}{\cdots}\right)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*(((3*A*d + B*(c + 4*d))*(Cos[e/2]
 + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*
E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^(
(-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)
*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d
]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((
I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))
*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2)*(Cos[e]
 + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4*d))*(Cos[
e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*
I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x
+ (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d
```

```
]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*
c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/S
qrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c
 + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt
[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(5/2)*(Cos[e
] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*Sqrt[d]*(-(B*c)
+ A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x
])^2) - ((2 - 2*I)*Sqrt[d]*(3*A*d + B*(c + 4*d))*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x]))))/(d^(3/2)*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

Maple [B] time = 2.075, size = 628, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4/a*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
a^2*c*d*(3*A*d+B*c+4*B*d)+arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1
/2))*a^2*d^2*(3*A*d+B*c+4*B*d)*cos(f*x+e)^2+3*A*(a-a*sin(f*x+e))^(3/2)*(a*(
c+d)*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^
3+B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^(3/2)
*(a*(c+d)*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c
^2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-4*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d
)^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-3*B*(a-a*sin
(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
d)^(1/2)*a*d^2)*(-a*(-1+sin(f*x+e)))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)
/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 16.4405, size = 3954, normalized size = 20.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
- (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c
*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d
^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A +
3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d +
d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*
d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x +
e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 +
(c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(
c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
 + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^
2 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2
*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3
*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*
d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 +
d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*c
os(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
 + 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(
f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))
, 1/8*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 -
 (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c*
d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d^
2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3
*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
```

```
2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d +
d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-
a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2
 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2*
A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3*
A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d
^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d
^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*co
s(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
+ 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f
*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")

[Out] Exception raised: TypeError
```

3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=374

$$\frac{2a^2\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}+\frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11Ad(c-\right.}{3465d^2f\sqrt{a\,\mathrm{s}}}$$

```
[Out] (4*a^2*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*
c*d + 56*d^2))*Cos[e + f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (8*a*(
5*c - d)*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f) + (4*(c + d)*(11*A*(c - 17*d)*d -
3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*
f) + (2*a^2*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(
c + d*Sin[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(3*B*(
c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[
e + f*x])^4)/(11*d*f)
```

Rubi [A] time = 0.920242, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2976, 2981, 2770, 2761, 2751, 2646}

$$\frac{2a^2\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a\sin(e+fx)+a}}+\frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11Ad(c-\right.}{3465d^2f\sqrt{a\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (4*a^2*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*
c*d + 56*d^2))*Cos[e + f*x])/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (8*a*(
5*c - d)*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d*f) + (4*(c + d)*(11*A*(c - 17*d)*d -
3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*
f) + (2*a^2*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(
c + d*Sin[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(3*B*(
c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a +
a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[
```

```
e + f*x])^4)/(11*d*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+\ldots}{11df}$$

$$= \frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+f\ldots}{99d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+\ldots}{693d^2f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{4(c+d)\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\mathrm{co}\ldots}{1155f}$$

$$= \frac{8a(5c-d)(c+d)\left(11A(c-17d)d-3B\left(c^2-9cd+\ldots\right.\right.}{3465df}$$

$$= \frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17d)d-3\ldots\right.}{3465d^2f\sqrt{a+a\sin(e+\ldots}}$$

Mathematica [A] time = 4.59295, size = 390, normalized size = 1.04

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8\left(11Ad\left(189c^2+351cd+137d^2\right)+3B\left(1287c^2d+231c^3\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(92400
*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A*c*d^2 + 1
77474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 + 351*c*d +
137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*Cos[2*(e + f*
x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 18480*A*c^3*Sin[
e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f*x] + 100188*B
*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B*c*d^2*Sin[e +
f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x] - 5940*B*c^2*d*S
in[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*c*d^2*Sin[3*(e +
f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e + f*x)] + 315*B*d
^3*Sin[5*(e + f*x)]))/(27720*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 1.015, size = 312, normalized size = 0.8

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^2\left(-1+\sin\left(fx+e\right)\right)\left(315\,B\left(\cos\left(fx+e\right)\right)^4\sin\left(fx+e\right)d^3+\left(-1485\,Acd^2-935\,Ad^3-1485\,Bc^2d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^
3+(-1485*A*c*d^2-935*A*d^3-1485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)
^2*sin(f*x+e)+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3+6
633*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+(385*A*d^3+1155*B*c*d^2+735
*B*d^3)*cos(f*x+e)^4+(-2079*A*c^2*d-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*
B*c^2*d-5676*B*c*d^2-2478*B*d^3)*cos(f*x+e)^2+5775*A*c^3+14553*A*c^2*d+1415
7*A*c*d^2+4499*A*d^3+4851*B*c^3+14157*B*c^2*d+13497*B*c*d^2+4431*B*d^3)/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^3, x)
```

Fricas [A] time = 1.98949, size = 1667, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d^
3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 132
*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33*(9
*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*c^3
+ 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1239*B
)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a*c^2*d
 + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e)^2 + (
231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A + 409*B)*a
*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(f*x + e)^
5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A + 47*B)*a
*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12*B)*a*d^3)*
cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (187*A + 294*
B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c^2*d + 33*(24
*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 + (231*(5*A + 9*
B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c*d^2 + 17*(143*
A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*c
os(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.294 $\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$

Optimal. Leaf size=294

$$\frac{2a^2\left(15c^2+10cd+7d^2\right)\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\cos(e+fx)}{315d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+}{63d^2f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (2*a^2*(15*c^2 + 10*c*d + 7*d^2)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^
2))*Cos[e + f*x])/(315*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (4*a*(5*c - d)*(3*
A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(315*d*f) + (2*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(3*B*c - 9*A*d - 10*B*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d^2*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*
f)
```

Rubi [A] time = 0.711904, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2976, 2981, 2761, 2751, 2646}

$$\frac{2a^2\left(15c^2+10cd+7d^2\right)\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\cos(e+fx)}{315d^2f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+}{63d^2f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```


```
[Out] (2*a^2*(15*c^2 + 10*c*d + 7*d^2)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^
2))*Cos[e + f*x])/(315*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (4*a*(5*c - d)*(3*
A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(315*d*f) + (2*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(3*B*c - 9*A*d - 10*B*d)
*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d^2*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*
f)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = -\frac{2aB\cos(e + fx)\sqrt{a + a\sin(e + fx)}(c + d\sin(e +}{9df}$$

$$= \frac{2a^2(3Bc - 9Ad - 10Bd)\cos(e + fx)(c + d\sin(e + f}{63d^2f\sqrt{a + a\sin(e + fx)}}$$

$$= \frac{2\left(3A(c - 13d)d - B\left(c^2 - 7cd + 34d^2\right)\right)\cos(e + fx}{105f}$$

$$= \frac{4a(5c - d)\left(3A(c - 13d)d - B\left(c^2 - 7cd + 34d^2\right)\right)\mathrm{co}}{315df}$$

$$= \frac{2a^2\left(15c^2 + 10cd + 7d^2\right)\left(3A(c - 13d)d - B\left(c^2 - 7c\right.\right.}{315d^2f\sqrt{a + a\sin(e + fx)}}$$

Mathematica [A] time = 2.2475, size = 267, normalized size = 0.91

$$-\frac{a\sqrt{a(\sin(e + fx) + 1)}\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(-4\left(9Ad(14c + 13d) + B\left(63c^2 + 234cd + 137d^2\right)\right)\cos(2(e +\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4200*
A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B*d^2 - 4*
(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e + f*x)] + 3
5*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*Sin[e + f*x]
 + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d^2*Sin[e + f
*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 90*A*d^2*Sin[3
*(e + f*x)] - 170*B*d^2*Sin[3*(e + f*x)]))/(1260*f*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]))
```

Maple [A] time = 1.083, size = 207, normalized size = 0.7

$$\frac{\left(2 + 2\sin\left(fx + e\right)\right)a^2\left(-1 + \sin\left(fx + e\right)\right)\left(\left(-45\,Ad^2 - 90\,Bcd - 85\,Bd^2\right)\sin\left(fx + e\right)\left(\cos\left(fx + e\right)\right)^2 + \left(105\,Ac^2 + 378\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*((-45*A*d^2-90*B*c*d-85*B*d^2)*sin
(f*x+e)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2+402*B*c*d+221
*B*d^2)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^2+(-126*A*c*d-117*A*d^2-63*B*c^2-234
*B*c*d-172*B*d^2)*cos(f*x+e)^2+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+858*
B*c*d+409*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [A] time = 1.84799, size = 1094, normalized size = 3.72

$$-\frac{2\left(35\,Bad^{2}\cos\left(fx+e\right)^{5}-5\left(18\,Bacd+(9\,A+10\,B)ad^{2}\right)\cos\left(fx+e\right)^{4}+84\,(5\,A+3\,B)ac^{2}+24\,(21\,A+19\,B)acd+4\,(5\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/315*(35*B*a*d^2*cos(f*x + e)^5 - 5*(18*B*a*c*d + (9*A + 10*B)*a*d^2)*cos
(f*x + e)^4 + 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*
B)*a*d^2 - (63*B*a*c^2 + 18*(7*A + 13*B)*a*c*d + (117*A + 172*B)*a*d^2)*cos
(f*x + e)^3 + (21*(5*A + 6*B)*a*c^2 + 6*(42*A + 43*B)*a*c*d + (129*A + 134*
B)*a*d^2)*cos(f*x + e)^2 + (21*(25*A + 21*B)*a*c^2 + 6*(147*A + 143*B)*a*c*
```

```
d + (429*A + 409*B)*a*d^2)*cos(f*x + e) - (35*B*a*d^2*cos(f*x + e)^4 + 84*(
5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*B)*a*d^2 + 5*(18*B
*a*c*d + (9*A + 17*B)*a*d^2)*cos(f*x + e)^3 - 3*(21*B*a*c^2 + 6*(7*A + 8*B)
*a*c*d + (24*A + 29*B)*a*d^2)*cos(f*x + e)^2 - (21*(5*A + 9*B)*a*c^2 + 6*(6
3*A + 67*B)*a*c*d + (201*A + 221*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.295 $\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$

Optimal. Leaf size=165

$$-\frac{8a^2(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2a(35Ac+2$$

```
[Out] (-8*a^2*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*a*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(105*f) - (2*(7*B*c + 7*A*d - 2*B*d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(7*a*f)
```

Rubi [A] time = 0.315633, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3023, 2751, 2647, 2646}

$$-\frac{8a^2(35Ac+21Ad+21Bc+19Bd)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{2(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{35f}-\frac{2a(35Ac+2$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```


```
[Out] (-8*a^2*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*a*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(105*f) - (2*(7*B*c + 7*A*d - 2*B*d)*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(5/2))/(7*a*f)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^{3/2}\left(Ac+(Bc+Ad)\sin(e+fx\right.$$

$$= -\frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af} + \frac{2\int(a+a}{}$$

$$= -\frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx)}{35f}$$

$$= -\frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+}}{105f}$$

$$= -\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2}{}$$

Mathematica [A] time = 1.07593, size = 144, normalized size = 0.87

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((140Ac+252Ad+252Bc+253Bd)\sin(e+fx)-6(7Ad+7}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(700*A
*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2*(e + f*
x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*d*Sin[3*(
e + f*x)]))/(210*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.996, size = 150, normalized size = 0.9

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^2\left(-1+\sin\left(fx+e\right)\right)\left(15\,Bd\left(\sin\left(fx+e\right)\right)^3+21\,Ad\left(\sin\left(fx+e\right)\right)^2+21\,Bc\left(\sin\left(fx+e\right)\right)^2+39\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(15*B*d*sin(f*x+e)^3+21*A*d*sin(f*
x+e)^2+21*B*c*sin(f*x+e)^2+39*B*sin(f*x+e)^2*d+35*A*sin(f*x+e)*c+63*A*sin(f
*x+e)*d+63*B*sin(f*x+e)*c+52*B*sin(f*x+e)*d+175*A*c+126*A*d+126*B*c+104*B*d
)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c), x)
```

Fricas [A] time = 1.7824, size = 660, normalized size = 4.

$$2\left(15\,Bad\cos\left(fx+e\right)^{4}+3\,(7\,Bac+(7\,A+13\,B)ad)\cos\left(fx+e\right)^{3}-28\,(5\,A+3\,B)ac-4\,(21\,A+19\,B)ad-(7\,(5\,A+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x + e
)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42*A
 + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*d)*
cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A + 19
*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B)*a*c
 + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=101

$$-\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

```
[Out] (-8*a^2*(5*A + 3*B)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*A + 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*B*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rubi [A] time = 0.0868601, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2647, 2646}

$$-\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] (-8*a^2*(5*A + 3*B)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(5
*A + 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*B*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(3/2))/(5*f)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}+\frac{1}{5}(5A+3B)\int (a+a\sin(e+f$$

$$= -\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f}-\frac{2B\cos(e+fx)(a+a\sin}{5f}$$

$$= -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}}-\frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+f}}{15f}$$

Mathematica [A] time = 0.401855, size = 101, normalized size = 1.

$$-\frac{a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(2(5A+9B)\sin(e+fx)+50A-3B\cos(2(e+fx))+39B)}{15f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(50*A
+ 39*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 9*B)*Sin[e + f*x]))/(15*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 1.089, size = 77, normalized size = 0.8

$$\frac{(2+2\sin(fx+e))a^2(-1+\sin(fx+e))\left(\sin(fx+e)(5A+9B)-3B(\cos(fx+e))^2+25A+21B\right)}{15f\cos(fx+e)}\frac{1}{\sqrt{a+a\sin(fx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*a^2*(-1+sin(f*x+e))*(sin(f*x+e)*(5*A+9*B)-3*B*cos(f*x+e
)^2+25*A+21*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [A] time = 1.65723, size = 347, normalized size = 3.44

$$\frac{2\left(3\,Ba\cos\left(fx+e\right)^{3}-(5\,A+6\,B)a\cos\left(fx+e\right)^{2}-(25\,A+21\,B)a\cos\left(fx+e\right)-4\,(5\,A+3\,B)a-\left(3\,Ba\cos\left(fx+e\right)^{2}+\right.}{15\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/15*(3*B*a*cos(f*x + e)^3 - (5*A + 6*B)*a*cos(f*x + e)^2 - (25*A + 21*B)*a
*cos(f*x + e) - 4*(5*A + 3*B)*a - (3*B*a*cos(f*x + e)^2 + (5*A + 9*B)*a*cos
(f*x + e) - 4*(5*A + 3*B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(
f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=153

$$\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}} - \frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+}}{3df}$$

```
[Out] (-2*a^(3/2)*(c - d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*
c - 3*A*d - 4*B*d)*Cos[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*
B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f)
```

Rubi [A] time = 0.502825, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2976, 2981, 2773, 208}

$$\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{3d^2f\sqrt{a\sin(e+fx)+a}} - \frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}} - \frac{2aB\cos(e+fx)\sqrt{a\sin(e+}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (-2*a^(3/2)*(c - d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*
c - 3*A*d - 4*B*d)*Cos[e + f*x])/(3*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*
B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
```

```
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}+\frac{2\int \frac{\sqrt{a+a\sin(e+fx)}\left(\frac{1}{2}a(Bc+3Ad)-\frac{1}{2}a(3}{c+d\sin(e+f}}{3d}$$

$$=\frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+f}}{3df}$$

$$=\frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}}-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+f}}{3df}$$

$$=-\frac{2a^{3/2}(c-d)(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}f}+\frac{2a^2(3Bc-3Ad-}{3d^2f\sqrt{a+a}}$$

Mathematica [B] time = 3.4313, size = 356, normalized size = 2.33

$$(a(\sin(e+fx)+1))^{3/2}\left(6\sqrt{d}(2Ad-2Bc+3Bd)\sin\left(\frac{1}{2}(e+fx)\right)-6\sqrt{d}(2Ad-2Bc+3Bd)\cos\left(\frac{1}{2}(e+fx)\right)+\frac{3(c-d)(Bc-Ad)\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e +
 f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(B*c - A*d)*(e + f
*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d
]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/
Sqrt[c + d] + (3*(c - d)*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*
x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[c + d] + 6*Sqrt[d]*(-2*B*c + 2*A*d + 3*
B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*(e + f*x))/2]))/(6*d^(5/2)*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] time = 1.621, size = 291, normalized size = 1.9

$$\frac{2+2\sin\left(fx+e\right)}{3\,d^{2}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(3\,A\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}d}{\sqrt{a\left(c+d\right)d}}\right)a^{2}cd-3\,A\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\ldots\right.}}{\sqrt{a\left(c\ldots\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(3*A*arctanh((-a*(-1+sin(f*x+
e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d-3*A*arctanh((-a*(-1+sin(f*x+e)))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^2*d^2-3*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^2*c^2+3*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)
*d)^(1/2))*a^2*c*d+B*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*d-3*A*(-a
*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*d+3*B*(-a*(-1+sin(f*x+e)))^(1/2
)*(a*(c+d)*d)^(1/2)*a*c-6*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*
d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c), x)
```

Fricas [B] time = 9.73292, size = 2086, normalized size = 13.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*
a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqr
t(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a
*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)
*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
- 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(
f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*a*d*cos(f*x + e)^2 - 3*
B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d
*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1/3*(3*(B*a*c^2
 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*cos(f*x +
e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt(-a/(c*d + d^2))
*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*
```

```
d + d^2))/(a*cos(f*x + e))) + 2*(B*a*d*cos(f*x + e)^2 - 3*B*a*c + (3*A + 4*
B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d*cos(f*x + e) + 3
*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*co
s(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=191

$$-\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a(Bc-Ad)\,c}{df(c}$$

```
[Out] -((a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqr
t[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d
)^(3/2)*f)) - (a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[
a + a*Sin[e + f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.552218, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2975, 2981, 2773, 208}

$$-\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{5/2}f(c+d)^{3/2}}-\frac{a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{d^2f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a(Bc-Ad)\,c}{df(c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -((a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqr
t[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(5/2)*(c + d
)^(3/2)*f)) - (a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/(d^2*(c + d)*f*Sqrt[
a + a*Sin[e + f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
```

```
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\frac{1}{2}a(Bc-3A\right.}{c-}}{}$$

$$= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{d(c+d)f(c+d\sin(e+f}$$

$$= -\frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\,\mathrm{si}}}{d(c+d)f(c+d\sin(e+f}$$

$$= -\frac{a^{3/2}\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f}$$

Mathematica [A] time = 4.88958, size = 381, normalized size = 1.99

$$(a(\sin(e+fx)+1))^{3/2}\left(\frac{(Ad(c+3d)+B(-3c^2-3cd+2d^2))\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)\right)-2\log\left(\sec^2\left(\frac{1}{4}(e+fx\right.\right.\right.}{(c+d)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((-(A*d*(c +
 3*d)) + B*(3*c^2 + 3*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] -
Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(3/2) + ((A*d*(c + 3*d) +
 B*(-3*c^2 - 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(
c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] +
Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 8*B*Sqrt[d]*Sin[(e + f*x)/2] - (4*Sq
rt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c +
d)*(c + d*Sin[e + f*x]))))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^3)
```

Maple [B] time = 1.839, size = 592, normalized size = 3.1

$$\frac{a\left(1+\sin\left(fx+e\right)\right)}{(c+d)\,d^2\left(c+d\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-\sin\left(fx+e\right)d\left(A\mathrm{Artanh}\left(d\sqrt{a-a\sin\left(fx+e\right)}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] a*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-sin(f*x+e)*d*(A*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e))
^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)
^(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d
^2+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-
a*sin(f*x+e))^(1/2)*d)-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d
```

```
^2+3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^3+3*B*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-2*B*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2+A*(a-a*sin(f*x+e))^(1/2)*(
a*(c+d)*d)^(1/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d^2-3*B*(a-
a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c^2-B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)
*d)^(1/2)*c*d)/d^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [B] time = 11.553, size = 3245, normalized size = 16.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^
3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a
*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c
^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
```

```
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2 - (A + B)*a*c
*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 -
(A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2
*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x
+ e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d
^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c^3 - (A - 6*B)*a*c^2*d
- (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2
- (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A
 + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*
a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B
)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f
*x + e))) - 2*(3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B
*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e)
 - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos
(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x
 + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f -
((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{a^2\left(Ad(c-5d)+B\left(3c^2+5cd-4d^2\right)\right)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a^{3/2}\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}$$

```
[Out] -(a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*ArcTanh[(Sqrt[a]*Sqrt
[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(5/2)*(c +
d)^(5/2)*f) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c
 + d)*f*(c + d*Sin[e + f*x])^2) + (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d -
4*d^2))*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Si
n[e + f*x]))
```

Rubi [A] time = 0.60951, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2975, 2980, 2773, 208}

$$\frac{a^2\left(Ad(c-5d)+B\left(3c^2+5cd-4d^2\right)\right)\cos(e+fx)}{4d^2f(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a^{3/2}\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4d^{5/2}f(c+d)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```


```
[Out] -(a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*ArcTanh[(Sqrt[a]*Sqrt
[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*d^(5/2)*(c +
d)^(5/2)*f) + (a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c
 + d)*f*(c + d*Sin[e + f*x])^2) + (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d -
4*d^2))*Cos[e + f*x])/(4*d^2*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Si
n[e + f*x]))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{\sqrt{a+a\sin(e+fx)}\left(-\frac{1}{2}a(Bc-5Ad\cdots\right.}{(c+d\cdots}}{2\cdots}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2\left(A(c-5d)d+B\left(3c^2+\cdots\right.\right.}{4d^2(c+d)^2f\sqrt{a+a\sin(e\cdots}}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2\left(A(c-5d)d+B\left(3c^2+\cdots\right.\right.}{4d^2(c+d)^2f\sqrt{a+a\sin(e\cdots}}$$

$$= -\frac{a^{3/2}\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \cdots$$

Mathematica [A] time = 5.1612, size = 416, normalized size = 1.88

$$\frac{(a(\sin(e+fx)+1))^{3/2}\left(-\frac{4\sqrt{d}\left(Ad(c+7d)+B\left(-5c^2-7cd+4d^2\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(c+d)^2(c+d\sin(e+fx))} + \frac{\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}\cdots\right.\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d
 + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)
/2]))]))/(c + d)^(5/2)) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(e +
 f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[
d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5
/2) - (8*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]*(A*d*(c + 7*d) + B*(-5*c
^2 - 7*c*d + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c +
 d*Sin[e + f*x]))))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [B] time = 2.233, size = 895, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)+arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*
cos(f*x+e)^2+A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+7*A*(a-a*sin(
f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c
*d+a*d^2)^(1/2))*a^2*c^3*d-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a^2*c^2*d^2-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2
))*a^2*c*d^3-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*
d^4-5*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-7*B*(a-a*sin(f*x+e))
^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+4*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*d^3-3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4-9*B*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-15*B*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-9*B*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-12*B*arctanh((a-a*sin(
f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+A*(a-a*sin(f*x+e))^(1/2)*(a*(c
+d)*d)^(1/2)*a*c^2*d-8*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-9
*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3+3*B*(a-a*sin(f*x+e))^(1/2
)*(a*(c+d)*d)^(1/2)*a*c^3+12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c
^2*d+5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-4*B*(a-a*sin(f*x+
e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3)*(-a*(-1+sin(f*x+e)))^(1/2)*(1+sin(f*x+e)
)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/cos(f*x+e)/(a+a*sin(f*x+
e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 19.132, size = 5040, normalized size = 22.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A
 + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3
+ (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2*d^
2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*a*c^
4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A +
 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*
B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2
 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*a*c^3*d
+ (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^3 + (A + 2*B)*a
*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*B*a*c^2*d - (A - 7*
B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 + (A + 7*B)*a*c
^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x + e) - (3*B*a*c^3 + (A + 2*
B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 - (5*B*a*c^2*d - (A
- 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5
*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d
^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c
*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 +
2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*
d^5 + d^6)*f)*sin(f*x + e)), 1/8*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
+ 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^
2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3
*d + (2*A + 21*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*
cos(f*x + e)^2 + (3*B*a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 +
(A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15
*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12
*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*
x + e)^2 + 2*(3*B*a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos
(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e
) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -
```

```
2*(3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^
3 + (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 + (A + 7*B)*a*c^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x +
 e) - (3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*
a*d^3 - (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f
*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4
*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4
*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(
f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*
c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=534

$$-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(13Ad\left(3c^2-38cd+355d^2\right)\right.}{}$$

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(13*A*d*(3*c^2 - 38*c*d + 355*d^2
) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x])/(45045*d^3
*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(13*A*d*(3*c^2 - 38
*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(45045*d^2*f) - (4*a*(c + d)*(13*A*d*(3*c^2 -
38*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^(3/2))/(15015*d*f) - (2*a^3*(13*A*d*(3*c^2 - 38*c
*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(9009*d^3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*(15
*B*c^2 - 39*A*c*d - 75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(1287*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 13*
A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)
/(143*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e
+ f*x])^4)/(13*d*f)
```

Rubi [A] time = 1.20306, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2976, 2981, 2770, 2761, 2751, 2646}

$$-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(13Ad\left(3c^2-38cd+355d^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (-4*a^3*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(13*A*d*(3*c^2 - 38*c*d + 355*d^2
) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x])/(45045*d^3
*f*Sqrt[a + a*Sin[e + f*x]]) - (8*a^2*(5*c - d)*(c + d)*(13*A*d*(3*c^2 - 38
*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(45045*d^2*f) - (4*a*(c + d)*(13*A*d*(3*c^2 -
```

```
38*c*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e +
f*x]*(a + a*Sin[e + f*x])^(3/2))/(15015*d*f) - (2*a^3*(13*A*d*(3*c^2 - 38*c
*d + 355*d^2) - B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(9009*d^3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*(15
*B*c^2 - 39*A*c*d - 75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*S
in[e + f*x])^4)/(1287*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 13*
A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)
/(143*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e
+ f*x])^4)/(13*d*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
```

```
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+}{13df}$$

$$= \frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+}}{143d^2f}$$

$$= -\frac{2a^3\left(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2\right)}{1287d^3f\sqrt{a+a\sin(e+}}$$

$$= -\frac{2a^3\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2\right.\right.}{9009d^3f\sqrt{a}}$$

$$= -\frac{4a(c+d)\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-\right.\right.}{}$$

$$= -\frac{8a^2(5c-d)(c+d)\left(13Ad\left(3c^2-38cd+355d^2\right)-B\right.}{}$$

$$= -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad\left(3c^2-38cd\right.\right.}{45045d^3}$$

Mathematica [C] time = 6.89669, size = 1565, normalized size = 2.93

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^3,x]
```

```
[Out] (B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78*B
*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*Cos
[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2
))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90
*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1
/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[
e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3
+ 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d
^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos[(3*(e + f
*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240
*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[(3*(e + f*x))/
2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 +
120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3
)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e + f*x))/2] - (1/3
20 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150
*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 + I/32
0)*Cos[(5*(e + f*x))/2] - (1/320 - I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d + 30*B*c^2*d + 3
0*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*
((1/224 + I/224)*Cos[(7*(e + f*x))/2] + (1/224 - I/224)*Sin[(7*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d +
30*B*c^2*d + 30*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e +
 f*x]))^(5/2)*((1/224 - I/224)*Cos[(7*(e + f*x))/2] + (1/224 + I/224)*Sin[(
7*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 +
 6*A*c*d + 15*B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/
288 - I/288)*d*Cos[(9*(e + f*x))/2] + (1/288 - I/288)*d*Sin[(9*(e + f*x))/2
]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c^2 + 6*A*c*d + 15*
B*c*d + 5*A*d^2 + 7*B*d^2)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/288 + I/288)*d
*Cos[(9*(e + f*x))/2] + (1/288 + I/288)*d*Sin[(9*(e + f*x))/2]))/(f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5) + ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e
+ f*x]))^(5/2)*((-1/704 + I/704)*d^2*Cos[(11*(e + f*x))/2] - (1/704 + I/704
```

```
)*d^2*Sin[(11*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) +
 ((6*B*c + 2*A*d + 5*B*d)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/704 - I/704)*d^
2*Cos[(11*(e + f*x))/2] - (1/704 - I/704)*d^2*Sin[(11*(e + f*x))/2]))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^5) - (B*d^3*(a*(1 + Sin[e + f*x]))^(5/2
)*Sin[(13*(e + f*x))/2])/(416*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 1.157, size = 374, normalized size = 0.7

$$\frac{\left(2+2\,\sin\left(fx+e\right)\right)a^{3}\left(-1+\sin\left(fx+e\right)\right)\left(\left(4095\,Ad^{3}+12285\,Bcd^{2}+11970\,Bd^{3}\right)\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+\left(-1930\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

```
[Out] 2/45045*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*((4095*A*d^3+12285*B*c*d^2+11970
*B*d^3)*sin(f*x+e)*cos(f*x+e)^4+(-19305*A*c^2*d-55770*A*c*d^2-31265*A*d^3-6
435*B*c^3-55770*B*c^2*d-93795*B*c*d^2-44860*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+
(42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3+181038*B
*c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)-3465*B*d^3*cos(f*x+e)^6+(1501
5*A*c*d^2+14560*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4
+(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^2-56810*A*d^3-25740*B*c^3-123981*B
*c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f*x+e)^2+138138*A*c^3+373230*A*c^2*d
+359502*A*c*d^2+116090*A*d^3+124410*B*c^3+359502*B*c^2*d+348270*B*c*d^2+113
818*B*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^3, x)
```

Fricas [A] time = 2.22997, size = 2237, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] 2/45045*(3465*B*a^2*d^3*cos(f*x + e)^7 - 315*(39*B*a^2*c*d^2 + (13*A + 27*B
)*a^2*d^3)*cos(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)
*a^2*c^2*d - 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3
- 35*(429*B*a^2*c^2*d + 39*(11*A + 32*B)*a^2*c*d^2 + 4*(104*A + 205*B)*a^2*
d^3)*cos(f*x + e)^5 + 5*(1287*B*a^2*c^3 + 429*(9*A + 19*B)*a^2*c^2*d + 39*(
209*A + 320*B)*a^2*c*d^2 + 2*(2080*A + 2813*B)*a^2*d^3)*cos(f*x + e)^4 + (1
287*(7*A + 20*B)*a^2*c^3 + 429*(180*A + 289*B)*a^2*c^2*d + 39*(3179*A + 437
0*B)*a^2*c*d^2 + (56810*A + 72109*B)*a^2*d^3)*cos(f*x + e)^3 - (429*(77*A +
 85*B)*a^2*c^3 + 429*(255*A + 263*B)*a^2*c^2*d + 39*(2893*A + 2965*B)*a^2*c
*d^2 + (38545*A + 39113*B)*a^2*d^3)*cos(f*x + e)^2 - 2*(429*(161*A + 145*B)
*a^2*c^3 + 429*(435*A + 419*B)*a^2*c^2*d + 39*(4609*A + 4465*B)*a^2*c*d^2 +
 (58045*A + 56909*B)*a^2*d^3)*cos(f*x + e) - (3465*B*a^2*d^3*cos(f*x + e)^6
 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)*a^2*c^2*d - 1248*(143*A
+ 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3 + 315*(39*B*a^2*c*d^2 + (
13*A + 38*B)*a^2*d^3)*cos(f*x + e)^5 - 35*(429*B*a^2*c^2*d + 39*(11*A + 23*
B)*a^2*c*d^2 + (299*A + 478*B)*a^2*d^3)*cos(f*x + e)^4 - 5*(1287*B*a^2*c^3
+ 429*(9*A + 26*B)*a^2*c^2*d + 507*(22*A + 37*B)*a^2*c*d^2 + (6253*A + 8972
*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(429*(7*A + 15*B)*a^2*c^3 + 429*(45*A + 53*
B)*a^2*c^2*d + 39*(583*A + 655*B)*a^2*c*d^2 + (8515*A + 9083*B)*a^2*d^3)*co
s(f*x + e)^2 + 2*(429*(49*A + 65*B)*a^2*c^3 + 429*(195*A + 211*B)*a^2*c^2*d
 + 39*(2321*A + 2465*B)*a^2*c*d^2 + (32045*A + 33181*B)*a^2*d^3)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e)
 + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.301 $\int (a + a \sin(e + fx))^{5/2}(A + B \sin(e + fx))(c + d \sin(e + fx))^2 \, dx$

Optimal. Leaf size=429

$$\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a\sin(e+fx)+a}} - \frac{2a^3\left(15c^2+10cd+7d^2\right)\left(11Ad\left(c^2-\right.\right.}{}$$

```
[Out] (-2*a^3*(15*c^2 + 10*c*d + 7*d^2)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^
3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x])/(3465*d^3*f*Sqrt[a + a*S
in[e + f*x]]) - (4*a^2*(5*c - d)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3
 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(3465*d^2*f) - (2*a*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d +
 169*c*d^2 - 710*d^3))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*d*f)
+ (2*a^3*(11*A*(3*c - 19*d)*d - B*(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(693*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B
*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^3)/(99*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])^3)/(11*d*f)
```

Rubi [A] time = 1.06576, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2976, 2981, 2761, 2751, 2646}

$$\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a\sin(e+fx)+a}} - \frac{2a^3\left(15c^2+10cd+7d^2\right)\left(11Ad\left(c^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,
x]
```

```
[Out] (-2*a^3*(15*c^2 + 10*c*d + 7*d^2)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^
3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x])/(3465*d^3*f*Sqrt[a + a*S
in[e + f*x]]) - (4*a^2*(5*c - d)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3
 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/
(3465*d^2*f) - (2*a*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d +
 169*c*d^2 - 710*d^3))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*d*f)
+ (2*a^3*(11*A*(3*c - 19*d)*d - B*(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]
*(c + d*Sin[e + f*x])^3)/(693*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B
```

```
*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x])^3)/(99*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])^3)/(11*d*f)
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2761

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m
 + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e\cdots}{11df}$$

$$= \frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e\cdots}}{99d^2f}$$

$$= \frac{2a^3\left(11A(3c-19d)d-B\left(15c^2-65cd+194d^2\right)\right)\mathrm{co}\cdots}{693d^3f\sqrt{a+a\sin(e+f\cdots}}$$

$$= -\frac{2a\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+1\cdots\right.\right.}{115\cdots}$$

$$= -\frac{4a^2(5c-d)\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-\cdots\right.\right.}{3\cdots}$$

$$= -\frac{2a^3\left(15c^2+10cd+7d^2\right)\left(11Ad\left(c^2-10cd+73d^2\right)\cdots\right.}{3465d^3f\sqrt{a+\cdots}}$$

Mathematica [B] time = 6.61678, size = 891, normalized size = 2.08

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(-277200A\cos\left(\frac{1}{2}(e+fx)\right)c^2-207900B\cos\left(\frac{1}{2}(e+fx)\right)c^2-46200A\cos\left(\frac{3}{2}(e+fx)\right)c^2-5082\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-277200*A*c^2*Cos[(e + f*x)/2] - 207900*B*c^
2*Cos[(e + f*x)/2] - 415800*A*c*d*Cos[(e + f*x)/2] - 360360*B*c*d*Cos[(e +
f*x)/2] - 180180*A*d^2*Cos[(e + f*x)/2] - 159390*B*d^2*Cos[(e + f*x)/2] - 4
6200*A*c^2*Cos[(3*(e + f*x))/2] - 50820*B*c^2*Cos[(3*(e + f*x))/2] - 101640
*A*c*d*Cos[(3*(e + f*x))/2] - 92400*B*c*d*Cos[(3*(e + f*x))/2] - 46200*A*d^
2*Cos[(3*(e + f*x))/2] - 43890*B*d^2*Cos[(3*(e + f*x))/2] + 5544*A*c^2*Cos[
```

```
(5*(e + f*x))/2] + 13860*B*c^2*Cos[(5*(e + f*x))/2] + 27720*A*c*d*Cos[(5*(e
 + f*x))/2] + 33264*B*c*d*Cos[(5*(e + f*x))/2] + 16632*A*d^2*Cos[(5*(e + f*
x))/2] + 17325*B*d^2*Cos[(5*(e + f*x))/2] + 1980*B*c^2*Cos[(7*(e + f*x))/2]
 + 3960*A*c*d*Cos[(7*(e + f*x))/2] + 9900*B*c*d*Cos[(7*(e + f*x))/2] + 4950
*A*d^2*Cos[(7*(e + f*x))/2] + 6435*B*d^2*Cos[(7*(e + f*x))/2] - 1540*B*c*d*
Cos[(9*(e + f*x))/2] - 770*A*d^2*Cos[(9*(e + f*x))/2] - 1925*B*d^2*Cos[(9*(
e + f*x))/2] - 315*B*d^2*Cos[(11*(e + f*x))/2] + 277200*A*c^2*Sin[(e + f*x)
/2] + 207900*B*c^2*Sin[(e + f*x)/2] + 415800*A*c*d*Sin[(e + f*x)/2] + 36036
0*B*c*d*Sin[(e + f*x)/2] + 180180*A*d^2*Sin[(e + f*x)/2] + 159390*B*d^2*Sin
[(e + f*x)/2] - 46200*A*c^2*Sin[(3*(e + f*x))/2] - 50820*B*c^2*Sin[(3*(e +
f*x))/2] - 101640*A*c*d*Sin[(3*(e + f*x))/2] - 92400*B*c*d*Sin[(3*(e + f*x)
)/2] - 46200*A*d^2*Sin[(3*(e + f*x))/2] - 43890*B*d^2*Sin[(3*(e + f*x))/2]
- 5544*A*c^2*Sin[(5*(e + f*x))/2] - 13860*B*c^2*Sin[(5*(e + f*x))/2] - 2772
0*A*c*d*Sin[(5*(e + f*x))/2] - 33264*B*c*d*Sin[(5*(e + f*x))/2] - 16632*A*d
^2*Sin[(5*(e + f*x))/2] - 17325*B*d^2*Sin[(5*(e + f*x))/2] + 1980*B*c^2*Sin
[(7*(e + f*x))/2] + 3960*A*c*d*Sin[(7*(e + f*x))/2] + 9900*B*c*d*Sin[(7*(e
+ f*x))/2] + 4950*A*d^2*Sin[(7*(e + f*x))/2] + 6435*B*d^2*Sin[(7*(e + f*x))
/2] + 1540*B*c*d*Sin[(9*(e + f*x))/2] + 770*A*d^2*Sin[(9*(e + f*x))/2] + 19
25*B*d^2*Sin[(9*(e + f*x))/2] - 315*B*d^2*Sin[(11*(e + f*x))/2]))/(55440*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A] time = 0.928, size = 257, normalized size = 0.6

$$\frac{\left(2+2\,\sin\left(fx+e\right)\right)a^{3}\left(-1+\sin\left(fx+e\right)\right)\left(315\,Bd^{2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+\left(-990\,Acd-1430\,Ad^{2}-495\,Bc^{2}-28\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*(315*B*d^2*sin(f*x+e)*cos(f*x+e)^
4+(-990*A*c*d-1430*A*d^2-495*B*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(
f*x+e)+(3234*A*c^2+8580*A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*
sin(f*x+e)+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-693*A*c^2-3960*A
*c*d-3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+10626*A*c^2+
19140*A*c*d+9218*A*d^2+9570*B*c^2+18436*B*c*d+8930*B*d^2)/cos(f*x+e)/(a+a*s
in(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c)^2, x)
```

Fricas [A] time = 1.96476, size = 1500, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a^
2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*c*
d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c*d
+ (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 + 22*(
180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (33*(7
7*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*a^2*d^
2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*B)*a^2*
c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(f*x + e)
^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(143*A + 125
*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x + e)^4 - 5*
(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d^2)*cos(f*x
 + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d + (583*A +
655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 + 22*(195*A +
211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=212

$$-\frac{16a^2(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{64a^3(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-64*a^3*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x])/(315*f*Sqrt[a +
a*Sin[e + f*x]]) - (16*a^2*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(315*f) - (2*a*(21*A*c + 15*B*c + 15*A*d + 13*B*
d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) - (2*(9*B*c + 9*A*d - 2
*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(63*f) - (2*B*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rubi [A] time = 0.367651, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3023, 2751, 2647, 2646}

$$-\frac{16a^2(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{315f}-\frac{64a^3(21Ac+15Ad+15Bc+13Bd)\cos(e+fx)}{315f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (-64*a^3*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x])/(315*f*Sqrt[a +
a*Sin[e + f*x]]) - (16*a^2*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(315*f) - (2*a*(21*A*c + 15*B*c + 15*A*d + 13*B*
d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) - (2*(9*B*c + 9*A*d - 2
*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(63*f) - (2*B*d*Cos[e + f*x]
*(a + a*Sin[e + f*x])^(7/2))/(9*a*f)
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx = \int (a + a\sin(e + fx))^{5/2}\left(Ac + (Bc + Ad)\sin(e + fx\right.$$

$$= -\frac{2Bd\cos(e + fx)(a + a\sin(e + fx))^{7/2}}{9af} + \frac{2\int (a + a}{}$$

$$= -\frac{2(9Bc + 9Ad - 2Bd)\cos(e + fx)(a + a\sin(e + fx)}{63f}$$

$$= -\frac{2a(21Ac + 15Bc + 15Ad + 13Bd)\cos(e + fx)(a +}{105f}$$

$$= -\frac{16a^2(21Ac + 15Bc + 15Ad + 13Bd)\cos(e + fx)\sqrt{a}}{315f}$$

$$= -\frac{64a^3(21Ac + 15Bc + 15Ad + 13Bd)\cos(e + fx)}{315f\sqrt{a + a\sin(e + fx)}} -$$

Mathematica [A] time = 4.20475, size = 202, normalized size = 0.95

$$-\frac{a^2\sqrt{a(\sin(e + fx) + 1)}\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)(-4(63Ac + 180Ad + 180Bc + 254Bd)\cos(2(e + fx)) + 235}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x]),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(747
6*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180*A*d + 25
4*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[e + f*x] +
 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e + f*x] - 90
*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3*(e + f*x)])
)/(1260*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 1.167, size = 152, normalized size = 0.7

$$\frac{\left(2 + 2\sin\left(fx + e\right)\right)a^3\left(-1 + \sin\left(fx + e\right)\right)\left((-45\,Ad - 45\,Bc - 130\,Bd)\sin\left(fx + e\right)\left(\cos\left(fx + e\right)\right)^2 + (294\,Ac + 390\,Ad\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] 2/315*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*((-45*A*d-45*B*c-130*B*d)*sin(f*x+
e)*cos(f*x+e)^2+(294*A*c+390*A*d+390*B*c+422*B*d)*sin(f*x+e)+35*B*d*cos(f*x
+e)^4+(-63*A*c-180*A*d-180*B*c-289*B*d)*cos(f*x+e)^2+966*A*c+870*A*d+870*B*
c+838*B*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(d\sin\left(fx+e\right)+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
 c), x)
```

Fricas [A] time = 1.76944, size = 900, normalized size = 4.25

$$-\frac{2\left(35\,Ba^2d\cos\left(fx+e\right)^5-5\left(9\,Ba^2c+(9\,A+19\,B)a^2d\right)\cos\left(fx+e\right)^4+96\,(7\,A+5\,B)a^2c+32\,(15\,A+13\,B)a^2d-\left(9\,(7\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] -2/315*(35*B*a^2*d*cos(f*x + e)^5 - 5*(9*B*a^2*c + (9*A + 19*B)*a^2*d)*cos(
f*x + e)^4 + 96*(7*A + 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d - (9*(7*A + 20*B
)*a^2*c + (180*A + 289*B)*a^2*d)*cos(f*x + e)^3 + (3*(77*A + 85*B)*a^2*c +
(255*A + 263*B)*a^2*d)*cos(f*x + e)^2 + 2*(3*(161*A + 145*B)*a^2*c + (435*A
 + 419*B)*a^2*d)*cos(f*x + e) - (35*B*a^2*d*cos(f*x + e)^4 + 96*(7*A + 5*B)
*a^2*c + 32*(15*A + 13*B)*a^2*d + 5*(9*B*a^2*c + (9*A + 26*B)*a^2*d)*cos(f*
```

```
x + e)^3 - 3*(3*(7*A + 15*B)*a^2*c + (45*A + 53*B)*a^2*d)*cos(f*x + e)^2 -
2*(3*(49*A + 65*B)*a^2*c + (195*A + 211*B)*a^2*d)*cos(f*x + e))*sin(f*x + e
))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=138

$$-\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7A+5B)\cos(e+fx)(a\sin(e+fx)+}{35f}$$

```
[Out] (-64*a^3*(7*A + 5*B)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```

Rubi [A] time = 0.112403, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2647, 2646}

$$-\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105f}-\frac{2a(7A+5B)\cos(e+fx)(a\sin(e+fx)+}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```


```
[Out] (-64*a^3*(7*A + 5*B)*Cos[e + f*x])/(105*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a
^2*(7*A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*f) - (2*a*(7*A +
 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(35*f) - (2*B*Cos[e + f*x]*(
a + a*Sin[e + f*x])^(5/2))/(7*f)
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
```

```
- b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}+\frac{1}{7}(7A+5B)\int (a+a\sin(e+$$

$$= -\frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f}-\frac{2B\cos(e+fx)(a+}{7f}$$

$$= -\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f}-\frac{2a(7A+5B)\cos(e}{}$$

$$= -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(}{105f}$$

Mathematica [A] time = 1.5225, size = 119, normalized size = 0.86

$$-\frac{a^2\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)((392A+505B)\sin(e+fx)-6(7A+20B)\cos(2(e+fx))}{210f\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(124
6*A + 1040*B - 6*(7*A + 20*B)*Cos[2*(e + f*x)] + (392*A + 505*B)*Sin[e + f*
x] - 15*B*Sin[3*(e + f*x)]))/(210*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.897, size = 99, normalized size = 0.7

$$\frac{\left(2+2\sin\left(fx+e\right)\right)a^3\left(-1+\sin\left(fx+e\right)\right)\left(-15B\left(\cos\left(fx+e\right)\right)^2\sin\left(fx+e\right)+(98A+130B)\sin\left(fx+e\right)+(-21A-}{105f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] 2/105*(1+sin(f*x+e))*a^3*(-1+sin(f*x+e))*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(98
*A+130*B)*sin(f*x+e)+(-21*A-60*B)*cos(f*x+e)^2+322*A+290*B)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [A] time = 1.61816, size = 483, normalized size = 3.5

$$\frac{2\left(15\,Ba^2\cos\left(fx+e\right)^4+3\,(7\,A+20\,B)a^2\cos\left(fx+e\right)^3-(77\,A+85\,B)a^2\cos\left(fx+e\right)^2-2\,(161\,A+145\,B)a^2\cos\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a^2*cos(f*x + e)^4 + 3*(7*A + 20*B)*a^2*cos(f*x + e)^3 - (77*A
+ 85*B)*a^2*cos(f*x + e)^2 - 2*(161*A + 145*B)*a^2*cos(f*x + e) - 32*(7*A +
 5*B)*a^2 + (15*B*a^2*cos(f*x + e)^3 - 3*(7*A + 15*B)*a^2*cos(f*x + e)^2 -
2*(49*A + 65*B)*a^2*cos(f*x + e) + 32*(7*A + 5*B)*a^2)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=218

$$\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{15d^3f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15d^2f}+\frac{2a}{}$$

```
[Out] (2*a^(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2)*Sqrt[c + d]*f) + (2*a^3*(5*A
*(3*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(15*d^3*f*Sqrt
[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(15*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*d*f)
```

Rubi [A] time = 0.884646, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2976, 2981, 2773, 208}

$$\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx)}{15d^3f\sqrt{a\sin(e+fx)+a}}+\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15d^2f}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```


```
[Out] (2*a^(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2)*Sqrt[c + d]*f) + (2*a^3*(5*A
*(3*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(15*d^3*f*Sqrt
[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(15*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*d*f)
```


Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
```

```
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = -\frac{2aB\cos(e + fx)(a + a\sin(e + fx))^{3/2}}{5df} + \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}\left(\frac{1}{2}a(3Bc+5Ad\right.}{c+d\sin}}{5a}$$

$$= \frac{2a^2(5Bc - 5Ad - 8Bd)\cos(e + fx)\sqrt{a + a\sin(e + fx)}}{15d^2f} - \frac{2aB\cos(e + f}{}$$

$$= \frac{2a^3\left(5A(3c - 7d)d - B\left(15c^2 - 35cd + 32d^2\right)\right)\cos(e + fx)}{15d^3f\sqrt{a + a\sin(e + fx)}} + \frac{2a^2(5Bc -}{}$$

$$= \frac{2a^3\left(5A(3c - 7d)d - B\left(15c^2 - 35cd + 32d^2\right)\right)\cos(e + fx)}{15d^3f\sqrt{a + a\sin(e + fx)}} + \frac{2a^2(5Bc -}{}$$

$$= \frac{2a^{5/2}(c - d)^2(Bc - Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c + d}f} + \frac{2a^3\left(5A(3c - 7d)\right.}{15}$$

<u>Mathematica</u> [B] time = 5.86886, size = 450, normalized size = 2.06

$$\frac{(a(\sin(e + fx) + 1))^{5/2}\left(30\sqrt{d}\left(Ad(5d - 2c) + B\left(2c^2 - 5cd + 5d^2\right)\right)\sin\left(\frac{1}{2}(e + fx)\right) - 30\sqrt{d}\left(Ad(5d - 2c) + B\left(2c^2 - 5cd +\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 - 5
*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[(3
*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] + (15*(c - d)^2*(B*c - A*
d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d
 + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)
/2]))]))/Sqrt[c + d] - (15*(c - d)^2*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e +
f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2
*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/Sqrt[c + d] + 30*Sqrt[d]*(A*d*(-
2*c + 5*d) + B*(2*c^2 - 5*c*d + 5*d^2))*Sin[(e + f*x)/2] - 5*d^(3/2)*(-2*B*
c + 2*A*d + 5*B*d)*Sin[(3*(e + f*x))/2] - 3*B*d^(5/2)*Sin[(5*(e + f*x))/2])
)/(30*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] time = 1.764, size = 543, normalized size = 2.5

$$\frac{2+2\sin\left(fx+e\right)}{15\,d^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-3\,B\left(a-a\sin\left(fx+e\right)\right)^{5/2}\sqrt{a\left(c+d\right)d}\,d^{2}+5\,A\left(a-a\sin\left(fx+e\right)\right)^{3/2}\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/15*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)
*(a*(c+d)*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2-1
5*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*ar
ctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arctanh(
(a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+e))^
(3/2)*(a*(c+d)*d)^(1/2)*a*c*d+20*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)
*a*d^2+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^3-3
0*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+15*B*ar
ctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2+15*A*(a-a*sin
(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-45*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+
d)*d)^(1/2)*a^2*d^2-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c^2+4
5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^2*c*d-60*B*(a-a*sin(f*x+e))^
(1/2)*(a*(c+d)*d)^(1/2)*a^2*d^2)/d^3/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(
f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) +
 c), x)
```

Fricas [B] time = 17.6338, size = 2961, normalized size = 13.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^
3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*cos
(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*
d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 -
2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 +
 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x +
 e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*
cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d
 + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)
*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f
*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B
*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32
*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 - (15*B*a
^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x + e) - (3
*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A +
32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x + e) +
d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A
*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d
^3)*cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 -
 A*a^2*d^3)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) +
 2*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35
*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 -
(15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x +
e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (
35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x
+ e) + d^3*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=265

$$-\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\ldots}{\sqrt{c+d}\sqrt{\ldots}}\right.}{d^{7/2}f(c+d)^{3/2}}$$

```
[Out] (a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2 + 5*c*d - 2*d^2))*ArcTanh[(Sqr
t[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2
)*(c + d)^(3/2)*f) - (a^3*(3*A*d*(3*c + d) - B*(15*c^2 - 5*c*d - 14*d^2))*C
os[e + f*x])/(3*d^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*(5*B*c - 3*A
*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d^2*(c + d)*f) + (a*(
B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c + d*Sin
[e + f*x]))
```

Rubi [A] time = 0.937835, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2975, 2976, 2981, 2773, 208}

$$-\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3f(c+d)\sqrt{a\sin(e+fx)+a}}+\frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\ldots}{\sqrt{c+d}\sqrt{\ldots}}\right.}{d^{7/2}f(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
2,x]
```



```
[Out] (a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2 + 5*c*d - 2*d^2))*ArcTanh[(Sqr
t[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(7/2
)*(c + d)^(3/2)*f) - (a^3*(3*A*d*(3*c + d) - B*(15*c^2 - 5*c*d - 14*d^2))*C
os[e + f*x])/(3*d^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]) - (a^2*(5*B*c - 3*A
*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d^2*(c + d)*f) + (a*(
B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c + d*Sin
[e + f*x]))
```


Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))} + \frac{\int \frac{(a+a\sin(e+fx))^{3/2}\left(-\frac{1}{2}a(3Bc\right.}{}}{}$$

$$= -\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f} + \frac{a(Bc-Ad)\,\mathrm{co}}{d(c+}$$

$$= -\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5Bc-3A}{}$$

$$= -\frac{a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}} - \frac{a^2(5Bc-3A}{}$$

$$= \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx}{\sqrt{c+d}\sqrt{a+a\sin(e+}}\right.}{d^{7/2}(c+d)^{3/2}f}$$

<u>Mathematica</u> [A] time = 5.89262, size = 460, normalized size = 1.74

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(\frac{3(c-d)\left(B\left(5c^2+5cd-2d^2\right)-Ad(3c+5d)\right)\left(2\log\left(\sqrt{d}\sqrt{c+d}\left(\tan^2\left(\frac{1}{4}(e+fx)\right)+2\tan\left(\frac{1}{4}(e+fx)\right)-1\right)+(c+d)\sec^2\left(\frac{1}{4}(e+fx)\right)\right)-2\log\left(\sec^2\left(\frac{1}{4}\right.\right.\right.}{(c+d)^{3/2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-12*Sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e
+ f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(-(A*d*(3*c + 5*d
)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Lo
g[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt
[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(3/2) + (3*(c - d)*(-(A*d*(3*
c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]
 + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e +
f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(3/2) + 12*Sqrt[d]*(-4*B*c + 2*A*d
 + 5*B*d)*Sin[(e + f*x)/2] - (12*(c - d)^2*Sqrt[d]*(-(B*c) + A*d)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])) - 4*B*d^(3/2)*
Sin[(3*(e + f*x))/2]))/(12*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
```

5)

Maple [B] time = 2.13, size = 933, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/3*a*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-sin(f*x+e)*d*(9*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d+6*A*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-15*A*arctanh((a-a*sin(f*
x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-15*a^2*arctanh((a-a*sin(f*x+e))^
(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3+21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^2*c*d^2-6*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*
d^2)^(1/2))*a^2*d^3+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+2*B*(a
-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c
+d)*d)^(1/2)*a*c*d-6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2+12*B*
(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-6*B*(a-a*sin(f*x+e))^(1/2)*(
a*(c+d)*d)^(1/2)*a*c*d-18*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2)
-9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-6*A*ar
ctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+15*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-2*B*(a-a*sin(f*x+
e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1
/2)*c*d^2+15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^
4-21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2+6*
B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+9*A*(a-a*
sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+3*A*(a-a*sin(f*x+e))^(1/2)*(a*(
c+d)*d)^(1/2)*a*d^3-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3+12*
B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+15*B*(a-a*sin(f*x+e))^(1
/2)*(a*(c+d)*d)^(1/2)*a*c*d^2)/d^3/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 19.1361, size = 4475, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 +
(3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*
d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^
2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*co
s(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2
 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c
^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x
 + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*
d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*
d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d
 + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e)
 + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*
cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e
)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^3 -
(9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2
*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*
a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10
*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*
a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a
^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*
(A + B)*a^2*c*d^2 - (3*A + 8*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a))/((c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*c
os(f*x + e) - (c^2*d^3 + 2*c*d^4 + d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) +
 (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin(f*x + e)), 1/6*(3*(5*B*a^2*c^4 - (3*A - 5
*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*
B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*
```

```
A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*
B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A
- 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A +
 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 +
(5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arcta
n(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^
2))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A -
 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*
x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*
cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 -
(3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d
+ 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d
^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*(A + B)*a^2*c*d^2 - (3*A + 8*B)*a
^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^4 + d^5
)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^2*d^3 + 2*c*d^4
+ d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) + (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin
(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")

[Out] Exception raised: TypeError
```

3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=308

$$\frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3f(c+d)^2\sqrt{a\sin(e+fx)+a}}-\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx}}{4d^2f(c+d)^2(c+d\sin(e+fx))}$$

```
[Out] -(a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 -
 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Si
n[e + f*x]])])/(4*d^(7/2)*(c + d)^(5/2)*f) + (a^3*(3*A*d*(c + 3*d) - B*(15*
c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(4*d^3*(c + d)^2*f*Sqrt[a + a*Sin[e +
f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c +
d)*f*(c + d*Sin[e + f*x])^2) - (a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4*d
^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*d^2*(c + d)^2*f*(c + d*Sin[e
 + f*x]))
```

Rubi [A] time = 0.97193, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2975, 2981, 2773, 208}

$$\frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3f(c+d)^2\sqrt{a\sin(e+fx)+a}}-\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx}}{4d^2f(c+d)^2(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^
3,x]
```

```
[Out] -(a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 -
 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Si
n[e + f*x]])])/(4*d^(7/2)*(c + d)^(5/2)*f) + (a^3*(3*A*d*(c + 3*d) - B*(15*
c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(4*d^3*(c + d)^2*f*Sqrt[a + a*Sin[e +
f*x]]) + (a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c +
d)*f*(c + d*Sin[e + f*x])^2) - (a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4*d
^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*d^2*(c + d)^2*f*(c + d*Sin[e
 + f*x]))
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{\int \frac{(a+a\sin(e+fx))^{3/2}\left(-\frac{1}{2}a(3B\right.}{(}}{}$$

$$= \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2\left(Ad(c+7d)-B\left(5c^2\right.\right.}{4d^2(}$$

$$= \frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\,\mathrm{cc}}{2d(c+}$$

$$= \frac{a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\,\mathrm{cc}}{2d(c+}$$

$$= -\frac{a^{5/2}\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\tanh}{4d^{7/2}(c+d)^{5/2}f}$$

Mathematica [A] time = 8.11959, size = 504, normalized size = 1.64

$$\frac{(a(\sin(e+fx)+1))^{5/2}\left(-\frac{4\sqrt{d}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(d\left(Ad\left(-5c^2-6cd+11d^2\right)+B\left(34c^2d+25c^3+cd^2+4d^3\right)\right)\sin(e+fx)-8Ac^2d^2-3Ac^3d+9Acd^3+2A\right.}{(c+d)^2(c+d\sin(e+fx))^2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-(A*d*(3*c^2 + 10*c*d + 19*d^2)) + B*(15*c
^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2
*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - S
qrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))]))/(c + d)^(5/2) + ((A*d*(3*c^2 + 10*
c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[
Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d
]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt
[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^4 - 3*A*c^3*d + 20*B*c^3*
d - 8*A*c^2*d^2 - B*c^2*d^2 + 9*A*c*d^3 + 10*B*c*d^3 + 2*A*d^4 + 4*B*d^4 -
4*B*d^2*(c + d)^2*Cos[2*(e + f*x)] + d*(A*d*(-5*c^2 - 6*c*d + 11*d^2) + B*(
25*c^3 + 34*c^2*d + c*d^2 + 4*d^3))*Sin[e + f*x]))/((c + d)^2*(c + d*Sin[e
```

```
+ f*x])^2)))/(16*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [B] time = 2.445, size = 1587, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*a*(8*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*d^4
+8*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d^2+16
*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c*d^3+16*B*(
-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^3*d+32*B*(-a*(-1
+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^2*d^2+16*B*(-a*(-1+sin
(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c*d^3-15*a^2*arctanh((-a*(-1
+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*B*c^5-11*A*(-a*(-1+sin(f*x+e)))^(3
/2)*(a*(c+d)*d)^(1/2)*d^4-4*B*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*
d^4-30*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)
*a^2*c^4*d-60*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin
(f*x+e)*a^2*c^3*d^2+40*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(
1/2))*sin(f*x+e)*a^2*c*d^4-3*A*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)
*a*c^3*d-13*A*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2+3*A*(-
a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^3+29*B*(-a*(-1+sin(f*x+e))
)^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-3*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d
)^(1/2)*a*c^2*d^2-13*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^3
+38*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^
2*c*d^4+3*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x
+e)^2*a^2*c^2*d^3+10*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/
2))*sin(f*x+e)^2*a^2*c*d^4-15*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+
d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-30*B*arctanh((-a*(-1+sin(f*x+e)))^(1/
2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-7*B*arctanh((-a*(-1+sin(f*
x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4+6*A*arctanh((-a*(-
1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2+20*A*arcta
nh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3-1
4*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*
c^2*d^3+3*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^4
*d+10*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2
+19*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3-9
*B*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d-2*B*(-a*(-1+sin(f*x+e
)))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+19*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)
*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+20*B*arctanh((-a*(-1+sin(f*x+e))
```

```
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+5*A*(-a*(-1+sin(f*x+e)))^
(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+6*A*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^
(1/2)*c*d^3+15*B*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3+13*A*(-
a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+15*B*(-a*(-1+sin(f*x+e)))^
(1/2)*(a*(c+d)*d)^(1/2)*a*c^4+4*B*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1
/2)*a*d^4-30*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*
c^4*d-7*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d
^2+20*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3
)*(-a*(-1+sin(f*x+e)))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+
e))^2/(c+d)^2/d^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 21.8934, size = 6692, normalized size = 21.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*((15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2
- 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2
*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^
4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B
)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19
*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d
 - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(11*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a
^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*
B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(
```

```
16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A -
 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*
x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A - 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*
c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d
+ d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*
a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x
+ e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3
+ (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a
/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x +
 e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f
*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d -
 d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e)
- c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3
*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d
^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a
^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2 - 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B
)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A -
 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x +
e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15
*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d
^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2
 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a))/((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*
c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^
4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^
2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*
(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2
*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e)), -1/8*((15*B*a^2*c^5 - 3*(A - 20*B)*
a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*
A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10
*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x +
 e)^3 - (30*B*a^2*c^4*d - 3*(2*A - 25*B)*a^2*c^3*d^2 - (23*A - 44*B)*a^2*c^
2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 +
 (15*B*a^2*c^5 - 3*(A - 10*B)*a^2*c^4*d - 2*(5*A - 11*B)*a^2*c^3*d^2 - 2*(1
1*A - 5*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^2*c*d^4 - (19*A + 20*B)*a^2*d^5)*co
s(f*x + e) + (15*B*a^2*c^5 - 3*(A - 20*B)*a^2*c^4*d - 2*(8*A - 41*B)*a^2*c^
3*d^2 - 6*(7*A - 4*B)*a^2*c^2*d^3 - 3*(16*A + 11*B)*a^2*c*d^4 - (19*A + 20*
B)*a^2*d^5 - (15*B*a^2*c^3*d^2 - 3*(A - 10*B)*a^2*c^2*d^3 - (10*A - 7*B)*a^
2*c*d^4 - (19*A + 20*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(15*B*a^2*c^4*d - 3*(A
- 10*B)*a^2*c^3*d^2 - (10*A - 7*B)*a^2*c^2*d^3 - (19*A + 20*B)*a^2*c*d^4)*c
os(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
 e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e)))
- 2*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*
A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^
3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2
```

```
- 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^
2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*
c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2
*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a
^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25
*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B
)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5
+ 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)
*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos
(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5
 + 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(
f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")

[Out] Exception raised: TypeError
```

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=284

$$-\frac{2d\left(7Ad(9c-d)+B\left(24c^2-15cd+31d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105af}-\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(-63c^2d\right.\right.}{105f\sqrt{a\sin(e}}$$

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(7*A*d*(21*c^2 - 12*c*d + 7*d^2) +
B*(36*c^3 - 63*c^2*d + 144*c*d^2 - 37*d^3))*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*d*(7*A*(9*c - d)*d + B*(24*c^2 - 15*c*d + 31*d^2))*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*a*f) - (2*(6*B*c + 7*A*d - B*d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(35*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*C
os[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 1.00144, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{2d\left(7Ad(9c-d)+B\left(24c^2-15cd+31d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{105af}-\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(-63c^2d\right.\right.}{105f\sqrt{a\sin(e}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e + f*x]],
x]
```


```
[Out] -((Sqrt[2]*(A - B)*(c - d)^3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(7*A*d*(21*c^2 - 12*c*d + 7*d^2) +
B*(36*c^3 - 63*c^2*d + 144*c*d^2 - 37*d^3))*Cos[e + f*x])/(105*f*Sqrt[a + a
*Sin[e + f*x]]) - (2*d*(7*A*(9*c - d)*d + B*(24*c^2 - 15*c*d + 31*d^2))*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(105*a*f) - (2*(6*B*c + 7*A*d - B*d)*Co
s[e + f*x]*(c + d*Sin[e + f*x])^2)/(35*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*C
os[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(7Ac-Bc+6Bd)-\right.}{\sqrt{a+a\sin(e}}}{7a}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)(}{7f\sqrt{a+}}$$

$$= -\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}} - \frac{2B\cos(e+fx)(}{7f\sqrt{a+}}$$

$$= -\frac{2d\left(7A(9c-d)d+B\left(24c^2-15cd+31d^2\right)\right)\cos(e+fx)\sqrt{a+a\sin(e+}}{105af}$$

$$= -\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(36c^3-63c^2d+144cd^2-37d^3\right)\right)\cos}{105f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{4\left(7Ad\left(21c^2-12cd+7d^2\right)+B\left(36c^3-63c^2d+144cd^2-37d^3\right)\right)\cos}{105f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\left(7Ad\left(21c^2-12cd+\right.\right.}{}$$

Mathematica [C] time = 0.87627, size = 375, normalized size = 1.32

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(105\left(4Ad\left(6c^2-3cd+2d^2\right)+B\left(-12c^2d+8c^3+24cd^2-5d^3\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((840 + 840*I)*(-1)^(3/4)*(A - B)*(c
 - d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 105*(4*A*
d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^2*d + 24*c*d^2 - 5*d^3))*Cos[(e
 + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c^2 - 6*c*d + 5*d^2))*Cos[(3*(e
+ f*x))/2] + 21*d^2*(6*B*c + 2*A*d - B*d)*Cos[(5*(e + f*x))/2] + 15*B*d^3*C
os[(7*(e + f*x))/2] + 105*(4*A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^
```

```
2*d + 24*c*d^2 - 5*d^3))*Sin[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c
^2 - 6*c*d + 5*d^2))*Sin[(3*(e + f*x))/2] + 21*d^2*(-2*A*d + B*(-6*c + d))*
Sin[(5*(e + f*x))/2] + 15*B*d^3*Sin[(7*(e + f*x))/2]))/(420*f*Sqrt[a*(1 + S
in[e + f*x])])
```

Maple [B] time = 1.624, size = 610, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/105*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(105*A*a^(7/2)*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-315*A*a^(7/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+315*A*a^(7/2)*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-105*A*a^(7/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3-105*B*a^(
7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3+315*B*
a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d-3
15*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*
d^2+105*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*d^3-30*B*d^3*(a-a*sin(f*x+e))^(7/2)+42*A*(a-a*sin(f*x+e))^(5/2)*a*d^3+12
6*B*(a-a*sin(f*x+e))^(5/2)*a*c*d^2+84*B*(a-a*sin(f*x+e))^(5/2)*a*d^3-210*A*
(a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-70*A*(a-a*sin(f*x+e))^(3/2)*a^2*d^3-210*B*
(a-a*sin(f*x+e))^(3/2)*a^2*c^2*d-210*B*(a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-140
*B*(a-a*sin(f*x+e))^(3/2)*a^2*d^3+630*A*c^2*d*a^3*(a-a*sin(f*x+e))^(1/2)+21
0*A*a^3*d^3*(a-a*sin(f*x+e))^(1/2)+210*B*c^3*a^3*(a-a*sin(f*x+e))^(1/2)+630
*B*a^3*c*d^2*(a-a*sin(f*x+e))^(1/2))/a^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/
f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^3}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) +
 a), x)
```

Fricas [B] time = 1.97671, size = 1543, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/210*(105*sqrt(2)*((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 -
 (A - B)*a*d^3 + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 - (
A - B)*a*d^3)*cos(f*x + e) + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)
*a*c*d^2 - (A - B)*a*d^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e
) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - si
n(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(15*B*d^3*cos(f*x + e
)^4 - 105*B*c^3 - 105*(3*A - 2*B)*c^2*d + 21*(10*A - 17*B)*c*d^2 - (119*A -
 92*B)*d^3 + 3*(21*B*c*d^2 + (7*A - B)*d^3)*cos(f*x + e)^3 - (105*B*c^2*d +
 21*(5*A - 4*B)*c*d^2 - 4*(7*A - 16*B)*d^3)*cos(f*x + e)^2 - (105*B*c^3 + 1
05*(3*A - B)*c^2*d - 21*(5*A - 16*B)*c*d^2 + 2*(56*A - 23*B)*d^3)*cos(f*x +
 e) + (15*B*d^3*cos(f*x + e)^3 + 105*B*c^3 + 105*(3*A - 2*B)*c^2*d - 21*(10
*A - 17*B)*c*d^2 + (119*A - 92*B)*d^3 - 3*(21*B*c*d^2 + (7*A - 6*B)*d^3)*co
s(f*x + e)^2 - (105*B*c^2*d + 21*(5*A - B)*c*d^2 - (7*A - 46*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin
(f*x + e) + a*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

[Out] Timed out

Giac [B] time = 2.12646, size = 2520, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/105*(210*sqrt(2)*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c*d^2 - 3*B
*c*d^2 - A*d^3 + B*d^3)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(
1/2*f*x + 1/2*e) + 1)) + ((((((((105*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) + 1
) + 315*A*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 105*B*a^3*c^2*d*sgn(tan
(1/2*f*x + 1/2*e) + 1) - 105*A*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 27
3*B*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 91*A*a^3*d^3*sgn(tan(1/2*f*x
+ 1/2*e) + 1) - 43*B*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1
/2*e)/a^12 - 105*(B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) + 1) + 3*A*a^3*c^2*d*s
gn(tan(1/2*f*x + 1/2*e) + 1) - 3*B*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1)
- 3*A*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 3*B*a^3*c*d^2*sgn(tan(1/2*f
*x + 1/2*e) + 1) + A*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - B*a^3*d^3*sgn(
tan(1/2*f*x + 1/2*e) + 1))/a^12)*tan(1/2*f*x + 1/2*e) + 7*(45*B*a^3*c^3*sgn
(tan(1/2*f*x + 1/2*e) + 1) + 135*A*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1)
- 75*B*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 75*A*a^3*c*d^2*sgn(tan(1/2
*f*x + 1/2*e) + 1) + 159*B*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 53*A*a
^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - 29*B*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e
) + 1))/a^12)*tan(1/2*f*x + 1/2*e) - 35*(9*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*
e) + 1) + 27*A*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 21*B*a^3*c^2*d*sgn
(tan(1/2*f*x + 1/2*e) + 1) - 21*A*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) +
 33*B*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 11*A*a^3*d^3*sgn(tan(1/2*f*
x + 1/2*e) + 1) - 11*B*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^12)*tan(1/2
*f*x + 1/2*e) + 35*(9*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) + 1) + 27*A*a^3*c^
2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 21*B*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e)
 + 1) - 21*A*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 33*B*a^3*c*d^2*sgn(t
an(1/2*f*x + 1/2*e) + 1) + 11*A*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - 11*
B*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^12)*tan(1/2*f*x + 1/2*e) - 7*(45
*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) + 1) + 135*A*a^3*c^2*d*sgn(tan(1/2*f*x
+ 1/2*e) + 1) - 75*B*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 75*A*a^3*c*d
^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 159*B*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e)
 + 1) + 53*A*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - 29*B*a^3*d^3*sgn(tan(1
```

```
/2*f*x + 1/2*e) + 1))/a^12)*tan(1/2*f*x + 1/2*e) + 105*(B*a^3*c^3*sgn(tan(1
/2*f*x + 1/2*e) + 1) + 3*A*a^3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 3*B*a^
3*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 3*A*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2
*e) + 1) + 3*B*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + A*a^3*d^3*sgn(tan(
1/2*f*x + 1/2*e) + 1) - B*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^12)*tan(
1/2*f*x + 1/2*e) - (105*B*a^3*c^3*sgn(tan(1/2*f*x + 1/2*e) + 1) + 315*A*a^3
*c^2*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 105*B*a^3*c^2*d*sgn(tan(1/2*f*x + 1/
2*e) + 1) - 105*A*a^3*c*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 273*B*a^3*c*d^2
*sgn(tan(1/2*f*x + 1/2*e) + 1) + 91*A*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1)
 - 43*B*a^3*d^3*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^12)/(a*tan(1/2*f*x + 1/2*e
)^2 + a)^(7/2) - (210*sqrt(2)*A*a^13*c^3*arctan(sqrt(a)/sqrt(-a)) - 210*sqr
t(2)*B*a^13*c^3*arctan(sqrt(a)/sqrt(-a)) - 630*sqrt(2)*A*a^13*c^2*d*arctan(
sqrt(a)/sqrt(-a)) + 630*sqrt(2)*B*a^13*c^2*d*arctan(sqrt(a)/sqrt(-a)) + 630
*sqrt(2)*A*a^13*c*d^2*arctan(sqrt(a)/sqrt(-a)) - 630*sqrt(2)*B*a^13*c*d^2*a
rctan(sqrt(a)/sqrt(-a)) - 210*sqrt(2)*A*a^13*d^3*arctan(sqrt(a)/sqrt(-a)) +
 210*sqrt(2)*B*a^13*d^3*arctan(sqrt(a)/sqrt(-a)) - 105*sqrt(2)*B*sqrt(-a)*s
qrt(a)*c^3 - 315*sqrt(2)*A*sqrt(-a)*sqrt(a)*c^2*d + 210*sqrt(2)*B*sqrt(-a)*
sqrt(a)*c^2*d + 210*sqrt(2)*A*sqrt(-a)*sqrt(a)*c*d^2 - 357*sqrt(2)*B*sqrt(-
a)*sqrt(a)*c*d^2 - 119*sqrt(2)*A*sqrt(-a)*sqrt(a)*d^3 + 92*sqrt(2)*B*sqrt(-
a)*sqrt(a)*d^3)*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(-a)*a^13))/f
```

3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=200

$$-\frac{4\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{\sqrt{2}(A-B)(\ldots}{\ldots}$$

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(5*A*(3*c - d)*d + B*(6*c^2 - 7*c*d
 + 7*d^2))*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*(4*B*c + 5*
A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*a*f) - (2*B*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.584511, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{4\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15af}-\frac{\sqrt{2}(A-B)(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -((Sqrt[2]*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
 + a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (4*(5*A*(3*c - d)*d + B*(6*c^2 - 7*c*d
 + 7*d^2))*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*d*(4*B*c + 5*
A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*a*f) - (2*B*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
```

```
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(5Ac-Bc+4Bd)+\frac{1}{2}\cdots\right.}{\sqrt{a+a\sin(e+f\cdots}}}{5a}$$

$$= -\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{1}{2}ac(5Ac-Bc+4Bd)+\left(\frac{1}{2}ac(4Bc+5Ad-B\cdots\right.}{\cdots}$$

$$= -\frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} - \frac{2B\cos(e+fx)(c\cdots}{5f\sqrt{a+a\cdots}}$$

$$= -\frac{4\left(5A(3c-d)d+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc+5Ad-\cdots}{\cdots}$$

$$= -\frac{4\left(5A(3c-d)d+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2d(4Bc+5Ad-\cdots}{\cdots}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{4\left(5A(3c-d)d+B\left(6c^2\cdots\right.\right.}{15f\sqrt{a+\cdots}}$$

Mathematica [C] time = 0.528206, size = 246, normalized size = 1.23

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(30\left(Ad(4c-d)+2B\left(c^2-cd+d^2\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)-30\left(Ad(4c-d)+2B\left(c^2-cd\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c -
 d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 30*(A*(4*c
- d)*d + 2*B*(c^2 - c*d + d^2))*Cos[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c +
d))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 30*(A*(4*c - d)*d
 + 2*B*(c^2 - c*d + d^2))*Sin[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c + d))*Si
n[(3*(e + f*x))/2] - 3*B*d^2*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[a*(1 + Sin[e
 + f*x])])
```

Maple [B] time = 1.362, size = 396, normalized size = 2.

$$-\frac{1+\sin\left(fx+e\right)}{15\,a^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(15\,Aa^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)c^{2}-30\,Aa^{5/2}\sqrt{2}\mathrm{Artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(15*A*a^(5/2)*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*a^(5/2)*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*a^(5/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*a^(5/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*a^(5/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*a^(5/2)*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a*sin(f
*x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x+e))^(3
/2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*A*a^2*c*d*(a-a*sin(f*x+e))^(
1/2)+30*B*a^2*c^2*(a-a*sin(f*x+e))^(1/2)+30*a^2*B*d^2*(a-a*sin(f*x+e))^(1/2
))/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) +
 a), x)
```

Fricas [B] time = 1.8325, size = 1119, normalized size = 5.6

$$-\frac{\frac{15\sqrt{2}\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2+\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2\right)\cos\left(fx+e\right)+\left((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2\right)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^2-(c}{}\right.}{\sqrt{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(15*sqrt(2)*((A - B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2 + ((A -
B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*cos(f*x + e) + ((A - B)*a*c^2 -
 2*(A - B)*a*c*d + (A - B)*a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos
(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x +
 e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(3*B*d^2*cos(
f*x + e)^3 - 15*B*c^2 - 10*(3*A - 2*B)*c*d + (10*A - 17*B)*d^2 - (10*B*c*d
+ (5*A - 4*B)*d^2)*cos(f*x + e)^2 - (15*B*c^2 + 10*(3*A - B)*c*d - (5*A - 1
6*B)*d^2)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - 15*B*c^2 - 10*(3*A - 2*B
)*c*d + (10*A - 17*B)*d^2 + (10*B*c*d + (5*A - B)*d^2)*cos(f*x + e))*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*
f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.81207, size = 1494, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/60*(120*sqrt(2)*(A*c^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)*arcta
n(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^
2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) + ((((
((15*B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 30*A*a^2*c*d*sgn(tan(1/2*f*x
 + 1/2*e) + 1) - 10*B*a^2*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 5*A*a^2*d^2*s
gn(tan(1/2*f*x + 1/2*e) + 1) + 13*B*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))*
tan(1/2*f*x + 1/2*e)/a^9 - 15*(B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 2*
A*a^2*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 2*B*a^2*c*d*sgn(tan(1/2*f*x + 1/2
*e) + 1) - A*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + B*a^2*d^2*sgn(tan(1/2*
f*x + 1/2*e) + 1))/a^9)*tan(1/2*f*x + 1/2*e) + 10*(3*B*a^2*c^2*sgn(tan(1/2*
f*x + 1/2*e) + 1) + 6*A*a^2*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 4*B*a^2*c*d
*sgn(tan(1/2*f*x + 1/2*e) + 1) - 2*A*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1)
+ 4*B*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^9)*tan(1/2*f*x + 1/2*e) - 10
*(3*B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 6*A*a^2*c*d*sgn(tan(1/2*f*x +
 1/2*e) + 1) - 4*B*a^2*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 2*A*a^2*d^2*sgn(
tan(1/2*f*x + 1/2*e) + 1) + 4*B*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^9)
*tan(1/2*f*x + 1/2*e) + 15*(B*a^2*c^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 2*A*a
^2*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 2*B*a^2*c*d*sgn(tan(1/2*f*x + 1/2*e)
 + 1) - A*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + B*a^2*d^2*sgn(tan(1/2*f*x
 + 1/2*e) + 1))/a^9)*tan(1/2*f*x + 1/2*e) - (15*B*a^2*c^2*sgn(tan(1/2*f*x +
 1/2*e) + 1) + 30*A*a^2*c*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - 10*B*a^2*c*d*sg
n(tan(1/2*f*x + 1/2*e) + 1) - 5*A*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1) + 1
3*B*a^2*d^2*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^9)/(a*tan(1/2*f*x + 1/2*e)^2 +
 a)^(5/2) - (120*sqrt(2)*A*a^10*c^2*arctan(sqrt(a)/sqrt(-a)) - 120*sqrt(2)*
B*a^10*c^2*arctan(sqrt(a)/sqrt(-a)) - 240*sqrt(2)*A*a^10*c*d*arctan(sqrt(a)
/sqrt(-a)) + 240*sqrt(2)*B*a^10*c*d*arctan(sqrt(a)/sqrt(-a)) + 120*sqrt(2)*
A*a^10*d^2*arctan(sqrt(a)/sqrt(-a)) - 120*sqrt(2)*B*a^10*d^2*arctan(sqrt(a)
/sqrt(-a)) - 15*sqrt(2)*B*sqrt(-a)*sqrt(a)*c^2 - 30*sqrt(2)*A*sqrt(-a)*sqrt
(a)*c*d + 20*sqrt(2)*B*sqrt(-a)*sqrt(a)*c*d + 10*sqrt(2)*A*sqrt(-a)*sqrt(a)
*d^2 - 17*sqrt(2)*B*sqrt(-a)*sqrt(a)*d^2)*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sq
rt(-a)*a^10))/f
```

3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=130

$$-\frac{2(3Ad+3Bc-2Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

```
[Out] -((Sqrt[2]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (2*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])
/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*a*f)
```

Rubi [A] time = 0.269964, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3023, 2751, 2649, 206}

$$-\frac{2(3Ad+3Bc-2Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
 a*Sin[e + f*x]])])/(Sqrt[a]*f)) - (2*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])
/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*a*f)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af} + \frac{2\int \frac{\frac{1}{2}a(3Ac+Bd)+\frac{1}{2}a(3Bc+3Ad-2Bd)\sin}{\sqrt{a+a\sin(e+fx)}}}{3a}$$

$$= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

$$= -\frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

$$= -\frac{\sqrt{2}(A-B)(c-d)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2(3Bc+3Ad-2Bd)\cos}{3f\sqrt{a+a\sin(e+f}}$$

Mathematica [C] time = 0.465693, size = 135, normalized size = 1.04

$$-\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(3Ad+3Bc+Bd\sin(e+fx)-Bd)-(6+6\ldots\right.}{3f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)*(c -
 d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x])))/(3*f*
Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 1.293, size = 232, normalized size = 1.8

$$-\frac{1+\sin\left(fx+e\right)}{3\,a^{2}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(3\,Aa^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)c-3\,Aa^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(3*A*a^(3/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-3*A*a^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-3*B*a^(3/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c+3*B*a^(3/2)*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-2*B*(a-a*sin(f*x+e))^(3/2)*d+6*
A*a*d*(a-a*sin(f*x+e))^(1/2)+6*B*a*c*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a
), x)
```

Fricas [B] time = 1.83923, size = 782, normalized size = 6.02

$$\frac{3\sqrt{2}\left((A-B)ac-(A-B)ad+((A-B)ac-(A-B)ad)\cos\left(fx+e\right)+((A-B)ac-(A-B)ad)\sin\left(fx+e\right)\right)\log\left(-\frac{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)-\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx\cdots\right.\right.}{\sqrt{a}}}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\cdots}\right.}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*cos
(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e)^2
- (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos
(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*d*co
s(f*x + e)^2 + 3*B*c + (3*A - 2*B)*d + (3*B*c + (3*A - B)*d)*cos(f*x + e) +
 (B*d*cos(f*x + e) - 3*B*c - (3*A - 2*B)*d)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1
)), x)
```

Giac [B] time = 1.58862, size = 720, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/3*(6*sqrt(2)*(A*c - B*c - A*d + B*d)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2
*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sq
rt(-a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) + ((((3*B*a*c*sgn(tan(1/2*f*x + 1/2*e
) + 1) + 3*A*a*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - B*a*d*sgn(tan(1/2*f*x + 1/
2*e) + 1))*tan(1/2*f*x + 1/2*e)/a^6 - 3*(B*a*c*sgn(tan(1/2*f*x + 1/2*e) + 1
) + A*a*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - B*a*d*sgn(tan(1/2*f*x + 1/2*e) +
1))/a^6)*tan(1/2*f*x + 1/2*e) + 3*(B*a*c*sgn(tan(1/2*f*x + 1/2*e) + 1) + A*
a*d*sgn(tan(1/2*f*x + 1/2*e) + 1) - B*a*d*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^
6)*tan(1/2*f*x + 1/2*e) - (3*B*a*c*sgn(tan(1/2*f*x + 1/2*e) + 1) + 3*A*a*d*
sgn(tan(1/2*f*x + 1/2*e) + 1) - B*a*d*sgn(tan(1/2*f*x + 1/2*e) + 1))/a^6)/(
a*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - (6*sqrt(2)*A*a^7*c*arctan(sqrt(a)/sqr
t(-a)) - 6*sqrt(2)*B*a^7*c*arctan(sqrt(a)/sqrt(-a)) - 6*sqrt(2)*A*a^7*d*arc
tan(sqrt(a)/sqrt(-a)) + 6*sqrt(2)*B*a^7*d*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt
(2)*B*sqrt(-a)*sqrt(a)*c - 3*sqrt(2)*A*sqrt(-a)*sqrt(a)*d + 2*sqrt(2)*B*sqr
t(-a)*sqrt(a)*d)*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(-a)*a^7))/f
```

3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.0703238, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2649, 206}

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```


```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + (A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= -\frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{(2(A-B))\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}$$
$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 0.219503, size = 106, normalized size = 1.34

$$\frac{2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(B\left(\sin\left(\frac{1}{2}(e+fx)\right)-\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-1)^{3/4}(A-B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1\right.\right.}{f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] time = 1.024, size = 128, normalized size = 1.6

$$-\frac{1+\sin\left(fx+e\right)}{af\cos\left(fx+e\right)}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sqrt{a}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{a}}\right)A-\sqrt{a}\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\,\text{s}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*A-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*B+2*(a-a*sin(f*x+e))^(1/2)*B)/a/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.94569, size = 572, normalized size = 7.24

$$-\frac{\frac{\sqrt{2}\left((A-B)a\cos\left(fx+e\right)+(A-B)a\sin\left(fx+e\right)+(A-B)a\right)\log\left(-\frac{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)-2\right)\sin\left(fx+e\right)+\frac{2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)}{\sqrt{a}}+3\cos\left(fx+e\right)+2}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)}{\sqrt{a}}+}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx+e\right)+af\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a)
*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*
sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e
) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) +
a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sin\left(e + fx\right)}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 1.47835, size = 297, normalized size = 3.76

$$\frac{2\left(\frac{\sqrt{2}(A-B)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{\frac{B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}-\frac{B}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}-\frac{\left(\sqrt{2}Aa\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}Ba\arctan \cdots\right.}{\cdots}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(2)*(A - B)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt
(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f
*x + 1/2*e) + 1)) + (B*tan(1/2*f*x + 1/2*e)/sgn(tan(1/2*f*x + 1/2*e) + 1) -
 B/sgn(tan(1/2*f*x + 1/2*e) + 1))/sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) - (sqr
t(2)*A*a*arctan(sqrt(a)/sqrt(-a)) - sqrt(2)*B*a*arctan(sqrt(a)/sqrt(-a)) -
sqrt(2)*B*sqrt(-a)*sqrt(a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(-a)*a))/f
```

3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqr
t[d]*Sqrt[c + d]*f)
```

Rubi [A] time = 0.282783, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*
Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*Sqr
t[d]*Sqrt[c + d]*f)
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{c-d} + \frac{(Bc-Ad)\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a(c-d)}$$

$$= -\frac{(2(A-B))\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{(c-d)f} - \frac{(2(Bc-Ad))\,\mathrm{Subst}\left(\int \right.}{(}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+}{\sqrt{c+d}\sqrt{a+a\sin(}\right.}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

Mathematica [C] time = 3.0525, size = 238, normalized size = 1.75

$$\frac{(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\sqrt[4]{-1}(Bc-Ad)\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])),x]
```

```
[Out] ((-1)^(3/4)*((2 + 2*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)
^(3/4)*(-1 + Tan[(e + f*x)/4])] + (-1)^(1/4)*(B*c - A*d)*(Log[Sec[(e + f*x)
/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])]
- Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*
Sin[(e + f*x)/2])]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c - d)*Sqrt[d
]*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [A] time = 1.713, size = 199, normalized size = 1.5

$$-\frac{1+\sin\left(fx+e\right)}{(c-d)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\frac{1}{\sqrt{a}}\right)\sqrt{a\,(c+d)\,d}A-2\,A\sqrt{a}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(2^(1/2)*arctanh(1/2*(-a*(-1+sin
(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*A-2*A*a^(1/2)*arctanh((-
a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*d-2^(1/2)*arctanh(1/2*(-a*(-1
+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*B+2*B*a^(1/2)*arctan
h((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c)/(c-d)/a^(1/2)/(a*(c+d)
*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)), x)
```

Fricas [B] time = 10.0327, size = 1831, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2
*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)
*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*si
n(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2
)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c
*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos
(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt
(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) -
2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) +
 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f), -1/2*
(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))
) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
- sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=207

$$-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)^2(c+d)^{3/2}}-\frac{\sqrt{2}(}{}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^2*f)) + ((A*d*(3*c + d) - B*(c^2 + c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(Sqrt[a]*(c - d)^2*Sqrt[d]*(c + d)^(3/2)*f) - ((B*c - A*d)*Cos[e +
f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.616985, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2984, 2985, 2649, 206, 2773, 208}

$$-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)^2(c+d)^{3/2}}-\frac{\sqrt{2}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),
x]
```


```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^2*f)) + ((A*d*(3*c + d) - B*(c^2 + c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f
*x]])])/(Sqrt[a]*(c - d)^2*Sqrt[d]*(c + d)^(3/2)*f) - ((B*c - A*d)*Cos[e +
f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```


Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
```

```
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}dx = -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(A(2c+d)-B(c+2d))}{\sqrt{a+a\sin(e+fx)}}}{a(c^2}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} + \frac{(A-B)\int \frac{1}{\sqrt{a+a\sin(e+}}}{(c-d)^2}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))} - \frac{(2(A-B))\operatorname{Subst}\left(\int\right.}{(c}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2f} + \frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right.}{\sqrt{a}(c-d)^2\sqrt{d}}$$

Mathematica [C] time = 6.79061, size = 374, normalized size = 1.81

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-\frac{\left(B\left(c^2+cd+2d^2\right)-Ad(3c+d)\right)\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)-2\log\left(\sec^2\right.\right.}{\sqrt{d}(c+d)^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) + B*
(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e +
f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2
])]))/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*d + 2*d^2))
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])]))/(Sqrt[d]*(c +
d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(
(c + d)*(c + d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] time = 2.365, size = 899, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] (1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)/a^(5/2)*(sin(f*x+e)*d*(3*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d+A*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-B*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2-B*arctanh((a-a*sin(f*x+e))^(1
/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d-2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d
/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-A*2^(1/2)*(a*(c+d)*d)^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-A*2^(1/2)*(a*(c+d)*d)^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+B*2^(1/2)*(a*(c+d)*
d)^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+B*2^(1/2
)*(a*(c+d)*d)^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
*d)+3*A*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^2*d
+A*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d^2-B*a^
(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^3-B*a^(5/2)*a
rctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^2*d-2*B*a^(5/2)*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d^2+A*a^(3/2)*(a-a*sin(
f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c*d-A*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*(a*(c+
d)*d)^(1/2)*d^2-A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*(a*(c+d)*d)^(1/2)*a^2*c^2-A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c*d-B*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*
(a*(c+d)*d)^(1/2)*c^2+B*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c*
d+B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)
^(1/2)*a^2*c^2+B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*(a*(c+d)*d)^(1/2)*a^2*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/
2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 29.2073, size = 4890, normalized size = 23.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*
c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B
)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A
 - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x + e)^3
- a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c
*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) +
c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*
d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2
 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 +
(2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e)
 + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x +
e))) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3
 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*
cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*
cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 +
 (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*co
s(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x
 + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1
)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*
d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (B*c^3*d - A*c
^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^4*d
^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^3*d^3 + a*c*d
^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c
*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e) + (a*c^
5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f)*sin(f*x +
 e)), 1/2*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 -
 (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A
 - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d -
(4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*
d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*
d - a*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*
d^2)*cos(f*x + e))) + sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A
 - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A
```

```
 - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A -
 B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A -
 B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A -
 B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e)
 - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin
(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(B*c^3*d - A*c^2*d^2 -
 B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (
B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^
3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*
c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x
 + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)
*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")

[Out] Exception raised: TypeError
```

3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=309

$$\frac{\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}+\frac{\left(Ad\left(15c^2+1\right.\right.}{}$$

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^3*f)) + ((A*d*(15*c^2 + 10*c*d + 7*d^2) - B*(3
*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[a]*(c - d)^3*Sqrt[d]*(c +
d)^(5/2)*f) - ((B*c - A*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a + a*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^2) + ((A*d*(7*c + d) - B*(3*c^2 + c*d + 4*d^2)
)*Cos[e + f*x])/(4*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

Rubi [A] time = 1.05417, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2984, 2985, 2649, 206, 2773, 208}

$$\frac{\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{4f\left(c^2-d^2\right)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}+\frac{\left(Ad\left(15c^2+1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3),
x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e
 + f*x]])])/(Sqrt[a]*(c - d)^3*f)) + ((A*d*(15*c^2 + 10*c*d + 7*d^2) - B*(3
*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/
(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*Sqrt[a]*(c - d)^3*Sqrt[d]*(c +
d)^(5/2)*f) - ((B*c - A*d)*Cos[e + f*x])/(2*(c^2 - d^2)*f*Sqrt[a + a*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^2) + ((A*d*(7*c + d) - B*(3*c^2 + c*d + 4*d^2)
)*Cos[e + f*x])/(4*(c^2 - d^2)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(A(4c+d)-B(c+4}{\sqrt{a+a\sin(e+}}}{2a}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{\left(Ad(7c+d)-B\right.}{4\left(c^2-d^2\right)^2 f\sqrt{a}}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{\left(Ad(7c+d)-B\right.}{4\left(c^2-d^2\right)^2 f\sqrt{a}}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)}{2\left(c^2-d^2\right)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{\left(Ad(7c+d)-B\right.}{4\left(c^2-d^2\right)^2 f\sqrt{a}}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3 f} + \frac{\left(Ad\left(15c^2+10cd+7d^2\right)-B\left(3c\right.\right.}{4\sqrt{}}$$

Mathematica [C] time = 10.6863, size = 847, normalized size = 2.74

$$\frac{(2+2i)(A-B)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{1}{4}(e+fx)\right)\left(\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e\right.\right.}{\left(\sqrt[4]{-1}c^3-3\sqrt[4]{-1}dc^2+3\sqrt[4]{-1}d^2c-\sqrt[4]{-1}d^3\right)f\sqrt{a(\sin(e+fx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x
])^3),x]
```

```
[Out] ((2 + 2*I)*(A - B)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e
+ f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(((-1
)^(1/4)*c^3 - 3*(-1)^(1/4)*c^2*d + 3*(-1)^(1/4)*c*d^2 - (-1)^(1/4)*d^3)*f*S
qrt[a*(1 + Sin[e + f*x])]) - ((-(A*d*(15*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3
+ 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log
[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(
e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(16*(c - d)^3*Sqrt[d]
*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((-(A*d*(15*c^2 + 10*c*d + 7
*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e +
f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/
2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(16
```

```
*(c - d)^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*(-(B*c*Cos[(e + f*x)/2]) + A*d*Cos[(e + f*x)/2
] + B*c*Sin[(e + f*x)/2] - A*d*Sin[(e + f*x)/2]))/(2*(c - d)*(c + d)*f*Sqrt
[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^2) + ((Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(-3*B*c^2*Cos[(e + f*x)/2] + 7*A*c*d*Cos[(e + f*x)/2] - B*c*d*
Cos[(e + f*x)/2] + A*d^2*Cos[(e + f*x)/2] - 4*B*d^2*Cos[(e + f*x)/2] + 3*B*
c^2*Sin[(e + f*x)/2] - 7*A*c*d*Sin[(e + f*x)/2] + B*c*d*Sin[(e + f*x)/2] -
A*d^2*Sin[(e + f*x)/2] + 4*B*d^2*Sin[(e + f*x)/2]))/(4*(c - d)^2*(c + d)^2*
f*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x]))
```

Maple [B] time = 3.1, size = 2275, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/4*(-16*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*
(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^4*c^2*d^2-4*A*2^(1/2)*arctanh(1/2*(-a*(-1+si
n(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^4*c^2*d^2+8*B*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*
a^4*c^3*d+4*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2
))*(a*(c+d)*d)^(1/2)*a^4*c^2*d^2-4*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)
))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^4*d^4+4*B*2^(1/2
)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)
*sin(f*x+e)^2*a^4*d^4+8*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^
(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^4*c*d^3-A*a^(7/2)*(-a*(-1+sin
(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+16*B*2^(1/2)*arctanh(1/2*(-a*(-1+
sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^4*c^2*d^
2-4*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c
+d)*d)^(1/2)*sin(f*x+e)^2*a^4*c^2*d^2-8*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f
*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^4*c*d^3+4*B
*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d
)^(1/2)*sin(f*x+e)^2*a^4*c^2*d^2+8*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)
))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^4*c*d^3-8*A*2^(1
/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/
2)*sin(f*x+e)*a^4*c^3*d+8*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*
2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^4*c^3*d-8*A*2^(1/2)*arctanh
(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+
e)*a^4*c*d^3-9*A*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c*d^3
+10*A*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f
```

```
*x+e)^2*c*d^4-3*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^
(1/2))*sin(f*x+e)^2*c^3*d^2-6*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*
d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c^2*d^3-19*B*a^(9/2)*arctanh((-a*(-1+sin(
f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c*d^4+30*A*a^(9/2)*arctanh
((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^3*d^2-12*B*a^
(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^
3*d^2-7*A*a^(5/2)*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+6*A*
a^(5/2)*(-a*(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3+3*B*a^(5/2)*(-a*
(-1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d-2*B*a^(5/2)*(-a*(-1+sin(f*x+
e)))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+3*B*a^(5/2)*(-a*(-1+sin(f*x+e)))^(3/2)
*(a*(c+d)*d)^(1/2)*c*d^3-3*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*c^5-6*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(
c+d)*d)^(1/2))*sin(f*x+e)*c^4*d+20*A*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(
1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^2*d^3+14*A*a^(9/2)*arctanh((-a*(-1+s
in(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c*d^4-4*A*2^(1/2)*arctanh
(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^4*c^4+
4*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d
)*d)^(1/2)*a^4*c^4+B*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c
^3*d-38*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*s
in(f*x+e)*c^2*d^3+B*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c^
2*d^2-B*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c*d^3+15*A*a^(
9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c
^2*d^3-8*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*
sin(f*x+e)*c*d^4+9*A*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c
^3*d-8*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a
*(c+d)*d)^(1/2)*a^4*c^3*d-6*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/
(a*(c+d)*d)^(1/2))*c^4*d-19*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/
(a*(c+d)*d)^(1/2))*c^3*d^2-4*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*c^2*d^3+A*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d
)^(1/2)*d^4-5*B*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*c^4+4*
B*a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(a*(c+d)*d)^(1/2)*d^4+A*a^(5/2)*(-a*(-
1+sin(f*x+e)))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-4*B*a^(5/2)*(-a*(-1+sin(f*x+e)))
^(3/2)*(a*(c+d)*d)^(1/2)*d^4+7*A*a^(9/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)
*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*d^5-4*B*a^(9/2)*arctanh((-a*(-1+sin(f*x+
e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*d^5+15*A*a^(9/2)*arctanh((-a*(
-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^4*d+10*A*a^(9/2)*arctanh((-a*(
-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^3*d^2+7*A*a^(9/2)*arctanh((-a*
(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^2*d^3)*(-a*(-1+sin(f*x+e)))^(
1/2)*(1+sin(f*x+e))/a^(9/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/(c
-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 59.8466, size = 9234, normalized size = 29.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/16*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A -
8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*
A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 -
(6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (35*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c
*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2
*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A
 - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B
)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 -
 (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d
^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c
^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)
*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*
cos(f*x + e)^2 + 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*
x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x
+ e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^
2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x +
 e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2
 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^
2 - 2*c*d - d^2)*sin(f*x + e))) - 8*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*
c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 +
 (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d
^5 + (A - B)*a*d^6)*cos(f*x + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3
```

```
*d^3 + 9*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e
)^2 + ((A - B)*a*c^5*d + 3*(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A -
 B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*
a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4
 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2
*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4
*d^2 + 3*(A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x
 + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e)
 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqr
t(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e
) - cos(f*x + e) - 2))/sqrt(a) + 4*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*
A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6
+ (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A
 - 4*B)*d^6)*cos(f*x + e)^2 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^
3*d^3 + (11*A - B)*c^2*d^4 + (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B
*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4
- (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A
 - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9
)*f*cos(f*x + e)^3 + (2*a*c^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 +
 6*a*c^3*d^6 + 3*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d
 - 2*a*c^6*d^3 + 2*a*c^2*d^7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d
^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*
d^9)*f + ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2
- 2*(a*c^7*d^2 - 3*a*c^5*d^4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c
^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f)*sin(f*x + e)), 1/8*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d
- 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (
7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4
 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (3
5*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2
 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)
*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 -
 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*
(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3
- (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*
(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x +
e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt
(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x +
e))) - 4*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*
d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a
*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x
 + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3*d^3 + 9*(A - B)*a*c^2*d^4
+ 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + ((A - B)*a*c^5*d + 3*
(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A - B)*a*c^2*d^4 + 3*(A - B)*a
```

```
*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d
^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A -
 B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 +
(A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4*d^2 + 3*(A - B)*a*c^3*d^3
+ 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x + e))*sin(f*x + e))*log(-(
cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x
 + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/
(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt
(a) + 2*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A -
 B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 + (3*B*c^4*d^2 - (7*A - B)*
c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*cos(f*x + e)^2
 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^3*d^3 + (11*A - B)*c^2*d^4
+ (A - 2*B)*c*d^5 - 2*A*d^6)*cos(f*x + e) - (5*B*c^5*d - (9*A + 2*B)*c^4*d^
2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A -
4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c
*d^5 + (A - 4*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^3 + (2*a*c
^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 + 6*a*c^3*d^6 + 3*a*c^2*d^7
- 2*a*c*d^8 - a*d^9)*f*cos(f*x + e)^2 - (a*c^8*d - 2*a*c^6*d^3 + 2*a*c^2*d^
7 - a*d^9)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*
d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f + ((a*c^6*d^3 - 3*a*
c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^7*d^2 - 3*a*c^5*d^
4 + 3*a*c^3*d^6 - a*c*d^8)*f*cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^
6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f)*sin
(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=283

$$\frac{d^2(15Ac-35Ad-51Bc+39Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{30a^2f}-\frac{(c-d)^2(A(c+11d)+3B(c-5d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+}{\sqrt{2}\sqrt{a\sin(e+}}\right.}{2\sqrt{2}a^{3/2}f}$$

```
[Out] -((c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(
Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(15*A*c^2 -
99*B*c^2 - 120*A*c*d + 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(15*a
*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(30*a^2*f) + ((5*A - 9*B)*d*Cos[e + f*x
]*(c + d*Sin[e + f*x])^2)/(10*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[
e + f*x]*(c + d*Sin[e + f*x])^3)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.999728, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{d^2(15Ac-35Ad-51Bc+39Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{30a^2f}-\frac{(c-d)^2(A(c+11d)+3B(c-5d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+}{\sqrt{2}\sqrt{a\sin(e+}}\right.}{2\sqrt{2}a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/2
),x]
```


```
[Out] -((c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(
Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(15*A*c^2 -
99*B*c^2 - 120*A*c*d + 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(15*a
*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*Cos
[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(30*a^2*f) + ((5*A - 9*B)*d*Cos[e + f*x
]*(c + d*Sin[e + f*x])^2)/(10*a*f*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*Cos[
e + f*x]*(c + d*Sin[e + f*x])^3)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```


Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(3B(c-2d)+\ldots\right.}{\sqrt{a+a\sin\ldots}}}{2\ldots}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+\ldots}{2f(a+a\sin(e+\ldots}$$

$$= \frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+\ldots}{2f(a+a\sin(e+\ldots}$$

$$= \frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f} + \frac{(5A\ldots}{\ldots}$$

$$= \frac{d\left(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2\right)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} \ldots$$

$$= \frac{d\left(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2\right)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}} \ldots$$

$$= -\frac{(c-d)^2(3B(c-5d)+A(c+11d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d\left(15A\ldots\right.}{\ldots}$$

Mathematica [C] time = 1.06042, size = 684, normalized size = 2.42

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((30+30i)(-1)^{3/4}(c-d)^2(A(c+11d)+3B(c-5d))\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*c
^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e + f*
x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 110*A
*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3*(e
+ f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e + f*x
))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2] + 30
*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*d^3*Cos
[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e + f*x)/2
] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*B*c^2*d*S
in[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[(e + f*x)/
2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (30 + 30*I)*
(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 -
180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*x))/2] + 210*B
*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2] - 123*B*d^3*Sin
[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*d^3*Sin[(5*(e +
f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(e + f*x))/2]))/(6
0*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 1.46, size = 1030, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/60*(sin(f*x+e)*(-40*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*d^3+360*A*c*d^2*a^(
5/2)*(a-a*sin(f*x+e))^(1/2)-120*A*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d
^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-120*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^
2+360*B*c^2*d*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-360*B*a^(5/2)*c*d^2*(a-a*sin(f
*x+e))^(1/2)+240*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arctanh(1/
```

```
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)-40*A*(a-a*sin(f*x+e))^
(3/2)*a^(3/2)*d^3+30*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c^3-90*A*(a-a*sin(f*x
+e))^(1/2)*a^(5/2)*c^2*d+450*A*c*d^2*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-150*A*a
^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-1
20*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^2-30*B*(a-a*sin(f*x+e))^(1/2)*a^(5/
2)*c^3+450*B*c^2*d*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-450*B*a^(5/2)*c*d^2*(a-a*
sin(f*x+e))^(1/2)+270*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)*(-a*(-1+sin(f*x+e
)))^(1/2)/a^(9/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(3/2), x)
```

Fricas [B] time = 2.03246, size = 1901, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/120*(15*sqrt(2)*(2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c
*d^2 + 2*(11*A - 15*B)*d^3 - ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A
- 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 + 3*(3*A
 - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e) + (2
*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c*d^2 + 2*(11*A - 15*
B)*d^3 + ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*
A - 15*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 +
 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) +
1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(12*B*d^
3*cos(f*x + e)^4 - 15*(A - B)*c^3 + 45*(A - B)*c^2*d - 45*(A - B)*c*d^2 + 1
5*(A - B)*d^3 + 4*(15*B*c*d^2 + (5*A - 3*B)*d^3)*cos(f*x + e)^3 - 4*(45*B*c
^2*d + 15*(3*A - 4*B)*c*d^2 - 4*(5*A - 9*B)*d^3)*cos(f*x + e)^2 - 15*((A -
B)*c^3 - 3*(A - 5*B)*c^2*d + 15*(A - B)*c*d^2 - (5*A - 9*B)*d^3)*cos(f*x +
e) + (12*B*d^3*cos(f*x + e)^3 + 15*(A - B)*c^3 - 45*(A - B)*c^2*d + 45*(A -
 B)*c*d^2 - 15*(A - B)*d^3 - 4*(15*B*c*d^2 + (5*A - 6*B)*d^3)*cos(f*x + e)^
2 - 60*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e)
- 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(c-d)(Ac+7Ad+3Bc-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d(3Ac-9A}{3a}$$

```
[Out] -((c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(3*A*c - 15*B*
c - 9*A*d + 13*B*d)*Cos[e + f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A
- 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((A - B)*Cos[
e + f*x]*(c + d*Sin[e + f*x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.575366, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$-\frac{(c-d)(Ac+7Ad+3Bc-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}+\frac{d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{6a^2f}+\frac{d(3Ac-9A}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) + (d*(3*A*c - 15*B*
c - 9*A*d + 13*B*d)*Cos[e + f*x])/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A
- 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(6*a^2*f) - ((A - B)*Cos[
e + f*x]*(c + d*Sin[e + f*x])^2)/(2*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

```
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(Ac+3Bc+4Ad\right.}{\sqrt{a+a\sin}}}{2a}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\frac{1}{2}ac(Ac+3Bc+4Ad-4Bd)+\left(-\frac{1}{2}a(3\right.}{}}{}$$

$$= \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+fx)(c+d}{2f(a+a\sin(e+f}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a}}{6a^2f}$$

$$= \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a}}{6a^2f}$$

$$= -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d(3Ac-1}{}$$

Mathematica [C] time = 0.734585, size = 357, normalized size = 1.76

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^2\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+f\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] - 3*(A - B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)*(
-1)^(3/4)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(
3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 6*d
*(-4*B*c - 2*A*d + 3*B*d)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^2 - 2*B*d^2*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - 6*d*(-4*B*c - 2*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2 - 2*B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[(3*(
e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 1.345, size = 694, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/12/a^(7/2)*(sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*d^2+24*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)+48*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-24*B*d^2*a^(3/2)*(
a-a*sin(f*x+e))^(1/2))+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^2+6*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-12*A*(a-a*sin(f*x+e)
)^(1/2)*a^(3/2)*c*d+30*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)-6*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2+60*B*c*d*a^(3/2)*(a
-a*sin(f*x+e))^(1/2)-30*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))*(-a*(-1+sin(f
*x+e)))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(3/2), x)
```

Fricas [B] time = 1.8714, size = 1422, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/24*(3*sqrt(2)*(2*(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2
- ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e)^2 + (
(A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e) + (2*(A
+ 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2 + ((A + 3*B)*c^2 + 2*(3
*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) + 4*(4*B*d^2*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A -
 B)*d^2 - 4*(6*B*c*d + (3*A - 4*B)*d^2)*cos(f*x + e)^2 - 3*((A - B)*c^2 - 2
*(A - 5*B)*c*d + 5*(A - B)*d^2)*cos(f*x + e) - (4*B*d^2*cos(f*x + e)^2 - 3*
(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 + 12*(2*B*c*d + (A - B)*d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 -
 a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{(Ac+3Ad+3Bc-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}-\frac{2Bd\cos(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -((A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((A - B)*(c - d)*Cos[e + f
*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)) - (2*B*d*Cos[e + f*x])/(a*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rubi [A] time = 0.279038, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3019, 2751, 2649, 206}

$$-\frac{(Ac+3Ad+3Bc-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}-\frac{2Bd\cos(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*f) - ((A - B)*(c - d)*Cos[e + f
*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)) - (2*B*d*Cos[e + f*x])/(a*f*Sqrt[a +
a*Sin[e + f*x]])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3B(c-d)+A(c+3d))-2aBd\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} + \frac{(Ac+3Bc+3Ad}{}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{(Ac+3Bc+3Ad}{}$$

$$= -\frac{(Ac+3Bc+3Ad-7Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c-d)\cos}{2f(a+a\sin(e+}$$

Mathematica [C] time = 0.435797, size = 246, normalized size = 1.85

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
- (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4
)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f*x)/
2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 0.986, size = 389, normalized size = 2.9

$$-\frac{1}{4f\cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\left(A\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{a}}\right)ac+3A\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}}{\sqrt{a}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+8*B*
(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d-2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+10*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)
*d)*(-a*(-1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3
/2), x)
```

Fricas [B] time = 1.80485, size = 1023, normalized size = 7.69

$$-\frac{\sqrt{2}\left(\left((A+3\,B)c+(3\,A-7\,B)d\right)\cos\left(fx+e\right)^{2}-2\,(A+3\,B)c-2\,(3\,A-7\,B)d-\left((A+3\,B)c+(3\,A-7\,B)d\right)\cos\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c
 - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A + 3
*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*sin(f
*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
 + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c - (A - B
)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e) - (A - B
)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
 + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(3/2), x)
```

Giac [B] time = 2.51031, size = 1077, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/2*(4*(B*d*tan(1/2*f*x + 1/2*e)/(a*sgn(tan(1/2*f*x + 1/2*e) + 1)) - B*d/(a
*sgn(tan(1/2*f*x + 1/2*e) + 1)))/sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(
```

```
2)*(A*c + 3*B*c + 3*A*d - 7*B*d)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x +
 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)
*a*sgn(tan(1/2*f*x + 1/2*e) + 1)) + 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sq
rt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*A*c - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*B*c - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e
) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*A*d + 3*(sqrt(a)*tan(1/2*f*x + 1/
2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*B*d + (sqrt(a)*tan(1/2*f*x + 1
/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*A*sqrt(a)*c - (sqrt(a)*tan(1/
2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*B*sqrt(a)*c - (sqrt(
a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*A*sqrt(a)*d
 + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*B*
sqrt(a)*d - (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 +
 a))*A*a*c + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2
+ a))*B*a*c + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2
 + a))*A*a*d - (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^
2 + a))*B*a*d + A*a^(3/2)*c - B*a^(3/2)*c - A*a^(3/2)*d + B*a^(3/2)*d)/(((s
qrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sq
rt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) -
a)^2*a*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] -((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x
])^(3/2))
```

Rubi [A] time = 0.0777001, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2750, 2649, 206}

$$-\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(2*Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x
])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}+\frac{(A+3B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a}\\ &= -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}-\frac{(A+3B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2af}\\ &= -\frac{(A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.193884, size = 150, normalized size = 1.72

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+(B-A)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)+(1+i)(-\right.}{2f(a(\sin(e+fx)+1))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A +
B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 1.084, size = 176, normalized size = 2.

$$-\frac{1}{4f\cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{a}}\right)a\left(A+3B\right)+A\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin\left(f\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a*(A+3*B)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+
2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*A-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*B)*(-a*
(-1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)
```

Fricas [B] time = 1.72679, size = 755, normalized size = 8.68

$$\frac{\sqrt{2}\left((A+3\,B)\cos\left(fx+e\right)^{2}-(A+3\,B)\cos\left(fx+e\right)-\left((A+3\,B)\cos\left(fx+e\right)+2\,A+6\,B\right)\sin\left(fx+e\right)-2\,A-6\,B\right)\sqrt{a}}{8\left(a^{2}f\cos\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*B
)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e
```

```
) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f
*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\sin\left(e + fx\right)}{\left(a\left(\sin\left(e + fx\right) + 1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] time = 1.90984, size = 599, normalized size = 6.89

$$\frac{\sqrt{2}(A+3B)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}+\frac{2\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+a}\right)^{3}A-3\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2)*(A + 3*B)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan
(1/2*f*x + 1/2*e) + 1)) + 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1
/2*f*x + 1/2*e)^2 + a))^3*A - 3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(
1/2*f*x + 1/2*e)^2 + a))^3*B + (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1
/2*f*x + 1/2*e)^2 + a))^2*A*sqrt(a) - (sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(
a*tan(1/2*f*x + 1/2*e)^2 + a))^2*B*sqrt(a) - (sqrt(a)*tan(1/2*f*x + 1/2*e)
- sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*A*a + (sqrt(a)*tan(1/2*f*x + 1/2*e) -
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*B*a + A*a^(3/2) - B*a^(3/2))/(((sqrt(a
)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)
```

```
*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - a)^2*
a*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=187

$$-\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}+\frac{2\sqrt{d}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(A-B)\cos(e+}{2f(c-d)(a\sin(e+f}$$

```
[Out] -((A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[
a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f) + (2*Sqrt[d]*(B*c -
A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e +
 f*x]])])/(a^(3/2)*(c - d)^2*Sqrt[c + d]*f) - ((A - B)*Cos[e + f*x])/(2*(c
- d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.589868, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2978, 2985, 2649, 206, 2773, 208}

$$-\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f(c-d)^2}+\frac{2\sqrt{d}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^2\sqrt{c+d}}-\frac{(A-B)\cos(e+}{2f(c-d)(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] -((A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[
a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^2*f) + (2*Sqrt[d]*(B*c -
A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e +
 f*x]])])/(a^(3/2)*(c - d)^2*Sqrt[c + d]*f) - ((A - B)*Cos[e + f*x])/(2*(c
- d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3Bc+A(c-4d))-\frac{1}{2}a(A-B)d\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,d}{2a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} - \frac{(d(Bc-Ad))\int \frac{\sqrt{a+a\sin(e+fx)}}{c+d\sin(e+fx)}\,dx}{a^2(c-d)^2} + -$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}} + \frac{(2d(Bc-Ad))\,\mathrm{Subst}\left(\int \frac{1}{ac+ad-dx^2}\,d\right.}{a(c-d)^2f}$$

$$= -\frac{(A(c-5d)+B(3c+d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^2f} + \frac{2\sqrt{d}(Bc-Ad)\,\mathrm{ta}}{a^{3/2}(c}$$

Mathematica [C] time = 3.00106, size = 419, normalized size = 2.24

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(B-A)(c-d)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/
4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A*d)*
(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d
] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-(B*c) + A*d)*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*
Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 1.467, size = 624, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(8*A*a^(3/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c
*d+a*d^2)^(1/2))*d^2-8*B*a^(3/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*
d^2)^(1/2))*c*d+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*(a*(c+d)*d)^(1/2)*a*c-5*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*c+B*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*d)+8*A*a^(3/2)*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2-8*B*a^(3/2)*arctanh((a-
a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d+A*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*c-5*A*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*d+3*B*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a
*c+B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d
)^(1/2)*a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*c-2*A*(a-a
*sin(f*x+e))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*d-2*B*(a-a*sin(f*x+e))^(1/2)*a
^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*(a*(c+d)*d)^(
1/2)*d)*(-a*(-1+sin(f*x+e)))^(1/2)/(a*(c+d)*d)^(1/2)/(c-d)^2/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 25.4829, size = 3671, normalized size = 19.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c +
 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*c
- 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e))*s
qrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*
(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*
a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x + e)^2
+ (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*d)*cos(f
*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
 + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^
2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
 + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)
*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x +
e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d
^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c
*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*
d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2
*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^
2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*
d^2)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x +
e)^2 - 2*(A + 3*B)*c + 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
+ e) - (2*(A + 3*B)*c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
 + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
- (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e
) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(2*B*a*c - 2*A*a*d - (B*a*c -
A*a*d)*cos(f*x + e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d +
 (B*a*c - A*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/
2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/
(d*cos(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*d)*cos(
f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/
((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d +
a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 -
 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)
*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=292

$$-\frac{\sqrt{d}\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(Ac-9Ad+3Bc+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)}}\right.}{2\sqrt{2}a^{3/2}f(c-d)^3}$$

```
[Out] -((A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f) - (Sqrt[d]*(A*d*(5
*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^3*(c + d)^(3/2
)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c +
d*Sin[e + f*x])) + (d*(B*(3*c + d) - A*(c + 3*d))*Cos[e + f*x])/(2*a*(c - d
)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.0175, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{d}\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)^3(c+d)^{3/2}}-\frac{(Ac-9Ad+3Bc+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)}}\right.}{2\sqrt{2}a^{3/2}f(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -((A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^3*f) - (Sqrt[d]*(A*d*(5
*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x
])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)^3*(c + d)^(3/2
)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c +
d*Sin[e + f*x])) + (d*(B*(3*c + d) - A*(c + 3*d))*Cos[e + f*x])/(2*a*(c - d
)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(Ac+3Bc-6A\ldots}{\sqrt{a+a\sin(\ldots}}}{\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c\ldots}{2a(c-d)^2(c+d\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c\ldots}{2a(c-d)^2(c+d\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c\ldots}{2a(c-d)^2(c+d\ldots}$$

$$= -\frac{(Ac+3Bc-9Ad+5Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3 f} - \frac{\sqrt{d}\left(Ad(5c+3\ldots\right.}{\ldots}$$

Mathematica [C] time = 9.07234, size = 542, normalized size = 1.86

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(\frac{\sqrt{d}\left(B\left(3c^2+3cd+2d^2\right)-Ad(5c+3d)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\sin\left(\frac{1}{2}(e\ldots\right.\right.\right.}{(c+d)^{3/2}}\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2]
+ 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-1)
```

```
^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + T
an[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-(A*d
*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - S
qrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)
^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*d)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*
Sin[e + f*x]))))/(4*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Maple [B] time = 2.48, size = 2049, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -1/4/a^(5/2)*(A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1
/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d-7*A*2^(1/2)*arctanh(1/2*(-a*(-1
+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^2*d+8
*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)
*d)^(1/2)*a*c^2*d+8*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2
)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c*d^2+13*B*2^(1/2)*arctanh(1/2*
(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*
c*d^2+11*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*
(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^2*d-17*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f
*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c*d^2-8*A*2^(
1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1
/2)*sin(f*x+e)^2*a*c*d^2+3*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)
*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d+20*A*a^(3/2)*arcta
nh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^2*d^2+5*B*2
^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^
(1/2)*a*c*d^2+5*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^
(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*d^3+5*B*2^(1/2)*arctanh(1/2*(-a*(-1
+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*d^3-9*A
*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d
)^(1/2)*sin(f*x+e)^2*a*d^3-9*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/
2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*d^3+3*B*2^(1/2)*arctanh(
1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e
```

```
)*a*c^3+A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(
a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^3+2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a
*(c+d)*d)^(1/2)*sin(f*x+e)*c^2*d+4*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*
(c+d)*d)^(1/2)*sin(f*x+e)*c*d^2-8*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e))
)^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*c^2*d-9*A*2^(1/2)*arctanh(1/2*
(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a*c*d^2-6*B*(
-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^2*d+4*B*(-
a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c*d^2-12*B*a^
(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^3*d-8*B*a^(
3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*d
^4+12*A*a^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin
(f*x+e)^2*d^4+12*A*a^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)
^(1/2))*sin(f*x+e)*d^4-8*B*a^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*
(c+d)*d)^(1/2))*sin(f*x+e)*d^4+2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c
+d)*d)^(1/2)*c^3-4*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*d
^3+3*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(
c+d)*d)^(1/2)*a*c^3+2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2
)*c*d^2-4*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*c^2*d+6*B*
(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*c*d^2+2*B*(-a*(-1+sin(
f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*d^3+32*A*a^(3/2)*arctan
h((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c*d^3-12*B*a^(
3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c
^2*d^2-12*B*a^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))
*sin(f*x+e)^2*c*d^3+20*A*a^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c
+d)*d)^(1/2))*sin(f*x+e)^2*c*d^3-12*B*a^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^
(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^3*d-24*B*a^(3/2)*arctanh((-a*(-1+si
n(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^2*d^2-20*B*a^(3/2)*arcta
nh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c*d^3-6*A*(-a
*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*d^3+A*2^(1/2)*
arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a
*c^3-2*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*c^3+20*A*a^(3
/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^2*d^2+12*A*a^
(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c*d^3-12*B*a^
(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^2*d^2-8*B*a
^(3/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c*d^3)*(-a*(
-1+sin(f*x+e)))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))/(c+d)/(c-d)^3/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 24.6809, size = 7567, normalized size = 25.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d
^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d
^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c
*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^
2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^
3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A
+ 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*
B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*
x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin
(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
 - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(6*B*a*c^3 - 2*(5*A - 6*B)*a*
c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A -
 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B
)*a*c^2*d - (13*A - 8*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3
*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*c
os(f*x + e) + (6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^
3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2
- (3*A - 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d
^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*
((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*c
os(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(
f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(
f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2
- c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*
d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c^3 - (A -
```

```
B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2
 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A + 3*B)*c*d
^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 +
 (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 +
 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2
*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d
 - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*
(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)
*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2
- (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^
5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3
+ a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*
(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)
*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3
 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x +
 e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A -
5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A
- 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9
*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A
 - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2)) - 4*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(
3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*
cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8
*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A -
 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d
- (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5
*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*
sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*((A - B)*
c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A
 + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A
 + 3*B)*c*d^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A -
 B)*c*d^2 + (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^
2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*
d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
- a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*
x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4
 - a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(
f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d
```

```
^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a
^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{\sqrt{d}\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(10c^2d+5c^3+13cd^2+4d^3\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(A(c-13d)+3B(c+3\ldots}{2\sqrt{2}a\ldots}$$

```
[Out] -((A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (Sqrt[d]*(A*d*(
35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*ArcT
anh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])
/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*
f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + (d*(B*(2*c + d) - A*
(c + 2*d))*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x])^2) + (d*(3*B*(3*c^2 + 3*c*d + 2*d^2) - A*(2*c^2 + 15*c
*d + 7*d^2))*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*
x]]*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.56209, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{d}\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(10c^2d+5c^3+13cd^2+4d^3\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}-\frac{(A(c-13d)+3B(c+3\ldots}{2\sqrt{2}a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3
),x]
```


```
[Out] -((A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sin[e + f*x]])])/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (Sqrt[d]*(A*d*(
35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*ArcT
anh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])
/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*
f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + (d*(B*(2*c + d) - A*
(c + 2*d))*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x])^2) + (d*(3*B*(3*c^2 + 3*c*d + 2*d^2) - A*(2*c^2 + 15*c
*d + 7*d^2))*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*
```

```
x]]*(c + d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(Ac+3Bc-8A\ldots}{\sqrt{a+a\sin\ldots}}}{\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2\ldots}{2a(c-d)^2(c+\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2\ldots}{2a(c-d)^2(c+\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2\ldots}{2a(c-d)^2(c+\ldots}$$

$$= -\frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2\ldots}{2a(c-d)^2(c+\ldots}$$

$$= -\frac{(A(c-13d)+3B(c+3d))\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4f} - \frac{\sqrt{d}\left(Ad\left(35c^2\ldots\right.\right.}{\ldots}$$

Mathematica [C] time = 13.2433, size = 1395, normalized size = 3.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^3),x]
```

```
[Out] ((1 + I)*(A*c + 3*B*c - 13*A*d + 9*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[
(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)/((2*(-1)^(1/4)*c^4 - 8*(-1)^(1/4)*c^3*d + 12*(-1)^(1/4)*c
^2*d^2 - 8*(-1)^(1/4)*c*d^3 + 2*(-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(3
/2)) + (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d
+ 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e +
f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2
])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(c + d)^(5/2)*f
*(a*(1 + Sin[e + f*x]))^(3/2)) - (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)
) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*
x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2]
 + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16
*(c - d)^4*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) + ((Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-8*A*c^4*Cos[(e + f*x)/2] + 8*B*c^4*Cos[(e + f*x)/2
] - 8*A*c^3*d*Cos[(e + f*x)/2] + 26*B*c^3*d*Cos[(e + f*x)/2] - 22*A*c^2*d^2
*Cos[(e + f*x)/2] + 6*B*c^2*d^2*Cos[(e + f*x)/2] - 10*A*c*d^3*Cos[(e + f*x)
/2] + 4*B*c*d^3*Cos[(e + f*x)/2] + 4*B*d^4*Cos[(e + f*x)/2] - 8*A*c^3*d*Cos
[(3*(e + f*x))/2] + 26*B*c^3*d*Cos[(3*(e + f*x))/2] - 40*A*c^2*d^2*Cos[(3*(
e + f*x))/2] + 31*B*c^2*d^2*Cos[(3*(e + f*x))/2] - 25*A*c*d^3*Cos[(3*(e + f
*x))/2] + 13*B*c*d^3*Cos[(3*(e + f*x))/2] + A*d^4*Cos[(3*(e + f*x))/2] + 2*
B*d^4*Cos[(3*(e + f*x))/2] + 2*A*c^2*d^2*Cos[(5*(e + f*x))/2] - 9*B*c^2*d^2
*Cos[(5*(e + f*x))/2] + 15*A*c*d^3*Cos[(5*(e + f*x))/2] - 9*B*c*d^3*Cos[(5*
(e + f*x))/2] + 7*A*d^4*Cos[(5*(e + f*x))/2] - 6*B*d^4*Cos[(5*(e + f*x))/2]
 + 8*A*c^4*Sin[(e + f*x)/2] - 8*B*c^4*Sin[(e + f*x)/2] + 8*A*c^3*d*Sin[(e +
 f*x)/2] - 26*B*c^3*d*Sin[(e + f*x)/2] + 22*A*c^2*d^2*Sin[(e + f*x)/2] - 6*
B*c^2*d^2*Sin[(e + f*x)/2] + 10*A*c*d^3*Sin[(e + f*x)/2] - 4*B*c*d^3*Sin[(e
 + f*x)/2] - 4*B*d^4*Sin[(e + f*x)/2] - 8*A*c^3*d*Sin[(3*(e + f*x))/2] + 26
*B*c^3*d*Sin[(3*(e + f*x))/2] - 40*A*c^2*d^2*Sin[(3*(e + f*x))/2] + 31*B*c^
2*d^2*Sin[(3*(e + f*x))/2] - 25*A*c*d^3*Sin[(3*(e + f*x))/2] + 13*B*c*d^3*S
in[(3*(e + f*x))/2] + A*d^4*Sin[(3*(e + f*x))/2] + 2*B*d^4*Sin[(3*(e + f*x)
)/2] - 2*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 9*B*c^2*d^2*Sin[(5*(e + f*x))/2]
- 15*A*c*d^3*Sin[(5*(e + f*x))/2] + 9*B*c*d^3*Sin[(5*(e + f*x))/2] - 7*A*d^
4*Sin[(5*(e + f*x))/2] + 6*B*d^4*Sin[(5*(e + f*x))/2]))/(16*(c - d)^3*(c +
d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c + d*Sin[e + f*x])^2)
```

Maple [B] time = 3.293, size = 4707, normalized size = 11.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] 1/4/a^(7/2)*(-a*(-1+sin(f*x+e)))^(1/2)*(63*A*2^(1/2)*arctanh(1/2*(-a*(-1+si
n(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c^2*d^3-
39*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+
d)*d)^(1/2)*sin(f*x+e)^2*a^2*c*d^4-33*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x
+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^2*c^3*d^2-57*
B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*
d)^(1/2)*sin(f*x+e)^2*a^2*c^2*d^3-3*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e
)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^3*a^2*c^3*d^2-15*B*
2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)
^(1/2)*sin(f*x+e)^3*a^2*c^2*d^3-21*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)
))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^3*a^2*c*d^4+60*B*a^(
5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^4
*d^2+99*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*s
in(f*x+e)*c^3*d^3+90*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d
)*d)^(1/2))*sin(f*x+e)*c^2*d^4+24*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1
/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c*d^5+3*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^
(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*d^5-119*A*a^(5/2)*arctanh((-a*(-1+sin(f*
x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c^2*d^4-80*A*a^(5/2)*arctanh
((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c*d^5+30*B*a^
(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*
c^4*d^2+75*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*sin(f*x+e)^2*c^3*d^3+108*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c^2*d^4-7*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/
2)*(a*(c+d)*d)^(1/2)*c^2*d^3-3*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d
)*d)^(1/2)*c*d^4-70*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)
*d)^(1/2))*sin(f*x+e)^2*c^3*d^3-5*A*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(
c+d)*d)^(1/2)*sin(f*x+e)*d^5+4*B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d
)*d)^(1/2)*sin(f*x+e)*d^5+11*A*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*
d)^(1/2)*c^2*d^3-6*A*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*c
*d^4-A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(
c+d)*d)^(1/2)*a^2*c^5-11*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(
1/2)*c^3*d^2-A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^3
-112*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(
f*x+e)*c^3*d^3-103*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*
d)^(1/2))*sin(f*x+e)*c^2*d^4+2*B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d
)*d)^(1/2)*c^2*d^3+B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*c
*d^4-3*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a
*(c+d)*d)^(1/2)*a^2*c^5-4*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^
(1/2)*sin(f*x+e)*d^5+21*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^
(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^2*c^3*d^2+61*A*2^(1/2)*arct
anh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f
*x+e)^2*a^2*c^2*d^3-6*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1
```

```
/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^2*c^4*d-2*A*2^(1/2)*arctanh(1
/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)
^2*a^2*c^4*d-7*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f
*x+e)*c*d^4-2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*
x+e)^2*c^3*d^2+2*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin
(f*x+e)^2*c^3*d^2+2*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*
sin(f*x+e)^2*c^2*d^3-2*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/
2)*sin(f*x+e)^2*c*d^4-51*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2
^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c^3*d^2-51*B*2^(1/2)*arcta
nh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*
x+e)*a^2*c^2*d^3-18*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2
)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c*d^4+9*A*2^(1/2)*arctanh(1/2*(
-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2
*c^4*d+47*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))
*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c^3*d^2+51*A*2^(1/2)*arctanh(1/2*(-a*(-1+
sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^2*c*d^
4+11*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(
c+d)*d)^(1/2)*sin(f*x+e)^3*a^2*c^2*d^3+25*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin
(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^3*a^2*c*d^4-A
*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d
)^(1/2)*sin(f*x+e)^3*a^2*c^3*d^2-21*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e
)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c^4*d+26*A*2^(1
/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/
2)*sin(f*x+e)*a^2*c*d^4-7*B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^
(1/2)*c^3*d^2-4*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*d^5-
19*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*
x+e)^2*d^6+12*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1
/2))*sin(f*x+e)^2*d^6-42*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*
(c+d)*d)^(1/2))*c^3*d^3-19*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*c^2*d^4+15*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*c^5*d-2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d
)^(1/2)*c^4*d-35*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)
^(1/2))*sin(f*x+e)^3*c^2*d^4+13*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+
d)*d)^(1/2)*c*d^4+11*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)
*c^4*d+B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d^2-42*A*
a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^
3*c*d^5+15*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2)
)*sin(f*x+e)^3*c^3*d^3+30*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a
*(c+d)*d)^(1/2))*sin(f*x+e)^3*c^2*d^4+39*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+
e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^3*c*d^5+63*B*a^(5/2)*arctanh((-a
*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*c*d^5+2*A*(-a*(-1
+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*d^5-35*A*a^(5/2)
*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^4*d^2
-38*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f
*x+e)*c*d^5-2*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*
```

x+e)^2*d^5+15*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*c^5*d-9*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^3*a^2*d^5+13*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^2*d^5+11*A*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^2*d^3-6*A*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c*d^4-A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c^5-7*B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^3*d^2+2*B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^2*d^3+B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c*d^4-3*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a^2*c^5+11*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^4*d+25*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^3*d^2-9*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a^2*d^5-15*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^4*d-21*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^3*d^2-9*B*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^2*d^3+2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*c*d^4-2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*c^2*d^3-17*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^3*d^2+A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^2*d^3+13*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*sin(f*x+e)^3*a^2*d^5-4*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^4*d+13*A*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^2*d^3+17*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c*d^4+13*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^4*d+7*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^3*d^2-9*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*c^2*d^3+30*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^4*d^2+39*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^3*d^3+12*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^2*d^4-2*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*c^5+3*A*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*d^5-35*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*c^4*d^2-19*A*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^3*d^6+12*B*a^(5/2)*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^3*d^6-5*A*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*d^5+4*B*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*(a*(c+d)*d)^(1/2)*d^5+2*B*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*(a*(c+d)*d)^(1/2)*c^5)/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 46.4561, size = 13168, normalized size = 32.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/16*(2*sqrt(2)*(2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c
^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d
^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13
*A - 9*B)*d^5)*cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A - 11*B)*c^3*d^2
 - (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15*B)*c^3*d^2 -
2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B)*d^5)*cos(f*
x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 -
 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c^3*d^2 -
 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d^5 - ((A
 + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13*A - 9*B
)*d^5)*cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3*d^2 - 36*(A
- B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^2 + (
(A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 - 2*(37*A - 3
3*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e))*sin(
f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(
```

56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d
^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*
(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^4 - (30*B*a*c^4*d
 - 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80*A - 63*B)*a*
c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^3 - (15*B*a*c^5 - 5*(7*A - 18*B)*
a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2*d^3 - (202*A
 - 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5)*cos(f*x + e)^2 + (15*B*a*c^5 - 5
*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A - 20*B)*a*c^2*d
^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e) + (30*B*a*c^
5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*
a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 - (15*B*a*c^3*d
^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^
5)*cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2 - (77*A - 69*
B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^2
 + (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*
A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x
 + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d +
7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 -
 c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d +
3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*
cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 +
d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d
 - d^2)*sin(f*x + e))) - 4*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*
d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^
3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^3
 + ((4*A - 13*B)*c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A
 - 4*B)*c*d^4 + 2*(2*A - B)*d^5)*cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11
*B)*c^4*d + (13*A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4
 - (5*A - 6*B)*d^5)*cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A -
 B)*c^3*d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A -
 9*B)*c^3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x
 + e)^2 - ((4*A - 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 -
(17*A - 7*B)*c*d^4 - (3*A - 4*B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a))/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^
5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d -
3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d
^6 + a^2*d^8)*f*cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6
*a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^
7 + 3*a^2*d^8)*f*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4
 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*
a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 -
a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*
x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a

^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c
^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e
) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f
)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(2
3*A - 27*B)*c^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(
13*A - 9*B)*d^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B
)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A -
11*B)*c^3*d^2 - (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B
)*d^5)*cos(f*x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15*
B)*c^3*d^2 - 2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B
)*d^5)*cos(f*x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27
*B)*c^3*d^2 - 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B
)*d^5)*cos(f*x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27
*B)*c^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9
*B)*d^5 - ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4
- (13*A - 9*B)*d^5)*cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3
d^2 - 36(A - B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f
*x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2
- 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f
*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*si
n(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e
) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)*
a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A
- 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a
*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^4 -
(30*B*a*c^4*d - 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80
*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^3 - (15*B*a*c^5 - 5*
(7*A - 18*B)*a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2
*d^3 - (202*A - 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5)*cos(f*x + e)^2 + (1
5*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A -
20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)
+ (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(
23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 -
(15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A
- 12*B)*a*d^5)*cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2
- (77*A - 69*B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*co
s(f*x + e)^2 + (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3
d^2 - 6(23*A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*
d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin
(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x +
e))) - 2*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*
c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 + 13*A*c^2
*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^3 + ((4*A - 13*B)*
c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A - 4*B)*c*d^4 + 2

```
*(2*A - B)*d^5)*cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11*B)*c^4*d + (13*
A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4 - (5*A - 6*B)*d^
5)*cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*
(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 +
13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*cos(f*x + e)^2 - ((4*A
- 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 - (17*A - 7*B)*c*d
^4 - (3*A - 4*B)*d^5)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))
/((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6
- 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 -
4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f*
cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 1
2*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f
*cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6
+ a^2*d^8)*f*cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*
a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*
a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^3 + 2*(a^
2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a
^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d
^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*cos(f*x + e) - 2*(a^2*c^8 -
 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=308

$$\frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{24a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-1\right.\right.}{16\sqrt{2}a^{5/2}f}$$

```
[Out] -((c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTa
nh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*
a^(5/2)*f) + (d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*d^
2))*Cos[e + f*x])/(24*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(9*A*c + 15*B*
c + 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(48*a^3*f) - ((
3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(16*a*
f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^
3)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 1.05919, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$\frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{24a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-1\right.\right.}{16\sqrt{2}a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -((c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTa
nh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*
a^(5/2)*f) + (d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*d^
2))*Cos[e + f*x])/(24*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (d^2*(9*A*c + 15*B*
c + 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(48*a^3*f) - ((
3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(16*a*
f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^
3)/(4*f*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+fx))^2\left(\frac{1}{2}a(3Ac+5Bc+\right.}{(a+a\sin}}{}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)}{}$$

$$= -\frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)}{}$$

$$= \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} - \frac{(3Ac}{}$$

$$= \frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} + \frac{d}{}$$

$$= \frac{d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} + \frac{d}{}$$

$$= -\frac{(c-d)\left(B\left(5c^2+62cd-163d^2\right)+3A\left(c^2+6cd+25d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{}}\right.}{16\sqrt{2}a^{5/2}f}$$

Mathematica [C] time = 1.78719, size = 523, normalized size = 1.7

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((3+3i)(-1)^{3/4}(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)/
2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c - d)
```

```
^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2 + 62
*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4
)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 16*B*d
^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*
I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2*A*d - 5*B
*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[(3*(e + f*x
))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 2.148, size = 1438, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/96*(60*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3+36*B*(a-a*sin(f*x+e))^(1/2)*
a^(3/2)*c^3-18*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3-126*A*(a-a*sin(f*x+e))^
(3/2)*a^(1/2)*d^3-30*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3+342*A*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+342*B*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-1350*B*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+90*A*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+2*sin(f*x+e)*
(9*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+45
*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+17
1*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-2
25*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+19
2*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3+15*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3+171*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d-675*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2+489*B*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+576*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*
c*d^2-384*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-64*B*(a-a*sin(f*x+e))^(3/2)*
a^(1/2)*d^3)+(-9*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*c^3-45*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c^2*d-171*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c*d^2+225*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*d^3-192*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-15*B*2^(1/2)*arctanh(1
```

```
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3-171*B*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+675*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^2-489*B*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^3+64*B*(a-a*sin(f*x+e))^(
3/2)*a^(1/2)*d^3-576*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2+384*B*(a-a*sin(
f*x+e))^(1/2)*a^(3/2)*d^3)*cos(f*x+e)^2+234*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2
)*c^2*d-378*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2+108*A*(a-a*sin(f*x+e))^(
1/2)*a^(3/2)*c^2*d-396*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2-396*B*(a-a*si
n(f*x+e))^(1/2)*a^(3/2)*c^2*d+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2*c^3-450*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*a^2*d^3+30*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*a^2*c^3+978*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*d^3+1836*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2-90*A*(a
-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2*d+234*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d
^2+612*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-1092*B*(a-a*sin(f*x+e))^(1/2)*a
^(3/2)*d^3+46*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3)*(-a*(-1+sin(f*x+e)))^(1
/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{3}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^
(5/2), x)
```

Fricas [B] time = 2.39947, size = 2394, normalized size = 7.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A - 7
5*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d
 + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A + 5
*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3
)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75
*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 12*(5*A
 + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*
B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)
*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*
B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B)*c^2*d - 36*(
A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*d^3)*cos(f*x +
e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A - 53*B)*c*d^2 +
(53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(A - 9*B)*c^2*d -
27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (32*B*d^3*cos(f*x +
 e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c*d^2 - 12*(A - B)*d
^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*((3*A + 5*B)*c^3 + 3
*(5*A - 13*B)*c^2*d - 3*(13*A - 85*B)*c*d^2 + (85*A - 157*B)*d^3)*cos(f*x +
 e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{d^2(A-9B)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e}{4f(a\,\mathrm{si}}$$

```
[Out] -((B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[(Sqrt
[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*
f) - ((c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/(16*a*f*(a + a
*Sin[e + f*x])^(3/2)) + ((A - 9*B)*d^2*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Si
n[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rubi [A] time = 0.578671, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2977, 2968, 3019, 2751, 2649, 206}

$$-\frac{\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}+\frac{d^2(A-9B)\cos(e+fx)}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{(A-B)\cos(e}{4f(a\,\mathrm{si}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -((B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[(Sqrt
[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*
f) - ((c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/(16*a*f*(a + a
*Sin[e + f*x])^(3/2)) + ((A - 9*B)*d^2*Cos[e + f*x])/(4*a^2*f*Sqrt[a + a*Si
n[e + f*x]]) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(4*f*(a + a*Si
n[e + f*x])^(5/2))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
```

```
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{(c+d\sin(e+fx))\left(\frac{1}{2}a(3Ac+5Bc+4A}{(a+a\sin(e}}{4a}$$

$$= -\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\frac{1}{2}ac(3Ac+5Bc+4Ad-4Bd)+\left(-\frac{1}{2}a(}{}}{}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c}{4f(a+a\sin(e}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+f}{4a^2f\sqrt{a+a\sin(e+f}}$$

$$= -\frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+f}{4a^2f\sqrt{a+a\sin(e+f}}$$

$$= -\frac{\left(B\left(5c^2+38cd-75d^2\right)+A\left(3c^2+10cd+19d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx}{\sqrt{2}\sqrt{a+a\sin(e+}}\right.}{16\sqrt{2}a^{5/2}f}$$

Mathematica [C] time = 1.133, size = 544, normalized size = 2.48

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((2+2i)(-1)^{3/4}\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*Cos[(e + f*x)/2] + 3*B*c^
2*Cos[(e + f*x)/2] + 6*A*c*d*Cos[(e + f*x)/2] + 10*B*c*d*Cos[(e + f*x)/2] +
 5*A*d^2*Cos[(e + f*x)/2] - 45*B*d^2*Cos[(e + f*x)/2] - 3*A*c^2*Cos[(3*(e +
 f*x))/2] - 5*B*c^2*Cos[(3*(e + f*x))/2] - 10*A*c*d*Cos[(3*(e + f*x))/2] +
26*B*c*d*Cos[(3*(e + f*x))/2] + 13*A*d^2*Cos[(3*(e + f*x))/2] - 69*B*d^2*Co
s[(3*(e + f*x))/2] + 16*B*d^2*Cos[(5*(e + f*x))/2] + 11*A*c^2*Sin[(e + f*x)
/2] - 3*B*c^2*Sin[(e + f*x)/2] - 6*A*c*d*Sin[(e + f*x)/2] - 10*B*c*d*Sin[(e
 + f*x)/2] - 5*A*d^2*Sin[(e + f*x)/2] + 45*B*d^2*Sin[(e + f*x)/2] + (2 + 2*
I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] +
```

```
Sin[(e + f*x)/2])^4 - 3*A*c^2*Sin[(3*(e + f*x))/2] - 5*B*c^2*Sin[(3*(e + f*
x))/2] - 10*A*c*d*Sin[(3*(e + f*x))/2] + 26*B*c*d*Sin[(3*(e + f*x))/2] + 13
*A*d^2*Sin[(3*(e + f*x))/2] - 69*B*d^2*Sin[(3*(e + f*x))/2] - 16*B*d^2*Sin[
(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 1.888, size = 982, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^2+10*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c*d+19*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*d^2+5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^2*c^2+38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a^2*c*d-75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*d^2+64*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+(-3*A*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-10*A*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-19*A*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-5*B*2^(1/2)*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-64*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)
)*cos(f*x+e)^2+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^2*c^2+20*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*a^2*c*d+38*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*d^2+20*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2+24*A*(a-a*sin(f*x+e))^(1/2)
*a^(3/2)*c*d-44*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-6*A*(a-a*sin(f*x+e))^(
3/2)*a^(1/2)*c^2-20*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d+26*A*(a-a*sin(f*x+
e))^(3/2)*a^(1/2)*d^2+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*c^2+76*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c*d-150*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*d^2+12*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-88*B*c*d*a^(3/2)*(
a-a*sin(f*x+e))^(1/2)+204*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-10*B*(a-a*si
n(f*x+e))^(3/2)*a^(1/2)*c^2+52*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-42*B*d^
2*(a-a*sin(f*x+e))^(3/2)*a^(1/2))*(-a*(-1+sin(f*x+e)))^(1/2)/a^(9/2)/(1+sin
(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^
(5/2), x)
```

Fricas [B] time = 2.33657, size = 1817, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*
cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)*d
^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x +
 e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*
x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2 - ((
3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^2 + 2
*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos
(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*s
in(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 - 4*(A - B)*c
^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d
- (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A - 9*B)*c*d - 9*
(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A - B)*c^2 + 8*
(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d - (13*A
 - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*
cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f +
(a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=151

$$-\frac{(3Ac+5Ad+5Bc+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)(c-d)\cos(e}{4f(a\sin(e+fx)+}$$

```
[Out] -((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*c + 5*B*c + 5*A*d - 13*B*
d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.28746, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2968, 3019, 2750, 2649, 206}

$$-\frac{(3Ac+5Ad+5Bc+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)(c-d)\cos(e}{4f(a\sin(e+fx)+}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] -((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((A - B)*(c - d)*Cos[e
 + f*x])/(4*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*c + 5*B*c + 5*A*d - 13*B*
d)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{Ac+(Bc+Ad)\sin(e+fx)+Bd\sin^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc+5Ad-5Bd)-4aBd\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}dx}{4a^2}$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} +$$

$$= -\frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} -$$

$$= -\frac{(3Ac+5Bc+5Ad+19Bd)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)(c-d)\,c}{4f(a+a\sin(e}$$

Mathematica [C] time = 0.767319, size = 267, normalized size = 1.77

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(3Ac+5Ad+5Bc-13Bd)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2]
- 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*B*
c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 1.544, size = 449, normalized size = 3.

$$-\frac{1}{\left(32+32\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(2\sin\left(fx+e\right)\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a^2\left(3\,Ac+5\,Ad+5\,Bc+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)*cos(f*x+e)^2+6*A*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+10*A*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+20*A*(a-a*sin(f*x+e
))^(1/2)*a^(3/2)*c+12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-6*A*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)*c-10*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d+10*B*2^(1/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+38*B*2^(1/2)*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+12*B*(a-a*sin(f*x+e))^(1/2
)*a^(3/2)*c-44*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-10*B*(a-a*sin(f*x+e))^(3/
2)*a^(1/2)*c+26*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d)*(-a*(-1+sin(f*x+e)))^(1
/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5
/2), x)
```

Fricas [B] time = 2.08343, size = 1343, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*(((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*
d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c + (5*
A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((3*A
+ 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(
f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin
(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2
*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
+ 4*(((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c - 4*(A -
 B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c - 4*(A - B)
*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] sage2
```

3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3A+5B)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

```
[Out] -((3*A + 5*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e +
f*x])^(5/2)) - ((3*A + 5*B)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2
))
```

Rubi [A] time = 0.10725, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(3A+5B)\cos(e+fx)}{16af(a\sin(e+fx)+a)^{3/2}}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((3*A + 5*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(16*Sqrt[2]*a^(5/2)*f) - ((A - B)*Cos[e + f*x])/(4*f*(a + a*Sin[e +
f*x])^(5/2)) - ((3*A + 5*B)*Cos[e + f*x])/(16*a*f*(a + a*Sin[e + f*x])^(3/2
))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{(3A+5B)\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(3A+5B)\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{32a^2}$$

$$= -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(3A+5B)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \cdots\right.}{16a^2f}$$

$$= -\frac{(3A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3A+5B)\cos(e+fx\cdots}{16af(a+a\sin(e+fx)\cdots}$$

Mathematica [C] time = 0.365738, size = 227, normalized size = 1.8

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-(3A+5B)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3+2(3A\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A
+ B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A + 5*B)*Sin[(e + f*x)/2]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2)*(-
1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4))
/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 1.338, size = 279, normalized size = 2.2

$$-\frac{1}{\left(32+32\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(2\sin\left(fx+e\right)\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a^{3}\left(3\,A+5\,B\right)-\sqrt{2}\mathrm{Art}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^3*(3*A+5*B)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a^3*(3*A+5*B)*cos(f*x+e)^2+20*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-6*A*(a-a
*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-10*B*(a-a*si
n(f*x+e))^(3/2)*a^(3/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^3)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)
```

Fricas [B] time = 2.11531, size = 1015, normalized size = 8.06

$$\sqrt{2}\left((3\,A+5\,B)\cos\left(fx+e\right)^3+3\,(3\,A+5\,B)\cos\left(fx+e\right)^2-2\,(3\,A+5\,B)\cos\left(fx+e\right)+\left((3\,A+5\,B)\cos\left(fx+e\right)^2-2\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 -
2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*co
s(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*cos(f
*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(
f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*
a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*B)*cos(
f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e) + a))/(
a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}-\frac{2d^{3/2}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}$$

```
[Out] -((B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(Sqrt[
a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]])/(16*Sqrt[2]*a^(5/2)*(
c - d)^3*f) - (2*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])
/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(a^(5/2)*(c - d)^3*Sqrt[c + d]*f)
 - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*
c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e +
f*x])^(3/2))
```

Rubi [A] time = 0.984072, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2978, 2985, 2649, 206, 2773, 208}

$$-\frac{\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{16\sqrt{2}a^{5/2}f(c-d)^3}-\frac{2d^{3/2}(Bc-Ad)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f(c-d)^3\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])),
x]
```



```
[Out] -((B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(Sqrt[
a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]])/(16*Sqrt[2]*a^(5/2)*(
c - d)^3*f) - (2*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])
/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(a^(5/2)*(c - d)^3*Sqrt[c + d]*f)
 - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)) - ((3*A*
c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e +
f*x])^(3/2))
```


Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-8Ad)-\frac{3}{2}a(A-B)d\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,d}{4a^2(c-d)}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+f}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+f}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+f}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{\left(B\left(5c^2-34cd-3d^2\right)+A\left(3c^2-14cd+43d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+f}}\right.}{16\sqrt{2}a^{5/2}(c-d)^3f}$$

Mathematica [C] time = 5.34184, size = 550, normalized size = 2.11

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2
] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*
(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2) + A
*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f
*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (8*d^(3/2)*(-(B*c) + A*d
)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c +
 d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4)/Sqrt[c + d] + (8*d^(3/2)*(B*c - A*d)*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]
*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e
```

```
+ f*x)/2])^4)/Sqrt[c + d]))/(16*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [B] time = 2.361, size = 1418, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] 1/32*(2*sin(f*x+e)*(64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a^(5/2)*d^3-64*B*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)
^(1/2))*c*d^2-3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*(a*(c+d)*d)^(1/2)*a^2*c^2+14*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c*d-43*A*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*2^(1/2)*a^2*d^2-5*B*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^2
+34*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*
d)^(1/2)*a^2*c*d+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a
*(c+d)*d)^(1/2)*2^(1/2)*a^2*d^2)+(-64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+64*B*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*
d/(a*c*d+a*d^2)^(1/2))*c*d^2+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^2-14*A*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c*d+43*A*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*2^(1/2)*a^2*d^2
+5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d
)^(1/2)*a^2*c^2-34*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*(a*(c+d)*d)^(1/2)*a^2*c*d-3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*2^(1/2)*a^2*d^2)*cos(f*x+e)^2+128*A*arctanh(
(a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3-128*B*a^(5/2)*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d^2-6*A*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*a^2*c^2+28*
A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(
1/2)*a^2*c*d-86*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c
+d)*d)^(1/2)*2^(1/2)*a^2*d^2-20*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*
a^(3/2)*c^2+72*A*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c*d-52*A*
a^(3/2)*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*d^2+6*A*(a*(c+d)*d)^(1/2)*
(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-28*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^
(3/2)*a^(1/2)*c*d+22*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2
-10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*
d)^(1/2)*a^2*c^2+68*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*(a*(c+d)*d)^(1/2)*a^2*c*d+6*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
```

```
1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)*2^(1/2)*a^2*d^2-12*B*a^(3/2)*(a-a*sin(f*x+e
))^(1/2)*(a*(c+d)*d)^(1/2)*c^2-8*B*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*
d)^(1/2)*c*d+20*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^2+10*B
*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-4*B*(a*(c+d)*d)^(1/2)
*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-6*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^
(3/2)*a^(1/2)*d^2)*(-a*(-1+sin(f*x+e)))^(1/2)/a^(9/2)/(a*(c+d)*d)^(1/2)/(1+
sin(f*x+e))/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 52.8022, size = 5966, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*(((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*c
os(f*x + e)^3 - 4*(3*A + 5*B)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2
 + 3*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)
^2 - 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x +
e) - (4*(3*A + 5*B)*c^2 - 8*(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A +
 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A
 + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*s
qrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x +
e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) - 32*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)
*cos(f*x + e)^3 - 3*(B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d
```

```
^2)*cos(f*x + e) + (4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e
)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))
*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2
*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^
2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^
2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x
+ e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2
 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^
2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (1
1*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A - 3*B)*c*d + (15*
A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2
 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2
- a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*
d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f
*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f + ((a^3
*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 -
 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c
^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e)), 1/64*(sqrt(2)*(((3*A + 5*B)
*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^3 - 4*(3*A + 5*B
)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d^2 + 3*((3*A + 5*B)*c^2 - 2*(7
*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 - 2*((3*A + 5*B)*c^2 - 2*
(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e) - (4*(3*A + 5*B)*c^2 - 8*
(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c
*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)
*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*
x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f
*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a
)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) +
64*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d
 - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*
d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*
cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e)))
 + 4*(4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*
(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A
 - 3*B)*c*d + (15*A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c
*d + 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^
2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*
c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e
```

```
)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Timed out
```

3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=395

$$-\frac{d\left(A\left(3c^2-16cd-35d^2\right)+B\left(5c^2+32cd+11d^2\right)\right)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{d^{3/2}\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\tanh^{-1}\left(\ldots\right.}{a^{5/2}f(c-d)^4(c+d)^{3/2}}$$

```
[Out] -((B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2))*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)
*(c - d)^4*f) + (d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2))*ArcT
anh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])
/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*
(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - ((3*A*c + 5*B*c - 15*A*d
 + 7*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])) - (d*(A*(3*c^2 - 16*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 11*
d^2))*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c
 + d*Sin[e + f*x]))
```

Rubi [A] time = 1.53571, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d\left(A\left(3c^2-16cd-35d^2\right)+B\left(5c^2+32cd+11d^2\right)\right)\cos(e+fx)}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}+\frac{d^{3/2}\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\tanh^{-1}\left(\ldots\right.}{a^{5/2}f(c-d)^4(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^2
),x]
```

```
[Out] -((B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2))*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)
*(c - d)^4*f) + (d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2))*ArcT
anh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])
/(a^(5/2)*(c - d)^4*(c + d)^(3/2)*f) - ((A - B)*Cos[e + f*x])/(4*(c - d)*f*
(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])) - ((3*A*c + 5*B*c - 15*A*d
 + 7*B*d)*Cos[e + f*x])/(16*a*(c - d)^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d
*Sin[e + f*x])) - (d*(A*(3*c^2 - 16*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 11*
d^2))*Cos[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c
```

```
 + d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-10\cdots}{(a+a\sin(e\cdots}}{\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))} - \frac{(3Ac+5\cdots}{16a(c-d)^2f(a+\cdots}$$

$$= -\frac{\left(B\left(5c^2-58cd-43d^2\right)+A\left(3c^2-22cd+115d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(\cdots}{\sqrt{2}\sqrt{a+a\sin\cdots}}\right.}{16\sqrt{2}a^{5/2}(c-d)^4f}$$

Mathematica [C] time = 12.2848, size = 1318, normalized size = 3.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^2),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 22*A*c*d - 58*B*c*d + 115*A*d^2 - 43*B*d^2)*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e +
 f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^4 - 6
4*(-1)^(1/4)*c^3*d + 96*(-1)^(1/4)*c^2*d^2 - 64*(-1)^(1/4)*c*d^3 + 16*(-1)^
(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(A*d*(7*c + 5*d) - B*
(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(
e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*
x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2
)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-(A*d*(7*c + 5*d)) + B*(5*c^2
 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*
x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^4*(c + d)^(3/2)*f*(a
*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-22*A
*c^3*Cos[(e + f*x)/2] + 6*B*c^3*Cos[(e + f*x)/2] + 40*A*c^2*d*Cos[(e + f*x)
/2] - 40*B*c^2*d*Cos[(e + f*x)/2] + 54*A*c*d^2*Cos[(e + f*x)/2] - 70*B*c*d^
2*Cos[(e + f*x)/2] + 24*A*d^3*Cos[(e + f*x)/2] + 8*B*d^3*Cos[(e + f*x)/2] -
 6*A*c^3*Cos[(3*(e + f*x))/2] - 10*B*c^3*Cos[(3*(e + f*x))/2] + 21*A*c^2*d*
Cos[(3*(e + f*x))/2] - 29*B*c^2*d*Cos[(3*(e + f*x))/2] + 54*A*c*d^2*Cos[(3*
(e + f*x))/2] - 86*B*c*d^2*Cos[(3*(e + f*x))/2] + 75*A*d^3*Cos[(3*(e + f*x)
)/2] - 19*B*d^3*Cos[(3*(e + f*x))/2] + 3*A*c^2*d*Cos[(5*(e + f*x))/2] + 5*B
*c^2*d*Cos[(5*(e + f*x))/2] - 16*A*c*d^2*Cos[(5*(e + f*x))/2] + 32*B*c*d^2*
Cos[(5*(e + f*x))/2] - 35*A*d^3*Cos[(5*(e + f*x))/2] + 11*B*d^3*Cos[(5*(e +
 f*x))/2] + 22*A*c^3*Sin[(e + f*x)/2] - 6*B*c^3*Sin[(e + f*x)/2] - 40*A*c^2
*d*Sin[(e + f*x)/2] + 40*B*c^2*d*Sin[(e + f*x)/2] - 54*A*c*d^2*Sin[(e + f*x
)/2] + 70*B*c*d^2*Sin[(e + f*x)/2] - 24*A*d^3*Sin[(e + f*x)/2] - 8*B*d^3*Si
n[(e + f*x)/2] - 6*A*c^3*Sin[(3*(e + f*x))/2] - 10*B*c^3*Sin[(3*(e + f*x))/
2] + 21*A*c^2*d*Sin[(3*(e + f*x))/2] - 29*B*c^2*d*Sin[(3*(e + f*x))/2] + 54
*A*c*d^2*Sin[(3*(e + f*x))/2] - 86*B*c*d^2*Sin[(3*(e + f*x))/2] + 75*A*d^3*
Sin[(3*(e + f*x))/2] - 19*B*d^3*Sin[(3*(e + f*x))/2] - 3*A*c^2*d*Sin[(5*(e
+ f*x))/2] - 5*B*c^2*d*Sin[(5*(e + f*x))/2] + 16*A*c*d^2*Sin[(5*(e + f*x))/
2] - 32*B*c*d^2*Sin[(5*(e + f*x))/2] + 35*A*d^3*Sin[(5*(e + f*x))/2] - 11*B
*d^3*Sin[(5*(e + f*x))/2]))/(64*(c - d)^3*(c + d)*f*(a*(1 + Sin[e + f*x]))^
(5/2)*(c + d*Sin[e + f*x]))
```

Maple [B] time = 3.569, size = 4092, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/32/a^(9/2)*(53*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)
))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^2+255*B*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*
a^2*c^2*d^2+19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^2-167*A*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^
2*c^2*d^2+187*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^
(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3+84*B*(a*(c+d)*d)^(1/2)*(-a*(-1+
sin(f*x+e)))^(1/2)*a^(3/2)*c*d^3-160*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d
/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^2-480*B*arctanh((-a*(-1+sin(
f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3-384*B*arct
anh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*
d^4-32*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*sin(f*x+e)^2*
d^4+448*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*s
in(f*x+e)*c^2*d^3+22*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)
*c^3*d-10*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*c^2*d^2-22
*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*c*d^3-5*B*(a*(c+d)*
d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^
2*c^4-480*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)
*sin(f*x+e)*c^2*d^3-288*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^
(1/2))*a^(5/2)*sin(f*x+e)*c*d^4-148*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e))
)^(1/2)*a^(3/2)*sin(f*x+e)*d^4+52*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^
(1/2)*a^(3/2)*sin(f*x+e)*d^4-160*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*
(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c^2*d^3-160*B*arctanh((-a*(-1+sin(f*x+
e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-320*B*arctanh((-
a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^2+22
4*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x
+e)^2*c^2*d^3+608*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)^2*c*d^4+38*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/
2)*a^(1/2)*sin(f*x+e)*d^4-22*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)
*a^(1/2)*sin(f*x+e)*d^4+544*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)
*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4-6*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e
)))^(3/2)*a^(1/2)*c^2*d^2+38*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)
*a^(1/2)*c*d^3-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)
))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4+84*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)
))^(1/2)*a^(3/2)*c^3*d+20*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^
(3/2)*c^2*d^2-52*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*c*d
^3-52*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*c^3*d-20*B*(a*
(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*c^2*d^2+224*A*arctanh((-a
*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-38*
A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*c^3*d-3*A*(a*(c+d)*d
)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin
```

```
(f*x+e)^2*a^2*c^4-10*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*
x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+43*B*(a*(c+d)*d)^(1/2)*2^(
1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2
*d^4+19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*
2^(1/2)/a^(1/2))*a^2*c^3*d-93*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d^2-12*B*(a*(c+d)*d)^(1/2)*(
-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*sin(f*x+e)*c^3*d-116*B*(a*(c+d)*d)^(1/2)*
(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^2-38*A*(a*(c+d)*d)^(1/2
)*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*sin(f*x+e)*c^2*d^2-230*A*(a*(c+d)*d)^(
1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*
x+e)^2*a^2*d^4+53*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e
)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^3*d+76*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x
+e)))^(1/2)*a^(3/2)*sin(f*x+e)*c*d^3-6*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+
e)))^(3/2)*a^(1/2)*sin(f*x+e)*c*d^3-6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1
/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^4+207*B*(a*
(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/
2))*sin(f*x+e)^2*a^2*c^2*d^2-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*
(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^3*d-3*A*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*s
in(f*x+e)^3*a^2*c^3*d+245*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+s
in(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^3+35*A*(a*(c+d)*d)^
(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f
*x+e)*a^2*c^3*d+101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x
+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c*d^3+13*A*(a*(c+d)*d)^(1/2)*
2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^
2*a^2*c^3*d-55*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^2*d^2-301*A*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*
a^2*c*d^3+43*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^3*d-93*A*(a*(c+d)*d)^(1/2)*2^(1/2)
*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c
*d^3-323*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)
*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3+101*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^3*d-64
*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+
e)^3*d^5+224*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5
/2)*c^2*d^3+160*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*c*d^4-160*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*
a^(5/2)*c^3*d^2-160*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2
))*a^(5/2)*c^2*d^3-64*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1
/2))*a^(5/2)*c*d^4-20*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2
)*c^4-32*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*d^4-115*A*(
a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(
1/2))*sin(f*x+e)*a^2*d^4+10*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*
a^(1/2)*sin(f*x+e)*c^3*d+22*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*
```

```
a^(1/2)*sin(f*x+e)*c^2*d^2-10*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2
)*a^(1/2)*sin(f*x+e)*c*d^3+43*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(
-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3-115*A*(a*(c+d)*d)^(1/2)*2^
(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*
a^2*d^4-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2
)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c^4+86*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arct
anh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*d^4+6*
A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3/2)*a^(1/2)*sin(f*x+e)*c^3*d+101
*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)
/a^(1/2))*a^2*c^2*d^2+84*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(
3/2)*sin(f*x+e)*c^2*d^2+84*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a
^(3/2)*sin(f*x+e)*c*d^3-20*A*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a
^(3/2)*sin(f*x+e)*c^3*d-115*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(-1
+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d^3-32*B*(a*(c+d)*d)^(1/2)*(-a*(
-1+sin(f*x+e)))^(1/2)*a^(3/2)*sin(f*x+e)^2*c^2*d^2+32*B*(a*(c+d)*d)^(1/2)*(
-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3+32*A*(a*(c+d)*d)^(1/2)
*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*sin(f*x+e)^2*c*d^3+43*B*(a*(c+d)*d)^(1/
2)*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^3*a^2*d^4-12*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(1/2)*a^(3/2)*c^4+
160*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f
*x+e)*d^5-64*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5
/2)*sin(f*x+e)*d^5+320*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(5/2)*sin(f*x+e)^2*d^5-128*B*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5+6*A*(a*(c+d)*d)^(1/2)*(-a*(-1+si
n(f*x+e)))^(3/2)*a^(1/2)*c^4+10*B*(a*(c+d)*d)^(1/2)*(-a*(-1+sin(f*x+e)))^(3
/2)*a^(1/2)*c^4+160*A*arctanh((-a*(-1+sin(f*x+e)))^(1/2)*d/(a*(c+d)*d)^(1/2
))*a^(5/2)*sin(f*x+e)^3*d^5)*(-a*(-1+sin(f*x+e)))^(1/2)/(a*(c+d)*d)^(1/2)/(
c+d*sin(f*x+e))/(c+d)/(1+sin(f*x+e))/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1
/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 98.914, size = 11614, normalized size = 29.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*(4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c
^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d
- (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*
x + e)^4 - ((3*A + 5*B)*c^4 - (13*A + 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2
+ 7*(43*A - 35*B)*c*d^3 + 2*(115*A - 43*B)*d^4)*cos(f*x + e)^3 - (3*(3*A +
5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317
*B)*c*d^3 + 5*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*
(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A
- 43*B)*d^4)*cos(f*x + e) + (4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37
*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 - ((3*A +
 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)
*d^4)*cos(f*x + e)^3 - ((3*A + 5*B)*c^4 - 2*(5*A + 19*B)*c^3*d + 4*(9*A - 6
5*B)*c^2*d^2 + 2*(197*A - 173*B)*c*d^3 + 3*(115*A - 43*B)*d^4)*cos(f*x + e)
^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16
*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(
a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos
(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*s
in(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2)) - 16*(20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*
B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 + (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (
5*A - 2*B)*a*d^4)*cos(f*x + e)^4 - (5*B*a*c^3*d - (7*A - 15*B)*a*c^2*d^2 -
(19*A - 12*B)*a*c*d^3 - 2*(5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (15*B*a*c^3*d
 - (21*A - 40*B)*a*c^2*d^2 - (50*A - 31*B)*a*c*d^3 - 5*(5*A - 2*B)*a*d^4)*c
os(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*
d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e) + (20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^
2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 - (5*B*a*c^2*d^2 - (7*
A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^3 - (5*B*a*c^3*d - (7*A
- 20*B)*a*c^2*d^2 - (26*A - 17*B)*a*c*d^3 - 3*(5*A - 2*B)*a*d^4)*cos(f*x +
e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*
A - 2*B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*co
s(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d
 + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*
x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x +
 e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x +
```

```
e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos
(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(4*(A - B)*c^4 - 8*(A - B
)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27
*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*cos(f*x + e)^3 + ((3
*A + 5*B)*c^4 - (15*A - 7*B)*c^3*d - (7*A - 15*B)*c^2*d^2 - (A + 23*B)*c*d^
3 + 4*(5*A - B)*d^4)*cos(f*x + e)^2 + ((7*A + B)*c^4 - 20*(A - B)*c^3*d - 2
*(13*A - 21*B)*c^2*d^2 - 4*(3*A + 13*B)*c*d^3 + (51*A - 11*B)*d^4)*cos(f*x
+ e) - (4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 -
 ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A -
 11*B)*d^4)*cos(f*x + e)^2 - ((3*A + 5*B)*c^4 - 12*(A - B)*c^3*d - 2*(13*A
- 21*B)*c^2*d^2 - 4*(5*A + 11*B)*c*d^3 + 5*(11*A - 3*B)*d^4)*cos(f*x + e))*
sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d^2 + 2*a^3
*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^4 - (a^3*c
^6 - a^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*a^3*c*d^5
+ 2*a^3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*c^4*d^2 +
16*a^3*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*x + e)^2 +
 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a
^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2
 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3*c^5*d - 3
*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos
(f*x + e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^2*d^4 - 8*
a^3*c*d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^
4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)
 - 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2
*a^3*c*d^5 + a^3*d^6)*f)*sin(f*x + e)), -1/64*(sqrt(2)*(4*(3*A + 5*B)*c^4 -
 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(
115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 10
1*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e)^4 - ((3*A + 5*B)*c^4 - (13*A
+ 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2 + 7*(43*A - 35*B)*c*d^3 + 2*(115*A -
 43*B)*d^4)*cos(f*x + e)^3 - (3*(3*A + 5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8
*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317*B)*c*d^3 + 5*(115*A - 43*B)*d^4)*co
s(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^
2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e) + (4*(3*A
+ 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)
*c*d^3 + 4*(115*A - 43*B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2
+ (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*cos(f*x + e)^3 - ((3*A + 5*B)*
c^4 - 2*(5*A + 19*B)*c^3*d + 4*(9*A - 65*B)*c^2*d^2 + 2*(197*A - 173*B)*c*d
^3 + 3*(115*A - 43*B)*d^4)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*
B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A - 43*B)
*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2
)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*
cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 -
 (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 32*(20*B*a*c^3*d -
4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 + (
```

```
5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e)^4 - (
5*B*a*c^3*d - (7*A - 15*B)*a*c^2*d^2 - (19*A - 12*B)*a*c*d^3 - 2*(5*A - 2*B
)*a*d^4)*cos(f*x + e)^3 - (15*B*a*c^3*d - (21*A - 40*B)*a*c^2*d^2 - (50*A -
 31*B)*a*c*d^3 - 5*(5*A - 2*B)*a*d^4)*cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*
A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e
) + (20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(
5*A - 2*B)*a*d^4 - (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4
)*cos(f*x + e)^3 - (5*B*a*c^3*d - (7*A - 20*B)*a*c^2*d^2 - (26*A - 17*B)*a*
c*d^3 - 3*(5*A - 2*B)*a*d^4)*cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)
*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*cos(f*x + e))*sin(f*
x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x
 + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) + 4*(4*(A - B)*c^4
- 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A - B)*d^4 - ((3*A + 5*B)*c^3*d -
(19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*cos(f*x +
e)^3 + ((3*A + 5*B)*c^4 - (15*A - 7*B)*c^3*d - (7*A - 15*B)*c^2*d^2 - (A +
23*B)*c*d^3 + 4*(5*A - B)*d^4)*cos(f*x + e)^2 + ((7*A + B)*c^4 - 20*(A - B)
*c^3*d - 2*(13*A - 21*B)*c^2*d^2 - 4*(3*A + 13*B)*c*d^3 + (51*A - 11*B)*d^4
)*cos(f*x + e) - (4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c*d^3 - 4*(A
- B)*d^4 - ((3*A + 5*B)*c^3*d - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3
 + (35*A - 11*B)*d^4)*cos(f*x + e)^2 - ((3*A + 5*B)*c^4 - 12*(A - B)*c^3*d
- 2*(13*A - 21*B)*c^2*d^2 - 4*(5*A + 11*B)*c*d^3 + 5*(11*A - 3*B)*d^4)*cos(
f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^5*d - 3*a^3*c^4*d
^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e)^
4 - (a^3*c^6 - a^3*c^5*d - 4*a^3*c^4*d^2 + 6*a^3*c^3*d^3 + a^3*c^2*d^4 - 5*
a^3*c*d^5 + 2*a^3*d^6)*f*cos(f*x + e)^3 - (3*a^3*c^6 - 4*a^3*c^5*d - 9*a^3*
c^4*d^2 + 16*a^3*c^3*d^3 + a^3*c^2*d^4 - 12*a^3*c*d^5 + 5*a^3*d^6)*f*cos(f*
x + e)^2 + 2*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2
*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*cos(f*x + e) + 4*(a^3*c^6 - 2*a^3*c^5*d - a
^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f - ((a^3
*c^5*d - 3*a^3*c^4*d^2 + 2*a^3*c^3*d^3 + 2*a^3*c^2*d^4 - 3*a^3*c*d^5 + a^3*
d^6)*f*cos(f*x + e)^3 + (a^3*c^6 - 7*a^3*c^4*d^2 + 8*a^3*c^3*d^3 + 3*a^3*c^
2*d^4 - 8*a^3*c*d^5 + 3*a^3*d^6)*f*cos(f*x + e)^2 - 2*(a^3*c^6 - 2*a^3*c^5*
d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f*co
s(f*x + e) - 4*(a^3*c^6 - 2*a^3*c^5*d - a^3*c^4*d^2 + 4*a^3*c^3*d^3 - a^3*c
^2*d^4 - 2*a^3*c*d^5 + a^3*d^6)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=519

$$-\frac{d\left(3A\left(-7c^2d+c^3-37cd^2-21d^3\right)+B\left(73c^2d+5c^3+79cd^2+35d^3\right)\right)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{d\left(A\left(3c^2-20cd-31d^2\right)+B\left(5c^2\right.\right.}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+\right.}$$

```
[Out] -((B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d + 73*d^2))*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)
*(c - d)^5*f) + (d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2) - B*(35*c^3 + 70
*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - (
(A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e
+ f*x])^2) - ((3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])/(16*a*(c - d)
^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(A*(3*c^2 - 20
*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (d*(3*A*(c^
3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^3)
)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c +
 d*Sin[e + f*x]))
```

Rubi [A] time = 2.15228, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$-\frac{d\left(3A\left(-7c^2d+c^3-37cd^2-21d^3\right)+B\left(73c^2d+5c^3+79cd^2+35d^3\right)\right)\cos(e+fx)}{16a^2f(c-d)^4(c+d)^2\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{d\left(A\left(3c^2-20cd-31d^2\right)+B\left(5c^2\right.\right.}{16a^2f(c-d)^3(c+d)\sqrt{a\sin(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^3
),x]
```


```
[Out] -((B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d + 73*d^2))*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(16*Sqrt[2]*a^(5/2)
*(c - d)^5*f) + (d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2) - B*(35*c^3 + 70
*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*a^(5/2)*(c - d)^5*(c + d)^(5/2)*f) - (
(A - B)*Cos[e + f*x])/(4*(c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e
```

```
+ f*x])^2) - ((3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])/(16*a*(c - d)
^2*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) - (d*(A*(3*c^2 - 20
*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x])/(16*a^2*(c - d)
^3*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) - (d*(3*A*(c^
3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^3)
)*Cos[e + f*x])/(16*a^2*(c - d)^4*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c +
 d*Sin[e + f*x]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-1}{(a+a\sin(}}{}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5}{16a(c-d)^2f(a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5}{16a(c-d)^2f(a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5}{16a(c-d)^2f(a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5}{16a(c-d)^2f(a}$$

$$= -\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} - \frac{(3Ac+5}{16a(c-d)^2f(a}$$

$$= -\frac{\left(B\left(5c^2-82cd-115d^2\right)+3A\left(c^2-10cd+73d^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{a}\cos(}{\sqrt{2}\sqrt{a+a\sin}}\right.}{16\sqrt{2}a^{5/2}(c-d)^5f}$$

Mathematica [C] time = 13.8827, size = 2103, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f
*x])^3),x]
```

```
[Out] ((1 + I)*(3*A*c^2 + 5*B*c^2 - 30*A*c*d - 82*B*c*d + 219*A*d^2 - 115*B*d^2)*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^5 -
80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 - 160*(-1)^(1/4)*c^2*d^3 + 80*
(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 + Sin[e + f*x]))^(5/2)) - (d^
(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2
+ 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*
(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[
```

```
(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5/2)*f*(a*(1 + S
in[e + f*x]))^(5/2)) + (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*
c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]
*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5
*(c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + ((Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(-44*A*c^5*Cos[(e + f*x)/2] + 12*B*c^5*Cos[(e + f*x)/2] + 84*A
*c^4*d*Cos[(e + f*x)/2] - 116*B*c^4*d*Cos[(e + f*x)/2] + 249*A*c^3*d^2*Cos[
(e + f*x)/2] - 433*B*c^3*d^2*Cos[(e + f*x)/2] + 385*A*c^2*d^3*Cos[(e + f*x)
/2] - 277*B*c^2*d^3*Cos[(e + f*x)/2] + 239*A*c*d^4*Cos[(e + f*x)/2] - 95*B*
c*d^4*Cos[(e + f*x)/2] + 47*A*d^5*Cos[(e + f*x)/2] - 51*B*d^5*Cos[(e + f*x)
/2] - 12*A*c^5*Cos[(3*(e + f*x))/2] - 20*B*c^5*Cos[(3*(e + f*x))/2] + 40*A*
c^4*d*Cos[(3*(e + f*x))/2] - 104*B*c^4*d*Cos[(3*(e + f*x))/2] + 261*A*c^3*d
^2*Cos[(3*(e + f*x))/2] - 581*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 781*A*c^2*d^
3*Cos[(3*(e + f*x))/2] - 665*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 579*A*c*d^4*C
os[(3*(e + f*x))/2] - 299*B*c*d^4*Cos[(3*(e + f*x))/2] + 79*A*d^5*Cos[(3*(e
 + f*x))/2] - 59*B*d^5*Cos[(3*(e + f*x))/2] + 12*A*c^4*d*Cos[(5*(e + f*x))/
2] + 20*B*c^4*d*Cos[(5*(e + f*x))/2] - 73*A*c^3*d^2*Cos[(5*(e + f*x))/2] +
217*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 353*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 3
97*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 419*A*c*d^4*Cos[(5*(e + f*x))/2] + 251*
B*c*d^4*Cos[(5*(e + f*x))/2] - 127*A*d^5*Cos[(5*(e + f*x))/2] + 75*B*d^5*Co
s[(5*(e + f*x))/2] + 3*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 5*B*c^3*d^2*Cos[(7*
(e + f*x))/2] - 21*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 73*B*c^2*d^3*Cos[(7*(e
+ f*x))/2] - 111*A*c*d^4*Cos[(7*(e + f*x))/2] + 79*B*c*d^4*Cos[(7*(e + f*x)
)/2] - 63*A*d^5*Cos[(7*(e + f*x))/2] + 35*B*d^5*Cos[(7*(e + f*x))/2] + 44*A
*c^5*Sin[(e + f*x)/2] - 12*B*c^5*Sin[(e + f*x)/2] - 84*A*c^4*d*Sin[(e + f*x
)/2] + 116*B*c^4*d*Sin[(e + f*x)/2] - 249*A*c^3*d^2*Sin[(e + f*x)/2] + 433*
B*c^3*d^2*Sin[(e + f*x)/2] - 385*A*c^2*d^3*Sin[(e + f*x)/2] + 277*B*c^2*d^3
*Sin[(e + f*x)/2] - 239*A*c*d^4*Sin[(e + f*x)/2] + 95*B*c*d^4*Sin[(e + f*x)
/2] - 47*A*d^5*Sin[(e + f*x)/2] + 51*B*d^5*Sin[(e + f*x)/2] - 12*A*c^5*Sin[
(3*(e + f*x))/2] - 20*B*c^5*Sin[(3*(e + f*x))/2] + 40*A*c^4*d*Sin[(3*(e + f
*x))/2] - 104*B*c^4*d*Sin[(3*(e + f*x))/2] + 261*A*c^3*d^2*Sin[(3*(e + f*x)
)/2] - 581*B*c^3*d^2*Sin[(3*(e + f*x))/2] + 781*A*c^2*d^3*Sin[(3*(e + f*x))
/2] - 665*B*c^2*d^3*Sin[(3*(e + f*x))/2] + 579*A*c*d^4*Sin[(3*(e + f*x))/2]
 - 299*B*c*d^4*Sin[(3*(e + f*x))/2] + 79*A*d^5*Sin[(3*(e + f*x))/2] - 59*B*
d^5*Sin[(3*(e + f*x))/2] - 12*A*c^4*d*Sin[(5*(e + f*x))/2] - 20*B*c^4*d*Sin
[(5*(e + f*x))/2] + 73*A*c^3*d^2*Sin[(5*(e + f*x))/2] - 217*B*c^3*d^2*Sin[(
5*(e + f*x))/2] + 353*A*c^2*d^3*Sin[(5*(e + f*x))/2] - 397*B*c^2*d^3*Sin[(5
*(e + f*x))/2] + 419*A*c*d^4*Sin[(5*(e + f*x))/2] - 251*B*c*d^4*Sin[(5*(e +
 f*x))/2] + 127*A*d^5*Sin[(5*(e + f*x))/2] - 75*B*d^5*Sin[(5*(e + f*x))/2]
+ 3*A*c^3*d^2*Sin[(7*(e + f*x))/2] + 5*B*c^3*d^2*Sin[(7*(e + f*x))/2] - 21*
A*c^2*d^3*Sin[(7*(e + f*x))/2] + 73*B*c^2*d^3*Sin[(7*(e + f*x))/2] - 111*A*
c*d^4*Sin[(7*(e + f*x))/2] + 79*B*c*d^4*Sin[(7*(e + f*x))/2] - 63*A*d^5*Sin
[(7*(e + f*x))/2] + 35*B*d^5*Sin[(7*(e + f*x))/2]))/(128*(c - d)^4*(c + d)^
2*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c + d*Sin[e + f*x])^2)
```

Maple [B] time = 5.305, size = 7322, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 61.2559, size = 19733, normalized size = 38.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*(4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(117*A - 413*B
)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(4
23*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A
+ 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A
 - 115*B)*d^6)*cos(f*x + e)^5 + (2*(3*A + 5*B)*c^5*d - 3*(13*A + 43*B)*c^4*
d^2 + 4*(63*A - 191*B)*c^3*d^3 + 6*(217*A - 241*B)*c^2*d^4 + 2*(831*A - 583
```

*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*cos(f*x + e)^4 - ((3*A + 5*B)*c^6 - 4*(3
*A + 13*B)*c^5*d + (75*A - 547*B)*c^4*d^2 + 8*(123*A - 203*B)*c^3*d^3 + 19*
(123*A - 115*B)*c^2*d^4 + 4*(525*A - 349*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*
cos(f*x + e)^3 - (3*(3*A + 5*B)*c^6 - 2*(21*A + 83*B)*c^5*d + (267*A - 1507
*B)*c^4*d^2 + 4*(669*A - 1045*B)*c^3*d^3 + (5871*A - 5383*B)*c^2*d^4 + 2*(2
523*A - 1667*B)*c*d^5 + 7*(219*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5
*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A - 413*B)*c^4*d^2 + 4*(177*A - 233*B
)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (219*A -
115*B)*d^6)*cos(f*x + e) + (4*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(1
17*A - 413*B)*c^4*d^2 + 16*(177*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^
2*d^4 + 8*(423*A - 271*B)*c*d^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*
d^2 - 24*(A + 3*B)*c^3*d^3 + 2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*
d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^4 - 2*((3*A + 5*B)*c^5*d - (21*A +
67*B)*c^4*d^2 + 2*(69*A - 173*B)*c^3*d^3 + 2*(285*A - 293*B)*c^2*d^4 + (627
*A - 427*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^3 - ((3*A + 5*B)*c^6
- 6*(A + 7*B)*c^5*d + 3*(11*A - 227*B)*c^4*d^2 + 12*(105*A - 193*B)*c^3*d^3
+ 3*(1159*A - 1119*B)*c^2*d^4 + 6*(559*A - 375*B)*c*d^5 + 5*(219*A - 115*B
)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A
- 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 +
2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e))*sin(f*x + e))*
sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2
*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2)) - 4*(140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108
*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c
d^5 - 4(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 -
(90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^5 + (70*B*a*c^4*
d^2 - 7*(18*A - 35*B)*a*c^3*d^3 - (369*A - 344*B)*a*c^2*d^4 - (348*A - 241*
B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^4 - (35*B*a*c^5*d - 21*(3*
A - 10*B)*a*c^4*d^2 - 2*(171*A - 226*B)*a*c^3*d^3 - 6*(98*A - 83*B)*a*c^2*d
^4 - (426*A - 281*B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (105
*B*a*c^5*d - 7*(27*A - 80*B)*a*c^4*d^2 - 6*(150*A - 191*B)*a*c^3*d^3 - 2*(7
29*A - 610*B)*a*c^2*d^4 - 3*(340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6
)*cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A -
121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 -
(39*A - 20*B)*a*d^6)*cos(f*x + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*
d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A
- 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)
*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^4 -
2*(35*B*a*c^4*d^2 - 21*(3*A - 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 -
3*(43*A - 29*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (35*B*a*c^5
d - 7(9*A - 40*B)*a*c^4*d^2 - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 38
6*B)*a*c^2*d^4 - (684*A - 455*B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*cos(f*x +
e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^
3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*

B)*a*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(4*(A - B)*c^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38*A - 31*B)*c^3*d^3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(19*A - 11*B)*d^6)*cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (31*A - 75*B)*c^4*d^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^2*d^4 + 20*(5*A - 3*B)*c*d^5 + (51*A - 31*B)*d^6)*cos(f*x + e)^2 + ((7*A + B)*c^6 - 2*(9*A - 17*B)*c^5*d - (79*A - 175*B)*c^4*d^2 - 28*(7*A - 3*B)*c^3*d^3 - (15*A + 121*B)*c^2*d^4 + 2*(107*A - 59*B)*c*d^5 + (87*A - 55*B)*d^6)*cos(f*x + e) - (4*(A - B)*c^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)^3 - 2*((3*A + 5*B)*c^5*d - 2*(11*A - 24*B)*c^4*d^2 - 4*(16*A - 7*B)*c^3*d^3 + 2*(3*A - 19*B)*c^2*d^4 + (61*A - 33*B)*c*d^5 + 2*(8*A - 5*B)*d^6)*cos(f*x + e)^2 - ((3*A + 5*B)*c^6 - 2*(5*A - 13*B)*c^5*d - 3*(25*A - 57*B)*c^4*d^2 - 4*(53*A - 25*B)*c^3*d^3 - (11*A + 125*B)*c^2*d^4 + 6*(37*A - 21*B)*c*d^5 + (83*A - 51*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 + 9*a^3*c*d^8 - 5*

a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*
c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a
^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2
 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^
2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e)), 1/64*(sqrt(2)*(4*(3*A + 5*B)
c^6 - 8(9*A + 31*B)*c^5*d + 4*(117*A - 413*B)*c^4*d^2 + 16*(177*A - 233*B
)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(423*A - 271*B)*c*d^5 + 4*(219*
A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A + 3*B)*c^3*d^3 + 2*(81*A - 13
7*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e)^5
 + (2*(3*A + 5*B)*c^5*d - 3*(13*A + 43*B)*c^4*d^2 + 4*(63*A - 191*B)*c^3*d^
3 + 6*(217*A - 241*B)*c^2*d^4 + 2*(831*A - 583*B)*c*d^5 + 3*(219*A - 115*B)
*d^6)*cos(f*x + e)^4 - ((3*A + 5*B)*c^6 - 4*(3*A + 13*B)*c^5*d + (75*A - 54
7*B)*c^4*d^2 + 8*(123*A - 203*B)*c^3*d^3 + 19*(123*A - 115*B)*c^2*d^4 + 4*(
525*A - 349*B)*c*d^5 + 3*(219*A - 115*B)*d^6)*cos(f*x + e)^3 - (3*(3*A + 5*
B)*c^6 - 2*(21*A + 83*B)*c^5*d + (267*A - 1507*B)*c^4*d^2 + 4*(669*A - 1045
*B)*c^3*d^3 + (5871*A - 5383*B)*c^2*d^4 + 2*(2523*A - 1667*B)*c*d^5 + 7*(21
9*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*
d + (117*A - 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)
*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*cos(f*x + e) + (4
*(3*A + 5*B)*c^6 - 8*(9*A + 31*B)*c^5*d + 4*(117*A - 413*B)*c^4*d^2 + 16*(1
77*A - 233*B)*c^3*d^3 + 4*(1197*A - 1013*B)*c^2*d^4 + 8*(423*A - 271*B)*c*d
^5 + 4*(219*A - 115*B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 24*(A + 3*B)*c^3*d^3 +
2*(81*A - 137*B)*c^2*d^4 + 24*(17*A - 13*B)*c*d^5 + (219*A - 115*B)*d^6)*co
s(f*x + e)^4 - 2*((3*A + 5*B)*c^5*d - (21*A + 67*B)*c^4*d^2 + 2*(69*A - 173
*B)*c^3*d^3 + 2*(285*A - 293*B)*c^2*d^4 + (627*A - 427*B)*c*d^5 + (219*A -
115*B)*d^6)*cos(f*x + e)^3 - ((3*A + 5*B)*c^6 - 6*(A + 7*B)*c^5*d + 3*(11*A
 - 227*B)*c^4*d^2 + 12*(105*A - 193*B)*c^3*d^3 + 3*(1159*A - 1119*B)*c^2*d^
4 + 6*(559*A - 375*B)*c*d^5 + 5*(219*A - 115*B)*d^6)*cos(f*x + e)^2 + 2*((3
*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A - 413*B)*c^4*d^2 + 4*(177*A -
 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + 2*(423*A - 271*B)*c*d^5 + (21
9*A - 115*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^
2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e)
 + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(140*
B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(14
1*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6
+ (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*
A - 20*B)*a*d^6)*cos(f*x + e)^5 + (70*B*a*c^4*d^2 - 7*(18*A - 35*B)*a*c^3*d
^3 - (369*A - 344*B)*a*c^2*d^4 - (348*A - 241*B)*a*c*d^5 - 3*(39*A - 20*B)*
a*d^6)*cos(f*x + e)^4 - (35*B*a*c^5*d - 21*(3*A - 10*B)*a*c^4*d^2 - 2*(171*
A - 226*B)*a*c^3*d^3 - 6*(98*A - 83*B)*a*c^2*d^4 - (426*A - 281*B)*a*c*d^5
 - 3*(39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (105*B*a*c^5*d - 7*(27*A - 80*B)*
a*c^4*d^2 - 6*(150*A - 191*B)*a*c^3*d^3 - 2*(729*A - 610*B)*a*c^2*d^4 - 3*(
340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6)*cos(f*x + e)^2 + 2*(35*B*a*
c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^3*d^3 - 2*(141*A -

112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^4 - 2*(35*B*a*c^4*d^2 - 21*(3*A - 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 - 3*(43*A - 29*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e)^3 - (35*B*a*c^5*d - 7*(9*A - 40*B)*a*c^4*d^2 - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 386*B)*a*c^2*d^4 - (684*A - 455*B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e))) - 4*(4*(A - B)*c^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38*A - 31*B)*c^3*d^3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(19*A - 11*B)*d^6)*cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (31*A - 75*B)*c^4*d^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^2*d^4 + 20*(5*A - 3*B)*c*d^5 + (51*A - 31*B)*d^6)*cos(f*x + e)^2 + ((7*A + B)*c^6 - 2*(9*A - 17*B)*c^5*d - (79*A - 175*B)*c^4*d^2 - 28*(7*A - 3*B)*c^3*d^3 - (15*A + 121*B)*c^2*d^4 + 2*(107*A - 59*B)*c*d^5 + (87*A - 55*B)*d^6)*cos(f*x + e) - (4*(A - B)*c^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*cos(f*x + e)^3 - 2*((3*A + 5*B)*c^5*d - 2*(11*A - 24*B)*c^4*d^2 - 4*(16*A - 7*B)*c^3*d^3 + 2*(3*A - 19*B)*c^2*d^4 + (61*A - 33*B)*c*d^5 + 2*(8*A - 5*B)*d^6)*cos(f*x + e)^2 - ((3*A + 5*B)*c^6 - 2*(5*A - 13*B)*c^5*d - 3*(25*A - 57*B)*c^4*d^2 - 4*(53*A - 25*B)*c^3*d^3 - (11*A + 125*B)*c^2*d^4 + 6*(37*A - 21*B)*c*d^5 + (83*A - 51*B)*d^6)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3*a^3*d^9)*f*cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a

```
^3*c*d^8 - a^3*d^9)*f*cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3
*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^
3*d^9)*f*cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c
^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 +
 9*a^3*c*d^8 - 5*a^3*d^9)*f*cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3
*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 +
4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8*
d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c
^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*sin(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=221

$$-\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}-8\sqrt{\cdots}$$

```
[Out] (-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)*Ap
pellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.337947, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2987, 2784, 139, 138}

$$-\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}-8\sqrt{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
- Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)*Ap
pellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
d*Sin[e + f*x])/(c + d))^n)
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}}\,dx, x$$

$$f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(}{-c-}\right.\right.}{f\sqrt{1-\sin(e+}}$$

$$= -\frac{8\sqrt{2}a^2BF_1\left(\frac{1}{2};-\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin}{c+}\right.}{f\sqrt{1+}}$$

Mathematica [F] time = 25.4958, size = 0, normalized size = 0.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F] time = 0.584, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^2\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left((A+2B)a^{2}\cos\left(fx+e\right)^{2}-2(A+B)a^{2}+\left(Ba^{2}\cos\left(fx+e\right)^{2}-2(A+B)a^{2}\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{2}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^
n, x)
```

3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=217

$$-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}-\frac{4\sqrt{2}a}{}$$

```
[Out] (-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*Appell
F1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)
]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c + d*Si
n[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.300444, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2968, 3017, 2755, 139, 138, 2784}

$$-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}-\frac{4\sqrt{2}a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```



```
[Out] (-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*Appell
F1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)
]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c + d*Si
n[e + f*x])/(c + d))^n)
```


Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3017

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Dist[A - C, I
nt[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*S
in[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C,
m}, x] && EqQ[A - B + C, 0] && !IntegerQ[2*m]
```

Rule 2755

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*S
qrt[1 - Sin[e + f*x]]), Subst[Int[((a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 -
(d*x)/c], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rubi steps

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx = \int (c + d\sin(e + fx))^n\,\big(aA + (aA + aB)\sin(e + fx) + a$$

$$= (a(A - B))\int (1 + \sin(e + fx))(c + d\sin(e + fx))^n\,dx +$$

$$= \frac{(a(A - B)\cos(e + fx))\,\text{Subst}\left(\int \frac{\sqrt{1+x}(c+dx)^n}{\sqrt{1-x}}\,dx, x, \sin(e\right.}{f\sqrt{1 - \sin(e + fx)}\sqrt{1 + \sin(e + fx)}}$$

$$= \frac{\left(a(A - B)\cos(e + fx)(c + d\sin(e + fx))^n\left(-\frac{c+d\sin(e+fx}{-c-d}\right.\right.}{f\sqrt{1 - \sin(e + fx)}\sqrt{}}$$

$$= -\frac{4\sqrt{2}aBF_1\left(\frac{1}{2}; -\frac{3}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx)), \frac{d(1-\sin(e+f}{c+d}\right.}{f\sqrt{1 + \sin}}$$

Mathematica [F] time = 9.05416, size = 0, normalized size = 0.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
 x]
```

Maple [F] time = 0.48, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx + e\right)\right)\left(A + B\sin\left(fx + e\right)\right)\left(c + d\sin\left(fx + e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(
f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n,
 x)
```

3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=221

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B\cos(e}{}$$

```
[Out] -((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/2,
-n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]
*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c + d))^n)
```

Rubi [A] time = 0.304206, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2987, 2784, 139, 138, 2665}

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B\cos(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```


```
[Out] -((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Sin[
e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/2,
-n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]
*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c + d))^n)
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

```
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx + \frac{B\int (c+d\sin(e+fx))^n\,dx}{a}$$

$$= \frac{((A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{(B\cos}{}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(}{}\right.}{af\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+}{af\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 4.92391, size = 0, normalized size = 0.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
),x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
), x]
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int \frac{\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=223

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}-\frac{B\cos(e+fx}{}$$

```
[Out] -((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]
))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 5/2
, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*
Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.341262, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2987, 2784, 139, 138}

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{5}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}-\frac{B\cos(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```


```
[Out] -((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]
))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 5/2
, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*
x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*
Sin[e + f*x])/(c + d))^n)
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
```

```
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n)/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx}{a}$$

$$= \frac{((A-B)\cos(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}} + \frac{(B\cos(\ldots}{\ldots}$$

$$= \frac{\left((A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \ldots\right.}{a^2 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= -\frac{BF_1\left(\frac{1}{2};\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin\ldots}{\sqrt{2}a^2 f\sqrt{1+\sin(e+fx)}}$$

Mathematica [F] time = 8.67493, size = 0, normalized size = 0.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2, x]
```

Maple [F] time = 0.714, size = 0, normalized size = 0.

$$\int \frac{(A+B\sin(fx+e))(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 -
 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
2, x)
```

3.332 $\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$

Optimal. Leaf size=427

$$\frac{2a^2(A-B)(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}} - \frac{2a^2(A-B)\cos}{df(2n}$$

```
[Out] (-2*a^2*(A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) + (2*a^2*B*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^(1 + n))/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]
]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 +
 n))/(d*f*(5 + 2*n)) + (2*a^2*(A - B)*(c - d*(5 + 4*n))*Cos[e + f*x]*Hyperg
eometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d
))^n) - (2*a^2*B*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e
 + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c
+ d*Sin[e + f*x])^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.914932, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2987, 2763, 21, 2776, 70, 69, 2981}

$$\frac{2a^2(A-B)(c-d(4n+5))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}} - \frac{2a^2(A-B)\cos}{df(2n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```


```
[Out] (-2*a^2*(A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) + (2*a^2*B*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^(1 + n))/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]
]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 +
 n))/(d*f*(5 + 2*n)) + (2*a^2*(A - B)*(c - d*(5 + 4*n))*Cos[e + f*x]*Hyperg
eometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d
```

```
))^n) - (2*a^2*B*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e
 + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c
+ d*Sin[e + f*x])^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx=(A-B)\int (a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^n\,d$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}-\frac{2a}{}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2a}{}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2a}{}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2a}{}$$

$$=-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}+\frac{2a}{}$$

Mathematica [A] time = 26.581, size = 245, normalized size = 0.57

$$-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left(-30(A+B)(c-d(4n+5))\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};-\frac{d(\sin(e+fx)-1)}{c+d}\right)+30(A+B\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^n,x]
```

```
[Out] -(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*n))*Hy
pergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + 6*B*d*(
3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/(c + d))
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hypergeometric
2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e + f*x]) +
 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(15*d*f*(3 + 2
*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{\frac{3}{2}} \left(A+B\sin\left(fx+e\right)\right) \left(c+d\sin\left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) +
 c)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(Ba\cos\left(fx+e\right)^{2}-(A+B)a\sin\left(fx+e\right)-(A+B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] Timed out
```

3.333 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=167

$$-\frac{2a\cos(e+fx)(Ad(2n+3)-B(c-2d(n+1)))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB}{a}$$

```
[Out] (-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) - (2*a*(A*d*(3 + 2*n) - B*(c - 2*d*(1 + n)))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[
e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/
(c + d))^n)
```

Rubi [A] time = 0.26483, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2981, 2776, 70, 69}

$$\frac{2a\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\,{}_2F_1\left(\frac{1}{2},-n;\frac{3}{2};\frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2aB}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
a*Sin[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[
e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/
(c + d))^n)
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(aAd(3+2\cdots}{\cdots}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{(a(aAd(3+\cdots}{\cdots}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{\left(a(aAd(3+\cdots\right.}{\cdots}$$

$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{2a(Bc-2B\cdots}{\cdots}$$

Mathematica [F] time = 8.21614, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x]
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sin\left(fx+e\right)}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)
^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**n, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=220

$$-\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+f$$

```
[Out] -(((A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c
+ d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)
]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[
a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (
d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] time = 0.402729, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {2987, 2788, 137, 136, 2776, 70, 69}

$$-\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},1;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+f$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]],
x]
```

```
[Out] -(((A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c
+ d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)
]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[
a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, (
d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a + a*Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
 x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx + \frac{B\int \sqrt{a+a\sin(e+fx)}(c+d\sin(e}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB}{}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx,\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{(A-B)F_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d}{}}}{(c-d)f(1+n)(1-\sin(e+fx))\sqrt{a+a\sin(}}$$

Mathematica [A] time = 5.4829, size = 244, normalized size = 1.11

$$\frac{\cos(e+fx)\sqrt{a(\sin(e+fx)+1)}(c+d\sin(e+fx))^n\left(\frac{4(A-B)\sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}}\left(\frac{c-d}{d\sin(e+fx)+d}+1\right)^{-n}F_1\left(-n-\frac{1}{2};-\frac{1}{2},-n;\frac{1}{2}-n;\frac{2}{\sin(e+fx)+1},\frac{d-c}{\sin(e+fx)d+}\right.}{2n+1}\right.}{4af(\sin(e+fx)-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B)
*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c +
 d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*(A -
```

```
B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d
+ d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)
*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\sqrt{a\sin\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)^{n}}{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x)
+ 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx + e\right) + A\right)\left(d\sin\left(fx + e\right) + c\right)^{n}}{\sqrt{a\sin\left(fx + e\right) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)\sqrt{}}{}$$

```
[Out] -((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Si
n[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c +
 d*Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e
+ f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Si
n[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a
 - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.469883, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2987, 2788, 137, 136}

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}F_1\left(n+1;\frac{1}{2},2;n+2;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B\cos(e+fx)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/2
),x]
```


```
[Out] -((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Si
n[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c +
 d*Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a +
a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e
+ f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Si
n[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a
 - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx = (A-B)\int \frac{(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)^2}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB}{}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{d(a-a\sin(e+fx))}{ac+ad}}\right)\text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax)^2\sqrt{\frac{ad}{ac+ad}-\frac{adx}{ac+ad}}}\,dx,\right.}{f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{BF_1\left(1+n;\frac{1}{2},1;2+n;\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+}{c+d}}}{(c-d)f(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+}}$$

Mathematica [B] time = 10.1795, size = 603, normalized size = 2.24

$$\frac{\sec(e+fx)(c+d\sin(e+fx))^n\left(aA(\sin(e+fx)+1)\left(a\sqrt{2-2\sin(e+fx)}(\sin(e+fx)+1)\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(1;\frac{1}{2},-n;2\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1
, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2
 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (
4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2
 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x]
)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]
), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (
c - d)/(d + d*Sin[e + f*x]))^n)) + a*A*(1 + Sin[e + f*x])*((a*AppellF1[1, 1
/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 -
2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n - (4*S
qrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*AppellF1[1/2 - n
, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])] +
 a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (
-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 + (c -
d)/(d + d*Sin[e + f*x]))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int \left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\left(a+a\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(d\sin\left(fx+e\right)+c\right)^{n}}{a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{n}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^
(3/2), x)
```

3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=351

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m\right.\right.\right.}{f(m+1)(}$$

[Out] ((d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 + m)*(A*c*d*(3 + m) + B*(c^2 + d^2 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (2^(1/2 + m)*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 + 5*m + m^2) + 2*c*d*(3 + 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))

Rubi [A] time = 0.986798, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2983, 2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m\right.\right.\right.}{f(m+1)(}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]

[Out] ((d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 + m)*(A*c*d*(3 + m) + B*(c^2 + d^2 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (2^(1/2 + m)*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 + 5*m + m^2) + 2*c*d*(3 + 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
```

```
1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx}{f(3+m)}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx}{f(3+m)}$$

$$= -\frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(}{af(2+m)(3+m)}$$

$$= \frac{\left(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+n\right.\right.}{f(1+m)(}$$

$$= \frac{\left(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+n\right.\right.}{f(1+m)(}$$

$$= \frac{\left(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+n\right.\right.}{f(1+m)(}$$

Mathematica [A] time = 7.64892, size = 300, normalized size = 0.85

$$-\frac{\csc^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^m(a(\sin(e+fx)+1))^m\left(-\frac{2}{7}(A-B)(c-d)^2\tan^7\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{7}{2},m+4;\frac{9}{2};-\tan^2\left(\frac{1}{4}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2,x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)^2*Hypergeometric2F1[1/2, 4 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan
[(2*e - Pi + 2*f*x)/4] - (2*(c + d)*(3*A*c + B*c - A*d - 3*B*d)*Hypergeomet
ric2F1[3/2, 4 + m, 5/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x
)/4]^3)/3 - (2*(c - d)*(A*(3*c + d) - B*(c + 3*d))*Hypergeometric2F1[5/2, 4
 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^5)/5 - (
2*(A - B)*(c - d)^2*Hypergeometric2F1[7/2, 4 + m, 9/2, -Tan[(2*e - Pi + 2*f
*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^7)/7))/f)
```

Maple [F] time = 2.762, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m} \left(A+B\sin\left(fx+e\right)\right) \left(c+d\sin\left(fx+e\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{2}\left(a\sin\left(fx+e\right)+a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Ac^{2}+2\,Bcd+Ad^{2}-\left(2\,Bcd+Ad^{2}\right)\cos\left(fx+e\right)^{2}-\left(Bd^{2}\cos\left(fx+e\right)^{2}-Bc^{2}-2\,Acd-Bd^{2}\right)\sin\left(fx+e\right)\right)\left(a\,s\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e)^2 - (B*d
^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e)
+ a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^2(a\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^
m, x)
```

3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=199

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+2)(cm+c+dm)+B\left(cm(m+2)+d\left(m^2+m+1\right)\right)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a}{f(m+1)(m+2)}$$

```
[Out] ((B*d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (2^(1/2 + m)*(A*(2 + m)*(c + c*m + d*m) + B*(c*m*(2 + m) + d*(
1 + m + m^2)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rubi [A] time = 0.361758, antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {2968, 3023, 2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd\left(m^2+m+1\right)\right)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((B*d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (2^(1/2 + m)*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*
(1 + m + m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m
)*(2 + m)) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a+a\sin(e+fx))^m\left(Ac+(Bc+Ad)\sin(e+fx)+\right.$$

$$= -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)} + \frac{\int (a+a\sin(}{}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx)}{f(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx)}{f(1+m)(2+m)}$$

$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx)}{f(1+m)(2+m)}$$

Mathematica [A] time = 3.40543, size = 212, normalized size = 1.07

$$-\frac{\csc^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^m(a(\sin(e+fx)+1))^m\left(-\frac{2}{5}(A-B)(c-d)\tan^5\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{5}{2},m+3;\frac{7}{2};-\tan^2\left(\frac{1}{4}(2e\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)*Hypergeometric2F1[1/2, 3 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(
2*e - Pi + 2*f*x)/4] - (4*(A*c - B*d)*Hypergeometric2F1[3/2, 3 + m, 5/2, -T
an[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^3)/3 - (2*(A - B)*(c
- d)*Hypergeometric2F1[5/2, 3 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(
2*e - Pi + 2*f*x)/4]^5)/5))/f)
```

Maple [F] time = 2.088, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(d\sin(fx+e)+c)(a\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(Bd\cos\left(fx+e\right)^2-Ac-Bd-(Bc+Ad)\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*si
n(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\left(A+B\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)
), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m,
 x)
```

3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=117

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{B\,c}{}$$

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rubi [A] time = 0.0809615, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2751, 2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(m+1)}-\frac{B\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f
*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{(A+Am+Bm)\int(a+a\sin(e+f}{1+m}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}+\frac{\left((A+Am+Bm)(1+\sin(e+fx))\right.}{}$$

$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)}-\frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)}{}$$

Mathematica [C] time = 1.76929, size = 275, normalized size = 2.35

$$-\frac{\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m\left(\frac{2\sqrt{2}A\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right){}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{(2m+1)\sqrt{1-\sin(e+fx)}}\right.}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I +
 E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1
 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeom
etric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*
(-1 + m^2)) + (2*Sqrt[2]*A*Cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometr
ic2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2
*f*x)/4])/((1 + 2*m)*Sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]
^(2*m)))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=191

$$-\frac{\sqrt{2}(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}-\frac{B2^{m+\frac{1}{2}}\cos(e}{}$$

```
[Out] -((Sqrt[2]*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m
)/((c - d)*d*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])) - (2^(1/2 + m)*B*Cos[e +
f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e
+ f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
```

Rubi [A] time = 0.291516, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2986, 2652, 2651, 2788, 137, 136}

$$-\frac{\sqrt{2}(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}-\frac{B2^{m+\frac{1}{2}}\cos(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```


```
[Out] -((Sqrt[2]*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m
)/((c - d)*d*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]])) - (2^(1/2 + m)*B*Cos[e +
f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e
+ f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
```


Rule 2986

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{B\int(a+a\sin(e+fx))^m\,dx}{d} - \frac{(Bc-Ad)\int\frac{(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx}{d}$$

$$= -\frac{\left(a^2(Bc-Ad)\cos(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)}\,dx,x,\sin(e+fx)\right)}{df\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} +$$

$$= -\frac{2^{\frac{1}{2}+m}B\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+}{df}$$

$$= -\frac{\sqrt{2}(Bc-Ad)F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right.}{(c-d)df(1+2m)\sqrt{1-\sin(e+fx)}}$$

Mathematica [B] time = 7.03589, size = 473, normalized size = 2.48

$$\frac{(a(\sin(e+fx)+1))^m\left(\frac{6(c+d)(Bc-Ad)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}\right.}{d(c+d\sin(e+fx))\left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\left(4dF_1\left(\frac{3}{2};\frac{1}{2}-m,2;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right)-(2m-1)(c+d)F_1\left(\frac{3}{2};\frac{3}{2}-m,1;\right.\right.\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*((Sqrt[2]*B*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 + m, 3/2 + m, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4])/((d + 2*d
*m)*Sqrt[1 - Sin[e + f*x]]) + (6*(c + d)*(B*c - A*d)*AppellF1[1/2, 1/2 - m,
 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Sec
[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*(c + d
*Sin[e + f*x])*(3*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*
f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (4*d*AppellF1[3/2,
1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4
]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e
+ Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Sin[(2*e -
Pi + 2*f*x)/4]^2))))/f
```

Maple [F] time = 1.326, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d\sin\left(fx+e\right)+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=293

$$\frac{\sqrt{2}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+\right.}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*(A*d*(c*(1 - m) - d*m) - B*(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m
, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]
*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*d*(c + d)*f*(1 + 2*m)*Sqrt
[1 - Sin[e + f*x]]) + (2^(1/2 + m)*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)
*(a + a*Sin[e + f*x])^m)/(d*(c^2 - d^2)*f) - ((B*c - A*d)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 0.61814, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2984, 2986, 2652, 2651, 2788, 137, 136}

$$\frac{\sqrt{2}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)(a\sin(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+\right.}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```


```
[Out] (Sqrt[2]*(A*d*(c*(1 - m) - d*m) - B*(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m
, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]
*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c - d)^2*d*(c + d)*f*(1 + 2*m)*Sqrt
[1 - Sin[e + f*x]]) + (2^(1/2 + m)*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)
*(a + a*Sin[e + f*x])^m)/(d*(c^2 - d^2)*f) - ((B*c - A*d)*Cos[e + f*x]*(a +
 a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```


Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
```

```
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2986

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{\int \frac{(a+a\sin(e+fx))^m(-a(Ac-Bd+}{c+d}}{a\left(\right.}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} - \frac{((Bc-Ad)m)\int(a+a\sin}{d\left(c^2-d^2\right)}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\left(a^2\left(Ad(c(1-m)-dm)\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}(Bc-Ad)m\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\mathrm{s}}{d\left(c^2-d^2\right)f}$$

$$= \frac{\sqrt{2}\left(Ad(c(1-m)-dm)-B\left(d^2-c^2m-cdm\right)\right)F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}\right.}{(c-d)^2d(c+d)f(1+2m}$$

Mathematica [B] time = 5.67102, size = 654, normalized size = 2.23

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos^2\left(\frac{1}{4}(2e+2f\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3*(
c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c
+ d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
 (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2) -
 (B*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1
/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*
x)/4]^2)/(c + d)] + (4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f
*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*A
ppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e -
Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi +
2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^2)
```

Maple [F] time = 1.797, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c+d\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
2, x)
```

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3} \, dx$

Optimal. Leaf size=467

$$\frac{\cos(e+fx)\left(B\left(2c^2d(1-m)m+c^3(1-m)m-cd^2\left(m^2-3m+3\right)+2d^3m\right)-Ad\left(c^2\left(-\left(m^2-3m+2\right)\right)+2cd(2-m)m-\right.\right.}{\sqrt{2}df(2m+1)(c-d)^3(c+d)^2\sqrt{1-\text{s}}}$$

```
[Out] ((B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3 - 3*m + m^2)) -
 A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m + m^2)))*AppellF1[
1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
 - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(Sqrt[2]*(c - d)^3*d*(c + d)^2
*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]) - (2^(-1/2 + m)*m*(A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin
[e + f*x])^m)/(d*(c^2 - d^2)^2*f) - ((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e
+ f*x])^m)/(2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/
(2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rubi [A] time = 1.34696, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2984, 2986, 2652, 2651, 2788, 137, 136}

$$\frac{\cos(e+fx)\left(B\left(2c^2d(1-m)m+c^3(1-m)m-cd^2\left(m^2-3m+3\right)+2d^3m\right)-Ad\left(c^2\left(-\left(m^2-3m+2\right)\right)+2cd(2-m)m-\right.\right.}{\sqrt{2}df(2m+1)(c-d)^3(c+d)^2\sqrt{1-\text{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```


```
[Out] ((B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3 - 3*m + m^2)) -
 A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m + m^2)))*AppellF1[
1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
 - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(Sqrt[2]*(c - d)^3*d*(c + d)^2
*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]) - (2^(-1/2 + m)*m*(A*d*(c*(3 - m) - d*
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1
/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin
[e + f*x])^m)/(d*(c^2 - d^2)^2*f) - ((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e
+ f*x])^m)/(2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((A*d*(c*(3 - m) - d*
```

```
m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/
(2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2986

```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
 m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
```

```
!IntegerQ[m]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} - \frac{\int \frac{(a+a\sin(e+fx))^m(-a(2Ac-2}{}}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(Ad(c(3-m)-dm)-I\right.}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(Ad(c(3-m)-dm)-I\right.}{}$$

$$= -\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2} + \frac{\left(Ad(c(3-m)-dm)-I\right.}{}$$

$$= -\frac{2^{-\frac{1}{2}+m}m\left(Ad(c(3-m)-dm)-B\left(2d^2+c^2(1-m)-cdm\right)\right)\cos(e+f}{}$$

$$= \frac{\left(B\left(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2\left(3-3m+m^2\right)\right)-Ad\left(2\right.\right.}{}$$

Mathematica [A] time = 6.1267, size = 654, normalized size = 1.4

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos^2\left(\frac{1}{4}(2e+2\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^3,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3*(
c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin
[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (12*d*AppellF1[3/2, 1/2 - m, 4, 5/2, C
```

```
os[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c
 + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2
, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2)
- (B*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(
2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*AppellF1[
1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)] + (8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*
f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*
AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e -
 Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi +
 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^3)
```

Maple [F] time = 2.198, size = 0, normalized size = 0.

$$\int \frac{\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)}{\left(c+d\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{3\,cd^{2}\cos\left(fx+e\right)^{2}-c^{3}-3\,cd^{2}+\left(d^{3}\cos\left(fx+e\right)^{2}-3\,c^{2}d-d^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)
^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
3, x)
```

3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=284

$$\frac{\sqrt{2}(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+}{c-}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (Sqrt[2]*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c
 + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d)*AppellF1[3/2 + m, 1/2, -3
/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*
m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rubi [A] time = 0.627923, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+}{c-}\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (Sqrt[2]*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c
 + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d)*AppellF1[3/2 + m, 1/2, -3
/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e
 + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*
m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
```

```
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = (A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{3/2}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{3/2}}{\sqrt{a-ax}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+f}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+a}{}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\,\text{s}}}$$

$$= \frac{\left(a(A-B)(ac-ad)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d}\right.}{\sqrt{2}f(a-a\sin(e+fx)}$$

$$= \frac{\sqrt{2}(A-B)(c-d)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin\right.}{f(1+2m}$$

Mathematica [B] time = 8.09756, size = 3281, normalized size = 11.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -((((-2*B*c*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^
2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 +
 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/2)*Sqrt[c + d -
 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x
)/2]^2)/(c + d)]) - (2*A*d*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e +
Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2
- f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + P
i/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/
```

```
2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(
-e + Pi/2 - f*x)/2]^2)/(c + d)]) + (B*d*AppellF1[5/2, (1 - 2*m)/2, -1/2, 7/
2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Co
s[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
)*Sin[(-e + Pi/2 - f*x)/2]^5*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
 + (-1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(5*Sqrt[(c +
d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]) - (2*B*d*AppellF1[3/2, (-1 -
2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2
]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^
2)^((-1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^
2)^((-1 - 2*m)/2 + (1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2
])/Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (3*B*d*(c + d)*
AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e
 + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(3 + 2*m)*(Cos[(-e +
 Pi/2 - f*x)/2]^2)^(1/2 + (-4 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(
-e + Pi/2 - f*x)/2]^2)^(3/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^
2])/(-3*(c + d)*AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]
^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -3/2 - m
, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c
 + d)] + (c + d)*(3 + 2*m)*AppellF1[3/2, -1/2 - m, -1/2, 5/2, Sin[(-e + Pi/
2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 -
f*x)/2]^2) - (6*B*c*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi
/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 -
f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 + (-2 - 2*m)/2)*Sin[(-e
 + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(1/2 + m)*Sqrt[c + d - 2
*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3
/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] +
 (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Si
n[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1 + 2*m)*AppellF1[3/2, 1/2 -
m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/
(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) - (6*A*d*(c + d)*AppellF1[1/2, -1/2 -
 m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)
/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(
1/2 + (-2 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^
2)^(1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*Appe
llF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + P
i/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e
+ Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1
+ 2*m)*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*S
in[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2) + (6*A*c*(
c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*S
in[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Co
s[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e +
 Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])
/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (
```

```
2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2, 1/2 - m, 1/2,
 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]
 + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*
x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2
]^2) + (3*B*d*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*
x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]
^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/
2 - f*x)/2]^2])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2
 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2
, 1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)
/2]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[
(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e
+ Pi/2 - f*x)/2]^2))*(a + a*Sin[e + f*x])^m)/(f*Cos[(-e + Pi/2 - f*x)/2]^(2
*m)))
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) +
 a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(Bd\cos\left(fx+e\right)^2-Ac-Bd-(Bc+Ad)\sin\left(fx+e\right)\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt(
d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sage2
```

3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -1/2, 5/2 + m, (1
+ Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*S
in[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[
e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Rubi [A] time = 0.554948, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```


```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2
, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sq
rt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin
[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -1/2, 5/2 + m, (1
+ Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*S
in[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[
e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
```

```
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx=(A-B)\int (a+a\sin(e+fx))^m\sqrt{c+d\sin(e+fx)}\,dx+\ldots$$

$$=\frac{\left(a^2(A-B)\cos(e+fx)\right)\mathrm{Subst}\left(\int\frac{(a+ax)^{-\frac{1}{2}+m}\sqrt{c+dx}}{\sqrt{a-ax}}\,dx,\ldots\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\ldots}$$

$$=\frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\mathrm{Subst}\left(\int\frac{(a+ax)^{\ldots}}{\ldots}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin\ldots}}$$

$$=\frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{c+d\sin(e+fx)}\ldots\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+\ldots}}$$

$$=\frac{\sqrt{2}(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx))\ldots\right.}{f(1+2m)\sqrt{1\ldots}}$$

<u>Mathematica</u> [B] time = 12.0641, size = 672, normalized size = 2.45

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\sqrt{c+d\sin(e\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*
x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*(((B*c - A*d)*AppellF1
[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi +
 2*f*x)/4]^2)/(c + d)])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2
*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*A
ppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e
- Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1
/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c
+ d)])*Cos[(2*e + Pi + 2*f*x)/4]^2) - (B*AppellF1[1/2, 1/2 - m, -3/2, 3/2,
```

```
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c
+ d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-6*d*Appell
F1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -3/2,
5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)
])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(
d*f)
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\sqrt{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)
^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)\sqrt{c+d\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e +
f*x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}+\ldots$$

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1/2, 5/2 + m, (1 +
Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*f*(3 + 2*m)*Sqrt[
1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.5435, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]],
x]
```



```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1/2, 5/2 + m, (1 +
Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin
[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*f*(3 + 2*m)*Sqrt[
1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = (A-B)\int \frac{(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}\sqrt{c+dx}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \frac{(aB}{}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{c+dx}}\,dx, x, \sin(e\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\operatorname{Subst}\left(\int \frac{(a+}{\sqrt{\frac{1}{2}-\frac{x}{2}}\sqrt{}}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin}}$$

$$= \frac{\sqrt{2}(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{1}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos}{f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin}}$$

Mathematica [B] time = 6.31583, size = 672, normalized size = 2.45

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos^2\left(\frac{1}{4}(2e+2\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
f*x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3
*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*
```

```
x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)
/4]^2) - (B*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)
*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2
*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c
+ d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]
^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2
))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*Sqrt[c + d*Sin[e + f*x]])
```

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\frac{1}{\sqrt{c+d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\left(A+B\sin\left(e+fx\right)\right)}{\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e +
f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{2}B}{}$$

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 3/2, 5/2 +
m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*(c - d)*f
*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```

Rubi [A] time = 0.545462, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}F_1\left(m+\frac{1}{2};\frac{1}{2},\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}+\frac{\sqrt{2}B}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/2
),x]
```


```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2,
 -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqr
t[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 3/2, 5/2 +
m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a
 + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(a*(c - d)*f
*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx = (A-B)\int \frac{(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx + \frac{B\int \frac{(a+a\sin(e+fx))^{1+m}}{(c+d\sin(e+fx))^{3/2}}\,dx}{a}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}} + \text{—} \quad (aB$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^{3/2}}\,dx, x, \sin(e\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{\left(a^3(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\sqrt{\frac{a(c+d\sin(e+fx))}{ac-ad}}\right)\operatorname{Subst}\left(\int \frac{(a+a}{\sqrt{\frac{1}{2}-\frac{x}{2}}\left(\frac{a}{ac-}\right.}\right.}{\sqrt{2}(ac-ad)f(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\sqrt{c+}}$$

$$= \frac{\sqrt{2}(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},\frac{3}{2};\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos}{(c-d)f(1+2m)\sqrt{1-\sin(e+fx)}\sqrt{c+d}}$$

Mathematica [B] time = 6.38498, size = 672, normalized size = 2.33

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m\left(\frac{}{\cos^2\left(\frac{1}{4}(2e+2f\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2,
Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(3
*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d
*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5
/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
 - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*
```

```
x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)
/4]^2) - (B*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (
2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*
AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e
 - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(
2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d
)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2,
(2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(
Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^(3/2))
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{d^{2}\cos\left(fx+e\right)^{2}-2\,cd\sin\left(fx+e\right)-c^{2}-d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^
(3/2), x)
```

3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=270

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2,
-((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c +
 d*Sin[e + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x
])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -n, 5/2 + m, (1 + Sin[e
+ f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*
((c + d*Sin[e + f*x])/(c - d))^n)
```

Rubi [A] time = 0.429505, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+\right.}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```


```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2,
-((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c +
 d*Sin[e + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x
])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -n, 5/2 + m, (1 + Sin[e
+ f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f
*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*
((c + d*Sin[e + f*x])/(c - d))^n)
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
```

```
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = (A-B)\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,d$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}}\,d\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+f}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{(a+a}{}\right.}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a+a\,\text{s}}}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\sqrt{\frac{a-a\sin(e+fx)}{a}}(c+d\sin(e+f\right.}{\sqrt{2}f(a-}$$

$$= \frac{\sqrt{2}(A-B)F_1\left(\frac{1}{2}+m;\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+f\right.}{}$$

Mathematica [B] time = 6.22371, size = 682, normalized size = 2.53

$$\frac{6(c+d)\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/
4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^n*(((B*c - A*d)*AppellF1[1
/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f
*x)/4]^2)/(c + d)])/(3*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + P
i + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*n*Appel
lF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e -
Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -n,
5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)
```

```
])*Cos[(2*e + Pi + 2*f*x)/4]^2) - (B*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Co
s[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c +
d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Cos[(2*e +
Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*(1 + n
)*AppellF1[3/2, 1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*
e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m,
-1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)
/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2
 - m))/(d*f)
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=277

$$\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{3}{2};\frac{1}{2},m+1;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}}$$

```
[Out] -((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])
^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((
c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1 + m
, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c
- d)*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```

Rubi [A] time = 0.482663, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2987, 2788, 132, 140, 139, 138}

$$\frac{\sqrt{2}B\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{3}{2};\frac{1}{2},m+1;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1\right.}{af(2m+3)(c-d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 -
m),x]
```


```
[Out] -((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x])
^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))/((
c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1 + m
, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e +
 f*x]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c
- d)*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```


Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
 + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n)/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
 n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^{-1-m}\,dx = (A-B)\int (a + a\sin(e+fx))^m (c + d\sin(e+fx))^{-}$$

$$= \frac{\left(a^2(A-B)\cos(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^{-}}{\sqrt{a-ax}}\right.}{f\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e}}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(}{2(c+}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(}{2(c+}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{(c-d)(}{2(c+}\right.}{}$$

Mathematica [B] time = 6.89663, size = 573, normalized size = 2.07

$$\frac{2\cot\left(\frac{1}{4}(2e+2fx+\pi)\right)\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}}(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^{-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1
 + Sin[e + f*x]))^m*((-3*B*(c + d)^2*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*
e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(d*(3*(c
+ d)*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(
2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (-4*d*m*AppellF1[3/2, 1/2 - m, 1 + m, 5/
2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]
+ (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, m, 5/2, Cos[(2*e + Pi + 2*f*x)/
4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]
^2)) - A*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*
x)/4]^2)/(c + d*Sin[e + f*x])]*(((c + d)*Cos[(2*e - Pi + 2*f*x)/4]^2)/(c +
d*Sin[e + f*x]))^(1/2 - m) + (B*c*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c
- d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x])]*(((c + d)*Cos[(2*e
- Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]))^(1/2 - m))/d)*(Sin[(2*e + Pi + 2*
f*x)/4]^2)^(1/2 - m))/((c + d)*f*(c + d*Sin[e + f*x])^m)
```

Maple [F] time = 0.44, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(
-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.348 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=132

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}\right.}{f(2m+1)}$$

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Si
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*((c + d*Sin[e + f*
x])/(c - d))^n)
```

Rubi [A] time = 0.200323, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3008, 140, 139, 138}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}\right.}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Si
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*((c + d*Sin[e + f*
x])/(c - d))^n)
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\mathrm{S}\cdots}{\cdots}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\cdots\right.}{f\sqrt{\cdots}}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{1}{2}+m;-\frac{1}{2},-n;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d}{\cdots}\right.}{\cdots}$$

Mathematica [F] time = 9.60983, size = 0, normalized size = 0.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n, x]
```

Maple [F] time = 0.434, size = 0, normalized size = 0.

$$\int \left(a-a\sin\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a\sin\left(fx+e\right)-a\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

3.349 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=139

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{1}{2};-\frac{1}{2},m+1;m+\frac{3}{2};\ldots\right)}{f(2m+1)(c-d)}$$

```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -(
(d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/((c - d)*f*(1 + 2*
m)*(c + d*Sin[e + f*x])^m)
```

Rubi [A] time = 0.230192, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3008, 140, 139, 138}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{1}{2};-\frac{1}{2},m+1;m+\frac{3}{2};\ldots\right)}{f(2m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 -
m),x]
```



```
[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -(
(d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/((c - d)*f*(1 + 2*
m)*(c + d*Sin[e + f*x])^m)
```


Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a - a\sin(e+fx))(a + a\sin(e+fx))^m(c + d\sin(e+fx))^{-1-m}\,dx = \frac{\left(\sec(e+fx)\sqrt{a - a\sin(e+fx)}\sqrt{a + a\sin(e+fx)}\right)}{}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a - a\sin(e+fx))\sqrt{a + a\sin(e+f}\right.}{}$$

$$= \frac{\left(\sqrt{2}a\sec(e+fx)(a - a\sin(e+fx))\sqrt{a + a\sin(e+}\right.}{}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{1}{2}+m;-\frac{1}{2},1+m;\frac{3}{2}+m;\frac{1}{2}(1+\sin(e+fx\right.}{}$$

Mathematica [F] time = 4.76157, size = 0, normalized size = 0.

$$\int (a - a\sin(e+fx))(a + a\sin(e+fx))^m(c + d\sin(e+fx))^{-1-m}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m),x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

Maple [F] time = 0.45, size = 0, normalized size = 0.

$$\int \left(a - a\sin\left(fx+e\right)\right)\left(a + a\sin\left(fx+e\right)\right)^m\left(c + d\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \left(a \sin \left(fx+e\right)-a\right)\left(a \sin \left(fx+e\right)+a\right)^{m}\left(d \sin \left(fx+e\right)+c\right)^{-m-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="maxima")
```

```
[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a \sin \left(fx+e\right)-a\right)\left(a \sin \left(fx+e\right)+a\right)^{m}\left(d \sin \left(fx+e\right)+c\right)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, al
gorithm="giac")
```

```
[Out] sage2
```

3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

```
[Out] -((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rubi [A] time = 0.171788, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2974}

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)*m + (
c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a+a\sin(e}{}$$

Mathematica [A] time = 0.664572, size = 39, normalized size = 1.

$$-\frac{\cos(e+fx)(a(\sin(e+fx)+1))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)
*m + (c + (c - d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Maple [F] time = 0.589, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{-2-m}\left(d-(c-d)\,m+(c+(c-d)\,m)\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f
*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.99149, size = 140, normalized size = 3.59

$$-\frac{\left(d\cos\left(fx+e\right)\sin\left(fx+e\right)+c\cos\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*s
in(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d-(c-d)*m+(c+(c-d)*
m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$

Optimal. Leaf size=40

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rubi [A] time = 0.171891, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02, Rules used = {2974}

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)*m + (
c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rule 2974

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Rubi steps

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+

Mathematica [A] time = 0.709647, size = 40, normalized size = 1.

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)
*m + (c + (c + d)*m)*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Maple [F] time = 0.566, size = 0, normalized size = 0.

$$\int \left(a-a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{-2-m}\left(d+(c+d)\,m+(c+(c+d)\,m)\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

```
[Out] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f
*x+e)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.82909, size = 142, normalized size = 3.55

$$-\frac{\left(d\cos\left(fx+e\right)\sin\left(fx+e\right)+c\cos\left(fx+e\right)\right)\left(-a\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d*
sin(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d+(c+d)*m+(c+(c+d)*
m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)
*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError
```

3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=199

$$-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+}{d^2f\left(c^2-d^2\right)(c+d\sin(e+}$$

```
[Out] -((b*(2*b*B*c - A*b*d - 2*a*B*d)*x)/d^3) - (2*(b*c - a*d)*(a*d^2*(A*c - B*d
) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)*f) - (b^2*B*Cos[e + f*x])/(d
^2*f) - ((b*c - a*d)^2*(B*c - A*d)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*
Sin[e + f*x]))
```

Rubi [A] time = 0.580276, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2988, 3023, 2735, 2660, 618, 204}

$$-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3f\left(c^2-d^2\right)^{3/2}}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+}{d^2f\left(c^2-d^2\right)(c+d\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -((b*(2*b*B*c - A*b*d - 2*a*B*d)*x)/d^3) - (2*(b*c - a*d)*(a*d^2*(A*c - B*d
) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e + f*x
)/2])/Sqrt[c^2 - d^2]])/(d^3*(c^2 - d^2)^(3/2)*f) - (b^2*B*Cos[e + f*x])/(d
^2*f) - ((b*c - a*d)^2*(B*c - A*d)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*
Sin[e + f*x]))
```

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
```

```
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-d\left(B(bc-ad)^2-Ad\left(a^2c+b^2c-2abd\right)\right)-b(bBc}{}}{}$$

$$= -\frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))} + \frac{\int \frac{-d^2\left(B(bc-ad)^2-\right.}{}}{}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos}{d^2\left(c^2-d^2\right)f(c+d\sin}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos}{d^2\left(c^2-d^2\right)f(c+d\sin}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{b^2B\cos(e+fx)}{d^2f} - \frac{(bc-ad)^2(Bc-Ad)\cos}{d^2\left(c^2-d^2\right)f(c+d\sin}$$

$$= -\frac{b(2bBc-Abd-2aBd)x}{d^3} - \frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(2Bc^3-Ac^2d\right.\right.}{d^3\left(c^2-\right.}$$

Mathematica [A] time = 1.5741, size = 188, normalized size = 0.94

$$\frac{\frac{2(bc-ad)\left(ad^2(Bd-Ac)+b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{3/2}} + b(e+fx)(2aBd+Abd-2bBc) + \frac{d(bc-ad)^2(Ad-Bc)\cos(e+fx)}{(c-d)(c+d)(c+d\sin(e+fx))}}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]
)^2,x]
```

```
[Out] (b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c) +
 B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e +
 f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (d*(
b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f
*x])))/(d^3*f)
```

Maple [B] time = 0.156, size = 1246, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2/f/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^2*tan(
1/2*f*x+1/2*e)*B*b^2+4/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/
(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*B*a*b+2/f*d^2/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)*A*a^2-4/f*d/(c*tan(1/2*f
*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*A*a*b+4/
f/d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*B*a*b*c^2-4
/f/d^2/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*B*a*b*c^3+2/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*A*a^2*c+8/f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/
2*e)+2*d)/(c^2-d^2)^(1/2))*B*a*b*c-2/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*
f*x+1/2*e)*d+c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)*B*a^2+2/f/d/(c*tan(1/2*f*x+1/2
*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*A*b^2*c^2-2/f/d^2/(c*tan(1/2*f*x+
1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^3*B*b^2-4/f*d/(c^2-d^2)^(3/2
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*a*b-2/f/d^2/(c
^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*b^
2*c^3+4/f/d^3/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-
d^2)^(1/2))*B*b^2*c^4-2/f*b^2/d^2*B/(1+tan(1/2*f*x+1/2*e)^2)+2/f*b^2/d^2*A*
arctan(tan(1/2*f*x+1/2*e))-6/f/d/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*b^2*c^2+2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(
1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*tan(1/2*f*x+1/2*e)*A*b^2-4/f/(c*tan(1/2*f*x
+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*A*a*b*c+4/f*b/d^2*B*arctan(ta
n(1/2*f*x+1/2*e))*a-4/f*b^2/d^3*B*arctan(tan(1/2*f*x+1/2*e))*c-2/f*d/(c^2-d
^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*a^2+4/
f/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*
A*b^2*c+2/f*d/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*A
*a^2-2/f/(c*tan(1/2*f*x+1/2*e)^2+2*tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*B*a^2*
c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23469, size = 2761, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^
2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*
B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b +
2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d
^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2
 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 +
(B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (
B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d - 4*B*b^2*
c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c
^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*
b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x
 + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4*B*b^2*c^4*d^2
+ 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 -
 (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b +
 A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d
^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2
+ 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(c^2
- d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*B*
b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*
b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2
)*c*d^5)*cos(f*x + e) + ((2*B*b^2*c^5*d - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 -
 (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2
)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*
```

```
d^5 + c*d^7)*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26617, size = 1048, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*d
^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x + 1
/2*e)^3 - 2*B*a*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*tan(1/2*f*
x + 1/2*e)^3 + B*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*tan(1/2*f
*x + 1/2*e)^3 - A*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*tan(1/2*f*x
+ 1/2*e)^2 - 2*B*a*b*c^3*d*tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*tan(1/2*f*x
 + 1/2*e)^2 + B*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d^2*tan(1/
2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c*d^3*tan(1
/2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*tan(1/2*f*x + 1/2*e) - 2*B*a*b*c^2*d^2*ta
n(1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^3*tan(1
/2*f*x + 1/2*e) + 2*A*a*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*B*b^2*c*d^3*tan(1/
2*f*x + 1/2*e) - A*a^2*d^4*tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4 - 2*B*a*b*c^3
*d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c^2*d^2 - A*a^2*
c*d^3)/((c^3*d^2 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2
*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (2*B*
b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f
```

3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=840

$$\frac{2(Ab-aB)(bc-ad)\sqrt{c+d\sin(e+fx)}\cos(e+fx)}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}-\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}\cos(}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

```
[Out] ((c - d)*Sqrt[c + d]*(2*A*b^2*c - 2*a*b*B*c - 2*a*A*b*d + 3*a^2*B*d - b^2*B
*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sq
rt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]
*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*S
qrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b
*Sin[e + f*x]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (Sqrt[c + d]*(3*b
*B*c + 2*A*b*d - 3*a*B*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a
 + b]*f) + (2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]]
)/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((2*A*b*(b*c - a*d) - B*(2*a
*b*c - 3*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b
^2)*f*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[a + b]*(2*A*b*(b*(c - 2*d) + a*d) -
 B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/
((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a
 - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^3*Sqrt[c + d
]*f)
```

Rubi [A] time = 3.15592, antiderivative size = 840, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.18, Rules used = {2989, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{2(Ab-aB)(bc-ad)\sqrt{c+d\sin(e+fx)}\cos(e+fx)}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}-\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}\cos(}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x])^
(3/2),x]
```

```
[Out] ((c - d)*Sqrt[c + d]*(2*A*b^2*c - 2*a*b*B*c - 2*a*A*b*d + 3*a^2*B*d - b^2*B
*d)*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sq
rt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]
*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*S
qrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b
*Sin[e + f*x]))/((a - b)*b^2*Sqrt[a + b]*(b*c - a*d)*f) + (Sqrt[c + d]*(3*b
*B*c + 2*A*b*d - 3*a*B*d)*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[
a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], (
(a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - S
in[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e
 + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^3*Sqrt[a
 + b]*f) + (2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]]
)/(b*(a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - ((2*A*b*(b*c - a*d) - B*(2*a
*b*c - 3*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2 - b
^2)*f*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[a + b]*(2*A*b*(b*(c - 2*d) + a*d) -
 B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/
((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a
 - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b^3*Sqrt[c + d
]*f)
```

<u>Rule 2989</u>

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((b*c - a*d)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c +
 d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)
*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1
)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*B)
*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1) -
a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a*A
*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

<u>Rule 3061</u>

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
```

```
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
```

```
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(a^2Bd^2+b^2c(Bc \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{\left(2Ab(bc-ad)-\cdots\right.}{\cdots}$$

$$= \frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}} - \frac{\left(2Ab(bc-ad)-\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(\cdots}{(a+b)(\cdots}\right.}{b^3\cdots}$$

$$= \frac{(c-d)\sqrt{c+d}\left(2Ab(bc-ad)-B\left(2abc-3a^2d+b^2d\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c\cdots}}{\sqrt{c+d}\sqrt{a\cdots}}\right.\right.}{(a\cdots}$$

Mathematica [B] time = 6.74381, size = 2012, normalized size = 2.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*(A*b^2*c*Cos[e + f*x] - a*b*B*c*Cos[e + f*x] - a*A*b*d*Cos[e + f*x] + a
^2*B*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b
*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b^2*
c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a +
 b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c)
+ a*d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*b^2*d^2
 + 2*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellip
ticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])
)/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e +
 f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(
a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x
)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a
+ b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-
(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x
]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
+ f*x]])) + 2*(-2*A*b^2*c*d + 2*a*b*B*c*d + 2*a*A*b*d^2 - 3*a^2*B*d^2 + b^2
*B*d^2)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]
]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcS
in[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x
])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]
])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqr
t[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c +
 d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*((((
a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Ellipti
cF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/
(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a
+ b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]
^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[S
qrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a
```

```
*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]])))/(b*d)))/(2*(a - b)*b*(a + b)*f)
```

Maple [B] time = 87.435, size = 6776582, normalized size = 8067.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
 a)^(3/2), x)
```

3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=630

$$\frac{2\sqrt{a+b}(c-d)(Ab-aB)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{bf(a-b)\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*(b*c
- a*d)*f) + (2*Sqrt[a + b]*(A*b - a*B)*(c - d)*EllipticF[ArcSin[(Sqrt[c + d
]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b*Sqrt[
c + d]*(b*c - a*d)*f) + (2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d))
, ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(b^2*Sqrt[c + d]*f)
```

Rubi [A] time = 0.886312, antiderivative size = 630, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {2992, 2811, 2795, 2818, 2996}

$$\frac{2\sqrt{a+b}(c-d)(Ab-aB)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{bf(a-b)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
```

```
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*b*Sqrt[a + b]*(b*c
- a*d)*f) + (2*Sqrt[a + b]*(A*b - a*B)*(c - d)*EllipticF[ArcSin[(Sqrt[c + d
]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a +
b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e +
f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*b*Sqrt[
c + d]*(b*c - a*d)*f) + (2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d))
, ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin
[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c
- a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a
*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x
]))/(b^2*Sqrt[c + d]*f)
```

Rule 2992

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> D
ist[B/b, Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Di
st[(A*b - a*B)/b, Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2),
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}\,dx}{b} + \frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx}{b}$$

$$= \frac{2\sqrt{a+b}B\Pi\left(\frac{(a+b)d}{b(c+d)};\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-}{(a+}}}{b^2\sqrt{c+d}f}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+f}{(a-b)b\sqrt{a+b}(bc}$$

Mathematica [B] time = 10.2793, size = 1871, normalized size = 2.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^
2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d)*
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi
/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]
))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
 + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)*((Sq
rt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-((
(a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/S
qrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e +
f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a
*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
 d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
)/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b
*d) + a*B*d)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e +
 f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE
[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e
+ f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e +
 f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])
]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)
*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)
*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*El
lipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*
x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[
e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^
2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(
-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), Arc
Sin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Si
n[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
```

```
+ d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]])))/(b*d)))/((a - b)*(a + b)*f)
```

Maple [B] time = 148.394, size = 3151745, normalized size = 5002.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{d\sin\left(fx+e\right)+c}}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{b^2\cos\left(fx+e\right)^2-2\,ab\sin\left(fx+e\right)-a^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(A + B\sin\left(e + fx\right)\right)\sqrt{c + d\sin\left(e + fx\right)}}{\left(a + b\sin\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))
**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\sin\left(fx + e\right) + A\right)\sqrt{d\sin\left(fx + e\right) + c}}{\left(b\sin\left(fx + e\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a
)^(3/2), x)
```

3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=417

$$\frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(}{(a-b)(}\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c -
a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c -
a*d)*f)
```

Rubi [A] time = 0.537495, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2998, 2818, 2996}

$$\frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(}{(a-b)(}\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d
*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/
((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d
)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b*c -
a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b
*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/
```

```
((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a +
 b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)
*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c + d]*(b*c -
a*d)*f)
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a-b} - \frac{(Ab-aB)\int \frac{1+\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d}}}{a-b}$$

$$= \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+}{(a-b)\sqrt{a+b}(bc}$$

Mathematica [B] time = 6.53657, size = 1919, normalized size = 4.6

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
 + f*x]]),x]
```

```
[Out] (-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a
^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*
(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a +
 b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c)
+ a*d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot[(-e +
 Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) +
 a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c
 + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt
[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)
)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*
d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d
))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 -
f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + P
i/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(A*b^2*d - a*b*B*d)*((Cos[e
 + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a -
 b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b
```

```
)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (
2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((
a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]
))/((c + d)*(a + b*Sin[e + f*x]))]]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)
*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))
]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + P
i/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x
]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[
c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^
2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]],
 (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/
2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
 + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(
-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]))))/(b*d)))/((a - b)*(a + b)*(-(b*c) + a*d)*f)
```

Maple [B] time = 1.39, size = 99082, normalized size = 237.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx+e\right)+c}}{2\,abd-\left(b^{2}c+2\,abd\right)\cos\left(fx+e\right)^{2}+\left(a^{2}+b^{2}\right)c-\left(b^{2}d\cos\left(fx+e\right)^{2}-2\,abc-\left(a^{2}+b^{2}\right)d\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*c
os(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{A+B\sin\left(e+fx\right)}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{c+d\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e
 + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x +
 e) + c)), x)
```

3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=544

$$-\frac{2\sec(e+fx)\left(A\left(a^2d^2+b^2\left(c^2-2d^2\right)\right)-B\left(a^2cd+ab\left(c^2-d^2\right)-b^2cd\right)\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)}{(a-b)}}}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3}$$

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (2*(A*(a^2*d^2 + b^2*(c^2 - 2*d^2)) - B
*(a^2*c*d - b^2*c*d + a*b*(c^2 - d^2)))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c -
 d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/
((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a
 - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqrt[a + b]*(c - d)*Sq
rt[c + d]*(b*c - a*d)^3*f) + (2*(A*b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)*E
llipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c
+ d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt
[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((
b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[
e + f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rubi [A] time = 1.37675, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3000, 2998, 2818, 2996}

$$\frac{2\sec(e+fx)\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-}{(a-}}}{f\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[c + d*Sin[e + f*x]]) + (2*(a^2*B*c*d - b^2*B*c*d - a^2*A*d^2 -
 A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*EllipticE[ArcSin[(Sqrt[c + d]*Sqr
t[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x])
```

```
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/(
(a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(Sqrt[a + b]*(c - d)*
Sqrt[c + d]*(b*c - a*d)^3*f) + (2*(A*b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)
*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[
c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sq
rt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(
((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Si
n[e + f*x]))/(Sqrt[a + b]*(c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} - \frac{2\int \frac{1}{2}\left(a^2A\right.}{}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{(Abc+b}{}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}} + \frac{2\left(a^2Bcd\right.}{}$$

Mathematica [B] time = 7.27942, size = 2236, normalized size = 4.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
- a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])
) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2 - d
^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*B*c^
3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 + b^3*B
*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*C
sc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]],
```

```
(2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2
]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
+ a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e
+ f*x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2*d - 2*a^
2*b*B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B*c*d^2 + 2
*a*b^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt[((c + d)*C
ot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(
-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b
*c) + a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(
c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)
*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e +
 Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc
[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*
Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b^3*c^2*d) + a
*b^2*B*c^2*d + a^2*b*B*c*d^2 - b^3*B*c*d^2 - a^2*A*b*d^3 + 2*A*b^3*d^3 - a*
b^2*B*d^3)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f
*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[A
rcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e +
f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f
*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x])]*
Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(
c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(
((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*Elli
pticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]
))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*
(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)
/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*(c + d)*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e
 + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSi
n[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[
(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[
e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c +
d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt
[c + d*Sin[e + f*x]])))/(b*d)))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) +
a*d)^2*f)
```

Maple [B] time = 2.608, size = 198381, normalized size = 364.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{b\sin\left(fx+e\right)+a}\sqrt{d\sin\left(fx\right.}}{b^2d^2\cos\left(fx+e\right)^4+4\,abcd+\left(a^2+b^2\right)c^2+\left(a^2+b^2\right)d^2-\left(b^2c^2+4\,abcd+\left(a^2+2\,b^2\right)d^2\right)\cos\left(fx+e\right)^2+2\left(ab\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^
```

```
2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 +
 a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x +
e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(3/2)), x)
```

3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=858

$$\frac{2d\left(A\left(\left(3c^2-4d^2\right)b^2+a^2d^2\right)-B\left(cda^2+3b\left(c^2-d^2\right)a-b^2cd\right)\right)\sqrt{a+b\sin(e+fx)}\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{2b(A}{\left(a^2-b^2\right)(bc-ad)f\sqrt{}}$$

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(3/2)) + (2*d*(A*(a^2*d^2 + b^2*(3*c^2 - 4*d^2
)) - B*(a^2*c*d - b^2*c*d + 3*a*b*(c^2 - d^2)))*Cos[e + f*x]*Sqrt[a + b*Sin
[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])
^(3/2)) + (2*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^2 - d^2) - a^3*
d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*(4*a^3*c*d^3 - 4
*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4)
))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*
Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[
-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*
Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) - (2
*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*d^2) - 6*a*b*d*(c^2 - d^2
)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*
c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)^3*f)
```

Rubi [A] time = 2.61628, antiderivative size = 858, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3000, 3055, 2998, 2818, 2996}

$$\frac{2d\left(A\left(\left(3c^2-4d^2\right)b^2+a^2d^2\right)-B\left(cda^2+3b\left(c^2-d^2\right)a-b^2cd\right)\right)\sqrt{a+b\sin(e+fx)}\cos(e+fx)}{3\left(a^2-b^2\right)(bc-ad)^2\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}+\frac{2b(A}{\left(a^2-b^2\right)(bc-ad)f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(3/2)) + (2*d*(A*(a^2*d^2 + b^2*(3*c^2 - 4*d^2
)) - B*(a^2*c*d - b^2*c*d + 3*a*b*(c^2 - d^2)))*Cos[e + f*x]*Sqrt[a + b*Sin
[e + f*x]])/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])
^(3/2)) + (2*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^2 - d^2) - a^3*
d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*(4*a^3*c*d^3 - 4
*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4)
))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqr
t[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*
Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[
-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*
Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c - a*d)^4*f) - (2
*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*d^2) - 6*a*b*d*(c^2 - d^2
)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*
c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(S
qrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]
*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e +
 f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*
x])))]*(c + d*Sin[e + f*x]))/(3*Sqrt[a + b]*(c - d)^2*(c + d)^(3/2)*(b*c -
a*d)^3*f)
```

Rule 3000

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
 + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
```

```
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2818

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c -
a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d
)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[
(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (
(a + b)*(c - d))/((a - b)*(c + d))])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*
Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

Rule 2996

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c
 + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)
*(c - d))])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} - \frac{2\int \frac{1}{2}}{}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d\left(A\right.}{}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d\left(A\right.}{}$$

$$= \frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} + \frac{2d\left(A\right.}{}$$

Mathematica [B] time = 8.65213, size = 2807, normalized size = 3.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x]
 - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x]
)) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c
^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*A*b*
c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e + f*x]
- 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e + f*x
]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c)
 + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4*c^4*d -
 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^4*B*c^3*d
^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 + 10*a^3*b*B
*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^4 - 4*a^4*B
*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*A*b^2*d^5 -
8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2 -
 f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x
)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/
((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*C
sc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a
+ b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a
```

```
 + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c
) + a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d + 9*a^2*b^2*B*c^4*
d - 6*b^4*B*c^4*d - 9*a^2*A*b^2*c^3*d^2 + 15*A*b^4*c^3*d^2 + 5*a^3*b*B*c^3*
d^2 - 11*a*b^3*B*c^3*d^2 - 5*a^3*A*b*c^2*d^3 + 11*a*A*b^3*c^2*d^3 - a^4*B*c
^2*d^3 - 7*a^2*b^2*B*c^2*d^3 + 2*b^4*B*c^2*d^3 + 4*a^4*A*c*d^4 + a^2*A*b^2*
c*d^4 - 8*A*b^4*c*d^4 - 5*a^3*b*B*c*d^4 + 8*a*b^3*B*c*d^4 + 5*a^3*A*b*d^5 -
 8*a*A*b^3*d^5 - 3*a^4*B*d^5 + 6*a^2*b^2*B*d^5)*((Sqrt[((c + d)*Cot[(-e + P
i/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a
*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c +
 d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-
(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]
)/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sq
rt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)
/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[
e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[a + b
*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(3*A*b^4*c^4*d - 3*a*b^3*B*c^
4*d - 6*a^2*b^2*B*c^3*d^2 + 6*b^4*B*c^3*d^2 + 9*a^2*A*b^2*c^2*d^3 - 15*A*b^
4*c^2*d^3 + a^3*b*B*c^2*d^3 + 5*a*b^3*B*c^2*d^3 - 4*a^3*A*b*c*d^4 + 4*a*A*b
^3*c*d^4 + 2*a^2*b^2*B*c*d^4 - 2*b^4*B*c*d^4 - 5*a^2*A*b^2*d^5 + 8*A*b^4*d^
5 + 3*a^3*b*B*d^5 - 6*a*b^3*B*d^5)*((Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])
/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi
/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/
2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]], (2*(-(b*c) + a*d))/((a - b)*(c + d
))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2
)/(a + b*Sin[e + f*x])]*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/
(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))
]) - (2*(-(b*c) + a*d)*((((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/
2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((
a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc
[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a +
b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a +
 b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*
d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) +
 a*d)/((a + b)*d), ArcSin[Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d
*Sin[e + f*x]))/(-(b*c) + a*d))]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c
+ d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2
 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[-(((a + b)*Csc[(-e
+ Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d))])/((a + b)*d*Sqrt[
a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])))/(b*d)))/(3*(a - b)*(a + b)*
(c - d)^2*(c + d)^2*(-(b*c) + a*d)^3*f)
```

Maple [B] time = 14.667, size = 827030, normalized size = 963.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{}{6\,abc^2d+2\,abd^3+\left(3\,b^2cd^2+2\,abd^3\right)\cos\left(fx+e\right)^4+\left(a^2+b^2\right)c^3+3\left(a^2+b^2\right)cd^2-\left(b^2c^3+6\,abc^2d+4\,abd^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 +
(a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3
+ 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c
^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6
*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^(5/2)), x)
```

3.358 $\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left((A+B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^n, x]
```

Rubi [A] time = 0.0929466, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin$$

Mathematica [A] time = 17.3612, size = 0, normalized size = 0.

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^
n, x]
```

Maple [A] time = 0.46, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\sin\left(fx+e\right)+A\right)\left(b\sin\left(fx+e\right)+a\right)^{m}\left(d\sin\left(fx+e\right)+c\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] sage2
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```python


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```